Regulating Externalities with Uncertainty, Investment, and Technology Spillovers

June 2010

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Abstract

This paper analyzes optimal and non-optimal regulation of environmental externalities with investment in technology and uncertainty about the severity of environmental damages. Investment creates the potential for technology spillovers so that optimal regulation must address externalities from both emissions and investment. Despite these multiple externalities and uncertainty, we show that an optimal outcome can be achieved with freely allocated permits where the allocation is a function of investment. The optimal allocation can also be achieved under emissions taxes and cap-and-trade with auctioned permits with non-linear pricing schemes that are a function of investment. Conventionally applied incentive-based mechanisms, whether taxes or cap-and-trade, which are not functions of investment fail to achieve an optimal outcome. Among non-optimal mechanisms, we show that emissions taxes and auctioned permits are preferable to grandfathered permits in a linear-quadratic model.

JEL: H23; Q55

Keywords: Cap-and-trade; emissions taxes; technology choice; uncertainty; technology spillovers

1 Introduction

The regulation of many environmental externalities has important dynamic elements. Stock pollutants, such as greenhouse gases, ozone depleting chemicals, PCBs, and nuclear waste have persistent environmental effects that can last decades, or millennia in the case of nuclear wastes. Even in the case of flow pollutants, where the biophysical effect of emissions in the environment dissipates quickly, there are important temporal dimensions to many environmental decisions. Investments in capital, including abatement equipment, are typically long-lived. Environmental regulations themselves, often the result of difficult political negotiations, typically are adjusted only periodically to reflect new circumstances. In this paper, we analyze a dynamic model of environmental regulation with investment in technology and uncertainty about severity of damages and compare the performance of alternative incentive-based regulatory mechanisms.

A realistic model of dynamic regulation must incorporate several features that complicate the analysis, including uncertainty, technological change, and strategic issues related to the timing of decisions. What the world will be like in the future is uncertain, but expectations of future states should be factored into current decision-making. Decisions should be updated when new information is available. Scientific advances over time change our understanding of the severity of various environmental threats. Investments by firms in R&D and abatement technology can shift the cost of emissions reductions. A good example of changing perceptions of damages and costs is the case of chlorofluorocarbons (CFCs). CFCs were in widespread use as a refrigerant for decades before it became clear that they posed a threat to the earth's ozone layer that shields living organisms from harmful ultraviolet radiation (Molina and Rowland 1974). Though there was initial industry opposition to regulation, DuPont's success in finding substitutes for CFCs made phasing out CFC production under the Montreal Protocol a relatively inexpensive proposition. Incentives to invest in abatement technology are determined in part by the expected stringency of future environmental regulation but future stringency is in turn likely to be a function of how costly it is to reduce emissions. These dynamics give rise to strategic interactions between regulators and regulated firms.

Technology change in one firm not only reduces its own abatement costs but may reduce other firms' abatement costs through technology spillovers. Such technology spillovers are observed in environmental and energy R&D (Margolis and Kammen 1999, Popp 2006) as well as in other areas (Jaffe 1986, Bloom et al. 2007). Competition among firms in environmental R&D can take several different forms. On the one hand, firms may invest in abatement technology to get tougher environmental rules in an attempt to raise rivals' costs (Salop and Scheffman 1983). After ARCO discovered a way to produce cleaner reformulated gasoline, the California Air Resources Board mandated that all oil companies sell gasoline causing fewer emissions.[3] Because ARCO had an advantage in producing such gasoline, the regulatory shift gave ARCO a competitive advantage. DuPont's discovery of substitutes for CFCs was major factor in strengthening the Montreal Protocol from a 50% reduction in CFC production by 1999 to a complete production ban. DuPont profited by shifting demand from CFCs to substitutes where DuPont held patents (Lyons and Maxwell 2004). On the other hand, firms sometimes collaborate in environmental R&D. The California Fuel Cell Partnership, a collaborative of auto makers and energy companies to develop hybrid cars, is one such example.

In this paper, we analyze a dynamic model of environmental regulation with endogenous choice of emissions abatement technology by regulated firms and uncertainty about the severity of environmental damages from emissions. In this model, regulated firms make two important decisions: i) how much to invest in technology that lowers the cost of emissions abatement, and ii) how much to abate emissions. The regulator also makes two important decisions: i) what type of regulatory policy to institute, and ii) how stringent to make environmental policy. We consider three environmental policy regimes: i) cap-and-trade with freely distributed permits, ii) cap-and-trade with auctioned permits, and iii) emissions taxes.

In the next section, we describe the structure of the model and characterize an optimal solution that minimizes the sum of investment costs, abatement costs, and expected costs of damage from pollution. An optimal solution requires making optimal decisions about both investment and abatement. In section 3, we solve for equilibrium in a game involving a regulator whose goal is to maximize social welfare (i.e., minimize the sum of investment, abatement, and damage costs) and N firms that minimize their individual costs (investment, abatement, and compliance costs). Because optimal decisions need to be made on two dimensions, investment and abatement, a policy to implement an optimal decision must have sufficient latitude to adjust on two margins. Under

[3] Cited from http://www.aqmd.gov/monthly/aprilcov.html.

cap-and-trade with free distribution of permits, the regulator can adjust both the total allowable emissions (the cap) and the initial allocation of permits among firms. By making the distribution of permits a function of investment by firms, the regulator can provide the optimal investment incentives. By setting the overall cap, the regulator can provide the optimal level of emissions abatement. Under auctioned permits or emissions taxes, the regulator can use non-linear pricing such as two-part tariffs to generate an optimal solution. For example, using a two-part tariff for emissions taxes the regulator can set a low tax on the first block of emissions where either the tax rate or the amount of the emissions in the first block is a function of investment, thereby generating correct investment incentive, while setting the tax equal to marginal damages on the second block, thereby generating the correct incentives for abatement.

Optimal policies require making policy a function of both emissions and investment levels. Conventional incentive-based policies (emissions taxes and cap-and-trade) typically are a function only of emissions. Observing investment levels of individual firms may be problematic and may preclude making policy conditional on investment. We therefore investigate conventional incentive-based policies that are not a function of the level of investments. These policies, however, cannot simultaneously achieve both optimal investment and abatement. Using a linear-quadratic model, we show that an emissions tax or permit auction is preferred to cap-and-trade with grandfathering of permits because of the large distortions in investment incentives under grandfathering.

Our results also show that firms may have an incentive to lower rivals' abatement costs under environmental regulation. Firms may have an incentive to encourage technology spillovers because this can result in more favorable environmental regulation. Under auctioned permits or emissions taxes, letting other firms use better technology lowers their abatement cost curves, which in turn leads to a lower optimal tax rate or permit price. Though spillovers imply that each firm may free ride on the other firms' investments, we find that the equilibrium investment can be increasing in the degree of technology spillovers. The result that firms may wish to encourage technology spillovers and lower rival's costs runs counter to the normal incentive to raise rival's cost that operates in strategic competition models (Salop and Scheffman 1983).

Prior literature has analyzed various aspects of dynamic environmental regulation including: i) regulating a stock pollutant, ii) uncertainty and learning, and iii) investment in abatement technology. Most prior papers focus on only one of these aspects. Hoel and Karp (2002) and

Newell and Pizer (2003) compare prices versus quantities for a stock pollutant and generally find that taxes are superior to quotas. Benford (1998), Moledina et al. (2003) and Karp and Zhang (2006) analyze dynamic models in which firms have private information about abatement costs or there is learning about environmental damages. Karp and Zhang (2006) find that the relative efficiency of taxes over standards increases as the regulator has more opportunities for learning about the level of environmental damage in a model with a stock pollutant and non-strategic firms. Other studies characterize optimal policy with learning and irreversibility (Kolstad 1996, Ulph and Ulph 1997, Kelly and Kolstad 1999, Pindyck 2000, Costello and Karp 2004, and Saphores 2004).

A fairly large literature exists on environmental regulation with technological change (see Jaffe et al. 2003 and Requate 2005a for surveys). Early work in this area analyzed incentives to adopt new technology assuming that regulation was fixed (e.g., Downing and White 1986, Milliman and Prince 1989, Jung et al. 1996). However, if innovation changes abatement costs then policy should be updated. Malik (1991) compares outcomes where policies are fixed (rules) versus where policies are updated (discretion) under environmental standards. Amacher and Malik (2002) conduct a similar analysis under emissions taxes. Several papers compare different policies when price-taking (non-strategic) firms choose innovation and where policy is updated to reflect the results of innovation (Biglaiser and Horowitz 1993, Kennedy 1999, Requate and Unold 2003, Requate 2005b). Karp and Zhang (2009) compare prices versus quantities for the case of a stock pollutant with innovation and price-taking firms and find a preference for price-based regulation. However, Krysiak (2008) finds that quantity-based regulations may be preferred to price-based regulation when firms choose the type of technology and well as the level of investment. More closely related papers to our paper consider investment by strategic firms that realize that their actions can affect future policy. Tarui and Polasky (2005) find that taxes are preferred to standards with quadratic costs and damages in a game with a single regulated firm. Biglaiser et al. (1995) compare outcomes under taxes and tradable permits and find an optimal solution when marginal damages are constant or with non-linear taxes set equal to non-constant marginal damages. Fischer et al. (2003) compare taxes, auctioned and grandfathered permits when a single firm can innovate and has imperfect control over appropriability (technology spillovers) and find ambiguous rankings among the policies. Our paper combines uncertainty about environmental damages, endogenous innovation with technology spillovers, strategic firms and regulatory policy that can be conditional on both innovation and knowledge of environmental damages and finds optimal solutions. We also rank outcomes of simple (conventional) policies that are not conditional on innovation.

Section 2 describes the game with investment in technology, technology spillovers and uncertainty about the damage function, and characterizes the optimal outcome. In Sections 3 and 4 we compare equilibrium outcomes under taxes, cap-and-trade with free permit distribution, and cap-and-trade with permit auction. Section 3 characterizes policies that support the optimal outcome. Section 4 characterizes outcomes of conventional cap-and-trade and tax policies when the regulator cannot make policy conditional on investment. In Section 5, we rank these suboptimal policies in terms of equilibrium welfare using a linear-quadratic model. Section 6 provides a brief discussion of results and suggestions for potential future research.

2 A model with uncertainty, investment, and technology spillovers

We model regulation of a dynamic externality as a game with a regulator and $N \geq 2$ firms. The firms generate emissions that cause environmental damages. Let k_i be firm i's investment in technology. Given investment profile $k = (k_1, \ldots, k_N)$, firm i's cost of choosing emission $x_i \geq 0$ is given by $C^i(x_i, z_i(k))$ where

$$z_i(k) = k_i + \lambda \sum_{j \neq i} k_j$$

represents the effective emission abatement capital available to firm i. Parameter λ measures the degree of technology spillovers across firms. This specification of spillovers follows standard models of technology innovation in industrial organization (e.g. Spence 1984). For all i, function C^i is convex with partial derivatives $C^i_x, C^i_z < 0$, $C^i_{xx}, C^i_{zz} > 0$ and $C^i_{xz} > 0$ given any (x_i, z_i) such that $0 \leq x_i \leq \bar{e}_i(z_i)$ and $z_i \geq 0$ where $\bar{e}_i(z_i)$ represents the status quo or business-as-usual emission level associated with no abatement. Assume $\frac{d\bar{e}_i}{dz} \leq 0$. Under the assumption $C^i_{xz} > 0$, the marginal abatement cost is decreasing in abatement capital z_i. Firm i's cost of investing k_i is given by $G_i(k_i)$ where $G'_i > 0$ and $G''_i \geq 0$. Total emissions $X = \sum_i x_i$ cause environmental damages $D(X; \varepsilon)$ given unknown damage parameter ε where $D_x > 0$ and $D_{xx} > 0$. The set Δ represents the possible realizations of the damage parameter ε. At the outset of the game, the regulator knows the probability distribution of ε. The regulator is assumed to observe firms' technology level and emissions.[4]

2.1 Order of moves

At the outset of the game, we assume there exists some type of regulatory regime (cap-and-trade or taxes). We then model the order of moves of the regulator and the regulated firms in order of their degree of permanence or degree of difficulty of changing the decision in a short period of time. The regulator and the firms choose their actions in the following order.

1. Each firm i simultaneously chooses investment k_i.
2. Uncertainty about the damage function is resolved, and the regulator learns the realization of ε.
3. Given (k, ε), the regulator sets policy (i.e., the tax rate or the number of permits to distribute).
4. Firms choose emissions simultaneously. If the regime includes tradeable permits, then emissions trading occurs simultaneously with choice of emissions.
5. Players receive their payoffs.

We view investments as fairly long-lived decisions so that this move is determined first. We assume that the regulator can change level of taxes or number of permits in response to learning about damages and on a shorter time scale than the useful life of investments. Finally, we assume that firms can choose emission levels on a very short time scale and can change in response to changes in policy.

We also consider a variant of the model in sections 4 and 5 in which, at the outset of the game, the regulator commits to the stringency of regulation as well as the regulatory regime. We call this variant of the model commitment policy (or rules). This policy will not in general be ex-post efficient as policy is fixed prior to learning about the severity of damages.

[4] We briefly discuss violations of these assumptions in Section 6.

2.2 Optimal investment and emissions abatement

The optimal investment and emissions plan minimizes the expected social cost of pollution control, which consists of investment costs, expected abatement costs, and expected damages:

$$\min_{k,x} \sum_{i=1}^{N} \left[G_i(k_i) + E\left\{C^i(x_i(\varepsilon), z_i(k))\right\}\right] + ED(X(\varepsilon); \varepsilon),$$

where k is the vector of N firms' investments and $x = \{x_i(\varepsilon)\}_{\varepsilon\in\Delta, i=1,\ldots,N}$ represents the state-contingent emission plan for N firms. Let (k^*, x^*) be the optimal solution, which is unique given the assumptions on G_i, C_i, and D. An interior solution equates the marginal abatement cost of each firm with the marginal damages:

$$-C^i_x(x^*_i(\varepsilon), z_i(k^*)) = D_x(X^*(\varepsilon); \varepsilon),$$

for all i and ε where $X^*(\varepsilon) \equiv \sum_i x^*_i(\varepsilon)$. Because emissions are chosen after the damage uncertainty is realized and investment is chosen, the optimal emission is a function of both damage realization ε and investments k. In contrast, investments must occur before the damage uncertainty is resolved. Hence, the optimal solution for investment satisfies

$$G'_i(k^*_i) + E\left[C^i_z(x^*_i(\varepsilon), z_i(k^*)) + \textstyle\sum_{j\neq i}\lambda C^j_z(x^*_j(\varepsilon), z_j(k^*))\right] = 0 \tag{1}$$

for all i. The marginal cost of investment by firm i, $G'_i(k^*_i)$, equals the expected marginal benefit of investment, which is the sum of the expected marginal benefit of reducing firm i's abatement cost, $-C^i_z$, and the expected marginal benefit of reducing other firms' abatement costs through the technology spillover effect, $-\sum_{j\neq i}\lambda C^j_z$.

3 Optimal Policies

In this section we solve for equilibrium given a regulatory regime. We consider the following three regulatory regimes:

Cap-and-trade with freely distributed permits The regulator sets the distribution of permits $q = (q_1, \dots, q_N)$, and the firms trade permits.

Cap-and-trade with permit auction The regulator sets Q, the total amount of emission permits to be auctioned.

Emissions Tax The regulator sets a tax rate on emissions $\tau > 0$.

The regulator chooses the stringency of a policy (the amount of permits to be auctioned or distributed to each firm or the tax rate) after the firms' investment choices are observed and the uncertainty about the damage parameter is resolved. The regulator minimizes the ex-post total cost of pollution control given investment and exogenous learning. Strategic firms choose investment to minimize the private cost of pollution control by taking into account the effect of investment on the regulator's policy decision.

3.1 Optimal cap-and-trade with freely distributed permits

We begin by showing that an optimal solution can be obtained under cap-and-trade with freely distributed permits given a correctly designed permit distribution scheme.

Under permit trading, we assume that firms engage in bargaining. An efficient outcome in bargaining over permit allocation is one in which the marginal abatement costs of all firms are equalized. An efficient outcome is the prediction for most bargaining games of complete information, as well as the Nash bargaining solution. In this paper, as in Montero (2002), we assume that an efficient outcome occurs. Though distribution of net benefits among firms may depend upon the bargaining power of the firms, we make the simplifying assumption that the equilibrium permit price is equal to the firms' marginal abatement costs and that each firm's net transfer equals the net payment for purchasing permits. With this assumption, we can characterize the optimal permit trading rule in a simple and intuitive way.[5] The main results of this section also hold under alternative assumptions about the firms' bargaining power and transfers.[6]

Under permit trading, the regulator sets the total permits $Q(k, \varepsilon)$ given investments k and damage realization ε in order to minimize the ex-post social cost of pollution control $\sum_i C^i(x_i, z_i(k))+$

[5] Hahn (1984) analyzed cap-and-trade schemes where a single firm has a market power and faces a competitive fringe. He showed that, in general, this case results in an inefficient outcome. In our model, all firms have market power, and therefore bargaining is the natural way to model the situation.

[6] The proof of the result given in Proposition 1 under the case of the Nash bargaining solution is available upon request.

$D(X;\varepsilon)$. The regulator also specifies distribution of permits to the firms $\{q_i(k,\varepsilon)\}$ such that the sum $\sum_i q_i(k,\varepsilon)$ equals the cap $Q(k,\varepsilon)$ for all (k,ε). In the investment stage, each firm chooses investment taking into account how it influences the permit distribution $\{q_i(k,\varepsilon)\}$ as well as the equilibrium permit price $p(k,\varepsilon)$.

The regulator minimizes the ex-post social cost of pollution control given investments k and a damage realization $\bar{\varepsilon}$:

$$\sum_{i=1}^{N}\Big[G_i(k_i)+C^i(x_i,z_i(k))\Big]+D(X;\bar{\varepsilon}).$$

Under the assumption of efficient bargaining, the marginal abatement cost of each firm is equal to the permit price, which equals the marginal damage from pollution. The result that efficient emissions decisions will be made given investment and the realization of damages will occur for all of the three policies considered (cap-and-trade with freely distribution permits, cap-and-trade with auctioned permits, and emissions taxes). Hence, letting $x^*(k,\varepsilon)$ be the ex-post optimal emission given (k,ε), we have

$$-C^i_x(x^*_i(k,\varepsilon),z_i(k))=D_x(X^*(k,\varepsilon);\varepsilon)\quad\text{for all } i,k,\varepsilon.$$

Thus the regulator's choice of permits depends upon the investment profile k and the realization of damages $\bar{\varepsilon}$ and total emissions will be efficient given k and $\bar{\varepsilon}$.

Given q_i, expectations of permit price p and other firm's investment k_{-i}, firm i chooses investment k_i to minimize its total expected cost:

$$G_i(k_i)+E[C^i(x_i(k,\varepsilon),z_i(k))+p(z,\varepsilon)(x_i(k,\varepsilon)-q_i(k,\varepsilon))]. \tag{2}$$

After applying the envelope theorem, we have the following first order condition for all i:

$$G_i'(k_i)+E\Bigg[C^i_z(x_i(k,\varepsilon),z_i(k))+\underbrace{\frac{\partial p(z,\varepsilon)}{\partial k_i}[x_i(k,\varepsilon)-q_i(k,\varepsilon)]-p(z,\varepsilon)\left(\frac{\partial q_i(k,\varepsilon)}{\partial k_i}\right)}_{(*)}\Bigg]=0. \tag{3}$$

In the above expression, the two terms above the underbrace (denoted by (*)) represent the strategic effect. The term $\frac{\partial p(z,\varepsilon)}{\partial k_i}[x_i(k,\varepsilon)-q_i(k,\varepsilon)]$ represents the *emissions payment effect* (Fischer et al.

2003). Note that $\partial p/\partial k_i < 0$ for all i (see Lemma 1 in the proof of Proposition 4). The sign of the emissions payment effect depends on whether firm i is a seller or a buyer of permits. A seller (a buyer) has an incentive to invest less (more) so that permit price stays high (low). (Under permit auction, all firms are buyers of permits.) The last term, $-p\frac{\partial q_i}{\partial k_i}$, represents the *permit allocation effect*. By investing, a firm will change the desired number of permits it holds.

We now present one of the main results of this paper.

Proposition 1 *Cap-and-trade with freely distributed permits can achieve the optimal outcome with a permit distribution rule* q^* *where*

(i) $\frac{\partial q_i^*(k,\varepsilon)}{\partial k_i} = \frac{D_{xx}(X^*(\varepsilon);\varepsilon)\frac{\partial X^*(k,\varepsilon)}{\partial k_i}[x_i^*()-q_i^*()]-\lambda\sum_{j\neq i}C_z^j(x_j^*(\varepsilon),z_j(k^*))}{D_x(X^*(\varepsilon);\varepsilon)}$ *and* $\frac{\partial^2 q_i^*(k;\varepsilon)}{\partial k_i^2} = 0$ *for all* k, ε*;*

(ii) $\sum_j q_j^*(k,\varepsilon) = X^*(k,\varepsilon)$ *for all* k, ε*; and*

(iii) each firm's objective function given the permit distribution (as a function of investments) is convex.

See the Appendix for the proof.

With condition (ii), the regulator sets the total emission cap at the ex-post optimal level given the firms' investment and realization of damage uncertainty. Together with a correctly specified permit distribution scheme that depends on the firms' investments (condition i), the regulator can induce the firms to choose the optimal investment level. The optimal emission and investment plan $\{k^*, x^*(\cdot)\}$ satisfies the necessary and sufficient condition of each firm's payoff maximization provided that each firm's objective function (2) is convex in investment.[7]

Inspection of the permit distribution rule q^* can be used to provide intuition for Proposition 1. We can write the permit distribution rule q^* as follows:

$$q_i^*(k,\varepsilon) = \alpha_i(\varepsilon) + \beta_i(\varepsilon)k_i + \gamma_i(\varepsilon)\sum_{j\neq i} k_j \geq 0,$$

for all k, ε. The coefficient β_i represents the marginal change in the permit allocation to firm i due to i's investment. In the limited case where $q_i^*(k^*,\varepsilon) = x_i^*(\varepsilon)$ (i.e. the emissions payment effect in

[7]As discussed in the proof in the Appendix, the firm's objective function is convex in investment under fairly weak assumptions.

(3) is zero), the optimal permit distribution q^* satisfies the following condition:

$$\frac{\partial q_i^*(k^*,\varepsilon)}{\partial k_i} = \beta_i(\varepsilon) = \frac{\lambda \sum_{j\neq i} \left| C_z^j(x_j^*(\varepsilon), z_j(k^*)) \right|}{D_x(X^*(\varepsilon);\varepsilon)} \quad \text{for all } i,\ k,\ \varepsilon, \tag{4}$$

The formula for the permit allocation rule (4) specifies that the marginal increase in the permit allocation for firm i equals the marginal contribution of firm i's investment on the reductions in the other firms' marginal abatement costs through technology spillovers discounted by the marginal damages of emissions. If $\lambda = 0$, then firm i's permit allocation should be independent of firm i's investment. If $\lambda > 0$, then the allocation to a firm should be increasing in the firm's own investment and decreasing in investment by the other firms. With this rule, the regulator rewards a firm conducting a larger investment for the contribution to reduce the other firms' abatement costs, thereby solving the discrepancy between private and social returns on investment.

In the presence of two types of externalities (pollution externality and technology spillovers), an optimal policy needs to address the firms' incentives at two margins. The above cap-and-trade system allows the regulator to address both of these margins by controlling the individual permit distribution as well as the total cap. In what follows, we discuss how other policy instruments (permit auction and emissions taxes) can address these two margins.

3.2 Optimal cap-and-trade with auctioned permits

The optimal outcome is also supportable under a permit auction scheme in which there are infra-marginal subsidies that depend on investment.

Under permit trading, the regulator sets the total permits $Q(k,\varepsilon)$ to be auctioned given k and ε in in an optimal manner (as was also done under cap-and-trade with free permit distribution). As a result, the marginal abatement cost of each firm is equalized with the marginal damage, which is also equal to the equilibrium permit price $p(\varepsilon,k)$. Let $R_i(\varepsilon,k)$ be the amount of subsidy to firm i when the investment profile is k. Firm i's private cost of pollution control is

$$G_i(k_i) + E[C^i(x_i(\varepsilon,k), z_i(k)) + p(\varepsilon,k)x_i(\varepsilon,k) - R_i(\varepsilon,k)],$$

under permit auction Q and the associated permit price p. With a correct specification of inframarginal subsidies, a permit auction can support the optimal outcome.

Proposition 2 *Suppose permit auction with cap Q and subsidies $\{R_i\}_{i=1}^N$ satisfies $Q(k,\varepsilon) = X^*(k,\varepsilon)$ for all (k,ε) and*

$$\frac{\partial R_i(k,\varepsilon)}{\partial k_i} = D_{xx}(X^*;\varepsilon)\cdot\frac{\partial X^*(k^*,\varepsilon)}{\partial k_i}x_i^*(k^*,\varepsilon) - \lambda\sum_{j\neq i}C_z^j\left(x_j^*(\varepsilon), z_j(k^*)\right) \quad \text{for all } i,\ k,\ \varepsilon. \tag{5}$$

If firm i's objective function given R_i is convex for all i, then cap-and-trade with permit auction and subsidy $\{R_i\}$ supports the optimal outcome.

See the Appendix for the proof. While the total permits induce the ex-post optimal emissions given investment k, the subsidy $\{R_i\}$ as a function of investments provides firms with the correct incentives to choose the optimal investment levels. In condition (5), both the first and the second terms are negative ($D_{xx} > 0$, $\partial X/\partial k_i < 0$, $x_i > 0$, $C_z^i < 0$). Hence, whether the subsidy R_i should be decreasing in investment k_i depends on the degree of spillovers λ. The subsidy should be decreasing in k_i if λ is small while it can be increasing in k_i if λ is large.

Proposition 2 describes the condition for subsidy in a generic form. In practice, the subsidy can be tied to the auction more explicitly. For example, instead of having a general subsidy system R, the regulator could set up two blocks of permits for auction. The first block is designated for preferential purchases at a discount price set by the regulator while the second block is set by a standard auction open to all firms. The logic behind the proposition works if each firm's access to the first block depends on the firms' investment choice. The price for preferential purchases, $\underline{p}$, can be set as a fraction of the market permit price or a predetermined level lower than the market price. In the latter case, set $\underline{p}$ at some level where

$$0 < \underline{p}(\varepsilon) < \min_k D_x(X^*(\varepsilon,k);\varepsilon),$$

for all $\varepsilon \in \Delta$. This assumption guarantees that the preferential purchase price is lower than the equilibrium permit price given by $p(k,\varepsilon)$. Firm i's private cost of pollution control is

$$G_i(k_i) + E[C^i(x_i(\varepsilon,k), z_i(k)) + p(\varepsilon,k)\{x_i(\varepsilon,k) - y_i(\varepsilon,k)\} + \underline{p}(\varepsilon)y_i(\varepsilon,k)].$$

The regulator can set $\{y_i\}$ to support the optimal outcome.[8]

3.3 Optimal taxes

A non-linear emissions tax can also support the optimal outcome. The logic for this result is virtually identical to the result that permit auctions can support the optimal outcome. Let $\tau(k, \varepsilon)$ be the tax rate given damage realization ε and investments k. Let $S_i(k, \varepsilon)$ be the subsidy given damage realization ε and the investment outcome k. Firm i's private cost of pollution control is

$$G_i(k_i) + E[C^i(x_i(k, \varepsilon), z_i(k)) + \tau(k, \varepsilon) x_i(k, \varepsilon) - S_i(k, \varepsilon)].$$

As in the case of cap-and-trade, all firms' marginal abatement costs are equalized with the marginal damage under taxes. The equilibrium tax and emissions satisfy

$$-C_x^i(x_i(k, \varepsilon), z_i(k)) = \tau(k, \varepsilon), \quad i = 1, \ldots, N, \quad \text{where } \tau(\varepsilon, k) = D_x(X^*(k, \varepsilon); \varepsilon) \quad \text{for all } k, \varepsilon.$$

Hence, the equilibrium tax rate is identical to the equilibrium permit price under cap-and-trade. Each firm's objective function is identical under both permit auction and emission tax given that the subsidies are the same. In fact, the emission tax supports the optimal outcome under the same condition on $\{S_i\}$ as the assumption on $\{R_i\}$ in Proposition 2.

Proposition 3 *Suppose emission tax τ with subsidy $\{S_i\}$ satisfies $\tau(k, \varepsilon) = D_x(X^*(k, \varepsilon); \varepsilon)$ for all (k, ε) and equation (5), with the left-hand side replaced with $\frac{\partial S_i(k,\varepsilon)}{\partial k_i}$. If firm i's objective function given S_i is convex for all i, then emissions tax with inframarginal subsidy $\{S_i\}$ supports the optimal outcome.*

Proposition 3 can be proved by replacing the equilibrium permit price p and subsidy $\{R_i\}$ in Proposition 2 with the equilibrium tax rate τ and subsidy $\{S_i\}$.

In practice, the regulator can give a subsidy on some inframarginal units of emissions, where the amount of emissions eligible for subsidy depends on the firms' investment profile. Given subsidy

[8] Such y_i must satisfy $\frac{\partial y_i}{\partial k_i} = \frac{D_{xx} \cdot \frac{\partial X^*(k;\varepsilon)}{\partial k_i} \left\{ x_i(k^*;\varepsilon) - y_i(k^*;\varepsilon) \right\} - \lambda \sum_{j \neq i} C_z^j \left(x_j^*(\varepsilon), z_j(k^*) \right)}{D_x(X^*(k^*;\varepsilon);\varepsilon) - \underline{p}(\varepsilon)}$ for all i, k, ε.

rate $s(\varepsilon)$ per eligible emissions $y_i(\varepsilon, k)$, firm i's private cost of pollution control is

$$G_i(k_i) + E[C^i(x_i(\varepsilon, k), z_i(k)) + \tau(\varepsilon, k)x_i(\varepsilon, k) - s(\varepsilon)y_i(\varepsilon, k)].$$

The regulator can specify the subsidy rates s and the eligible emissions $\{y_i\}$ in order to support the optimal outcome.[9]

Instead of a tax credit or inframarginal subsidy, the regulator can also use emissions tax with exemptions. Consider an emissions tax coupled with inframarginal unit subsidy: each firm receives some tax exemptions where the amount of exempt emissions depends on how much the firm has invested. If $\underline{x}_i$ units of emissions are tax-exempt, then firm i's private cost of pollution control is

$$G_i(k_i) + E\left[C^i(x_i(\varepsilon, k), z_i(k)) + \tau(\varepsilon, k)\left\{x_i(\varepsilon, k) - \underline{x}_i(\varepsilon, k)\right\}\right].$$

The tax exemption supports the optimal outcome if

$$\underline{x}_i(\varepsilon, k) \leq x_i(\varepsilon, k), \quad \frac{\partial \underline{x}_i(k^*; \varepsilon)}{\partial k_i} = \frac{\frac{\partial \tau(k^*;\varepsilon)}{\partial k_i}\left\{x_i(\varepsilon, k) - \underline{x}_i(\varepsilon, k)\right\} - \lambda \sum_{j \neq i} C_z^j(x_j^*(\varepsilon), z_j(k^*))}{D_x(X^*(\varepsilon); \varepsilon)}.$$

4 Non-optimal conventional policies

All of the optimal policies discussed in section 3 are two-dimensional so that they can provide correct incentives to firms for both investment and emissions choices. This section demonstrates that simple one-dimensional policies typically put into practice, a cap-and-trade with grandfathering, a cap-and-trade with a linear permit auction, and a linear emissions tax, will not achieve the optimal outcome.

4.1 Cap-and-trade with grandfathered permits

Under cap-and-trade, a common approach in practice is to allocate permits on the basis of past emissions ("grandfathering"). The SO_2 cap-and-trade program under the US Clean Air Act and the EU Emissions Trading Scheme have permit distributions based primarily on grandfathering.[10]

[9] The eligibility y_i must satisfy $\frac{\partial y_i(k^*;\varepsilon)}{\partial k_i} = \frac{\frac{\partial \tau(k^*;\varepsilon)}{\partial k_i} x_i(k^*;\varepsilon) - \lambda \sum_{j \neq i} C_z^j(x_j^*(\varepsilon), z_j(k^*))}{s(\varepsilon)}$.

[10] See Ellerman et al. (2000) for discussion on the distribution of permits in the SO_2 cap-and-trade program in the US.

Under grandfathering, each firm's allocation of permits is a fixed share of total permits, with the share for each firm set by its proportion of past emissions:

$$q_i = \alpha_i \sum_{j=1}^{N} q_j \text{ for some fixed share } \alpha_i > 0 \text{ such that } \textstyle\sum_{j=1}^{N} \alpha_j = 1.$$

From Proposition 1 we can show the following corollary.

Corollary 1 *Grandfathering does not support the optimal outcome.*

4.2 Linear permit auction and linear emissions tax

Under a linear permit auction or emissions tax there is a single price for all permits or emissions. Under cap-and-trade with a linear permit auction, let $Q(k,\varepsilon)$ be the cap set by the regulator. The regulator will set $Q(k,\varepsilon) = X^*(k,\varepsilon)$ for all (k,ε). Let $p(k,\varepsilon)$ be the equilibrium auction price of permits given damage realization ε and the investment profile k. Firm i's private cost of pollution control given (k,ε) is

$$G_i(k_i) + E[C^i(x_i, z_i(k)) + p(k,\varepsilon)x_i].$$

The equilibrium price and emissions satisfy

$$p(k,\varepsilon) = D_x(X^*(k,\varepsilon);\varepsilon), \quad C^i_x(x^*_i(k,\varepsilon), z_i(k)) + D_x(X^*(k,\varepsilon);\varepsilon) = 0.$$

Under linear emissions tax τ with no subsidy, firm i's private cost of pollution control given (k,ε) is

$$G_i(k_i) + E[C^i(x_i, z_i(k)) + \tau(k,\varepsilon)x_i].$$

The equilibrium tax rate and emissions satisfy

$$\tau(k,\varepsilon) = D_x(X^*(k,\varepsilon);\varepsilon), \quad C^i_x(x^*_i(k,\varepsilon), z_i(k)) + D_x(X^*(k,\varepsilon);\varepsilon) = 0.$$

Under linear permit auction and linear emissions tax, each firm considers the effects of its investment on the equilibrium tax rates or the price of permits.

Let k^* be the optimal investment profile and k^{PA}, k^T the equilibrium investment profiles under cap-and-trade with linear permit auction and linear emissions tax.

Proposition 4 *Cap-and-trade with linear permit auction and linear emissions tax satisfy the following properties.*

(i) Linear emissions tax and cap-and-trade with linear auction are equivalent, and hence $k^T = k^{PA}$.

(ii) If the firms are strategic and if there are no technology spillovers ($\lambda = 0$)*, then* $k_i^T > k_i^*$ *for all* i.

(iii) If $\lambda > 0$*, then both* $k_i^T < k_i^*$ *and* $< k_i^T > k_i^*$ *are possible.*

(iv) If there are many firms where each is non-strategic, then linear permit auction and linear emissions tax support the same equilibrium outcome, which is efficient if and only if there are no technology spillovers.

See the Appendix for the proof. The equilibrium permit prices under auctioned permits and the equilibrium tax rates depend on the firms' investment profile in the same way. Because they are equivalent from the firms' point of view, the equilibria under the two policy instruments support the same outcome (part i).

The logic behind over-investment under taxes when λ is small (part ii) is explained in Malik (1991), Kennedy and Laplante (1999) and Tarui and Polasky (2005). The equilibrium tax rate and the equilibrium permit price under auction are lower given larger investments. Hence, each firm has an incentive to increase investment in order to induce the regulator to set a lower emissions target. Whether the equilibrium investment exceeds the optimal level depends on the balance between the under-investment incentives due to spillovers and over-investment incentives for lowering the tax rates or the permit prices (part iii).

Given a large number of firms, distortions due to the firms' strategic behavior to influence the regulator's policy decision will disappear. However, the externality due to technology spillovers remains to be corrected to achieve optimal investment.[11]

[11] Karp (2006) demonstrates that uniform emissions standards and permit trading are not necessarily equivalent because multiple equilibria could occur under standards given a large number of firms.

4.3 Policies under commitment

So far we have assumed that the regulator chooses policy stringency (i.e., the number of permits or the emissions tax rate) after the firms choose investment and damage uncertainty is resolved. Under this assumption, the firms can act strategically to influence the regulator's policy decision. An alternative is a "commitment policy," where the regulator chooses the type and the stringency of policy before the firms make investment and emissions decisions. Though the regulator does not incorporate learning under commitment policy so that ex-post emissions given investment and the cost of damages is not likely to be optimal, the regulator removes the incentive for firms to change investment to manipulate the number of permits issued or the tax rate set. Under commitment policy, since policy does not depend on the resolution of damage uncertainty, the firm's choices of investment or emissions also do not depend on the resolution of damage uncertainty. Firm i's private cost of pollution control given its emissions x_i and an investment profile k is:

$$\begin{cases} G_i(k_i) + C^i(x_i, z_i(k)) + \tau x_i & \text{given emissions tax rate } \tau; \\ G_i(k_i) + C^i(x_i, z_i(k)) + p x_i & \text{given cap-and-trade with permit auction;} \\ G_i(k_i) + C^i(x_i, z_i(k)) + p\{x_i - q_i\} & \text{given cap-and-trade with permit distribution } q_i; \end{cases}$$

where p denotes the equilibrium permit price given total permits Q under auction and given permit distribution q under free permit trading. Firm i's equilibrium investment satisfies

$$\begin{cases} G_i'(k_i) + C_z^i = 0 & \text{given emissions tax rate } \tau; \\ G_i'(k_i) + C_z^i + \frac{\partial p}{\partial k_i} x_i(Q, k) = 0 & \text{given cap-and-trade with permit auction;} \\ G_i'(k_i) + C_z^i + \frac{\partial p}{\partial k_i}[x_i(Q, k) - q_i] = 0 & \text{given cap-and-trade with permit distribution } q_i; \end{cases} \tag{6}$$

Under each policy option, the regulator chooses policy to minimize the sum of emissions abatement costs and expected damages taking the firms' best response investment and emissions $\{k_i(\cdot), x_i(\cdot)\}$ as given:

$$\min_y \sum_i [G_i(k_i(y)) + C^i(x_i(y), z_i(k(y)))] + ED(X(y); \varepsilon).$$

The following proposition describes the equilibrium outcomes under each commitment policy.

Proposition 5 *Commitment policies with strategic firms and interior solutions have the following* *properties.*

(i) Without technology spillovers ($\lambda = 0$), the emissions tax is preferred to the other policies and strictly preferred to permit auction.

(ii) Without technology spillovers and damage uncertainty (i.e. $\lambda = Var(\varepsilon) = 0$), the equilibrium outcome under the emissions tax is efficient.

(iii) For any $\lambda \geq 0$ and ε with any $Var(\varepsilon) > 0$, cap-and-trade with free permit distribution and the emissions tax are equivalent if and only if the permit distribution coincides with the equilibrium emissions allocation under the tax.

When N is large and no firm acts strategically, all commitment policies achieve the same outcome. The outcome is inefficient if $\lambda > 0$ or if damage is uncertain.

See the Appendix for the proof. Parts (i) and (ii) hold because the emissions tax supports the second-best allocation that minimizes the expected social cost given that emissions do not depend upon the realization of damages. The best that the regulator can do under commitment is to choose investments and state-independent emissions that minimize expected social costs, by equating the marginal abatement cost of each firm with the expected marginal damages, and the marginal invest cost with the expected marginal benefit of investment. In the absence of technology spillovers, the emissions tax can implement this second-best outcome (Part ii).

The difference between investment choices under cap-and-trade with free permit distribution and emissions taxes arises because firms have an incentive to influence permit prices when they buy or sell permits. Because more investment implies a lower permit price, a firm expecting to be a seller (buyer) of permits has an incentive lower (raise) investment to raise (lower) permit price. If the permit distribution coincides with the firm's equilibrium emissions, then there is no trading and no incentive to manipulate permit prices through investment. In this case the emissions tax and cap-and-trade with free distribution are equivalent (Part iii).

Finally, when there are many firms so that firms ignore the effect of their own choices on permit prices (non-strategic firms), all commitment policies yield the same equilibrium outcome (Part iv).

5 Linear-quadratic example

In this section we use a linear-quadratic example to compare outcomes under the non-optimal policies discussed in section 4 with the optimal outcome that can be achieved under the policies considered in section 3. In this section we assume the following functional forms for investment cost, emissions control cost and damages from pollution:

$$G_i(k_i) = \frac{r}{2}k_i^2, \quad i = 1, \ldots, N. \tag{7}$$

$$C^i(x_i, z_i) = \frac{(\bar{e}_i - x_i - az_i)^2}{2\theta}, \quad i = 1, \ldots, N. \tag{8}$$

$$D(X; \varepsilon) = \frac{\varepsilon X^2}{2}, \quad \text{where } \underline{\varepsilon} \leq \varepsilon \leq \bar{\varepsilon} \text{ and } 0 < \underline{\varepsilon} < \bar{\varepsilon}. \tag{9}$$

5.1 Comparison of tax and permit allocation rules

Let K^* be the optimal total investment of all firms. Let K^T and K^G be the equilibrium total investment of all firms under the linear tax (which will be identical to the total investment of all firms under the linear permit auction) and under cap-and-trade with grandfathering. Let TC^* be the social costs of pollution under the optimal outcome and TC^T, TC^G the equilibrium social costs of pollution under each policy:

$$TC^J \equiv \sum_i [G_i(k^J) + EC^i(x(k^J, \varepsilon), z_i(k^J))] + ED(X(k^J, \varepsilon); \varepsilon), \quad J = T, G.$$

Proposition 6 *Let the functions $\{G_i\}, \{C_i\}$ and D be given by equations (7), (8) and (9). Then:*

(i) Equilibrium investment

$$\begin{cases} K^G < K^* \textit{for all } \lambda \geq 0; \\ k_i^T > k_i^* \textit{ for all firm } i \textit{ if } \lambda < \hat{\lambda} \equiv \frac{\mu}{(N\phi - \mu)(N-1)}, \\ \quad \textit{where } \phi \equiv E\left[\frac{\varepsilon}{1+N\theta\varepsilon}\right], \ \mu \equiv E\left[\frac{\varepsilon}{(1+N\theta\varepsilon)^2}\right] \textit{ and } \hat{\lambda} > 0; \\ k_i^T < k_i^* \textit{ for all firm } i \textit{ if } \lambda > \hat{\lambda}. \end{cases}$$

(ii) Equilibrium social cost

$$\begin{cases} TC^T = TC^* \text{ if } \lambda = \hat{\lambda};\ TC^T > TC^* \text{ if } \lambda \neq \hat{\lambda}; \\ TC^T < TC^G \text{ for all } \lambda \geq 0. \end{cases}$$

See the Appendix for the proof. With the linear-quadratic specification, cap-and-trade with grandfathered permits results in under-investment (at the aggregate level for N firms) regardless of the degree of technology spillovers. Emissions taxes will result in over-investment when spillover effects are small and under-investment when spillover effects are large.

We compare the welfare loss under linear emissions tax and cap-and-trade with grandfathered permits relative to the optimal outcome in Figure 1. The two panels are based on the same parameter values except for the number of firms. In each panel, the horizontal axis measures the degree of technology spillovers. The linear emissions tax has lower welfare losses than cap-and-trade with grandfathering regardless of the degree of technology spillovers and regardless of the number of firms. As technology spillovers increase, the performance of grandfathering worsens for both $N = 2$ and $N = 10$. Under emissions taxes, performance improves slightly with an increase in technology spillovers for $N = 2$ but worsens for $N = 10$. The welfare losses tend to be far larger with $N = 10$ than with $N = 2$. The scale of welfare losses in panel (b) with $N = 10$ increases to over 50% while the welfare losses in panel (a) with $N = 2$ are less than 1%. Larger welfare losses with a larger number of firms occur because the technology spillover effects are amplified when the number of firms is large. The welfare loss under conventional policies is not simply due to market power by a small number of firms but can be sizable even if the number of regulated firms is large.

5.2 Comparison of flexible and commitment policies

We compare the equilibrium social costs under flexible policies in which the stringency of emissions policy is contingent on investment and the realization of damage function and commitment policies (introduced in section 4.3) in which the stringency of policy is chosen prior to investment or the realization of the damage function.

Figures 2, 3 and 4 compare linear emissions taxes, cap-and-trade with permit auction, and cap-and-trade with free permit distribution under flexibility and commitment. In the four panels for all three figures, the values of all parameters are the same except for N (the number of firms) and $\text{Var}(\varepsilon)$ (the variance of the damage parameter). Because commitment policies do not incorporate learning about damage uncertainty, flexible policies perform better comparatively when the degree of uncertainty is large (panels c and d relative to panels a and b).

For emissions taxes (Figure 2), flexibility is preferred to commitment except under low variance of damage and small values of technology spillover and even here the gap between the policies is small. When variance or technology spillovers are large the welfare gap between flexible and commitment policy can be large. Larger uncertainty about environmental damages increases expected losses under commitment but leaves expected losses under flexible policy largely unchanged. The incentive to invest is larger under flexible linear emissions tax than it is under commitment, and this tends to yield better performance when technology spillovers are large.

[Figures 1, 2, 3, 4]

Under permit auction, when $N = 2$, flexible policy tends to perform much better than commitment policy even when the degree of uncertainty is small (Figure 3 panel a). However, for $N = 10$, commitment may be preferred to flexible policy (panels c and d). Panel (d) shows that commitment may be preferred to flexibility even when there is large uncertainty about damages if the technology spillover is large enough. Firms' equilibrium investment is smaller than the optimal level under flexible permit auction when there are large technology spillovers (Proposition 6). The strategic incentive to under-invest to gain more permits is absent under commitment policy. With large spillover effects the gain from increased investment can more than offset the losses associated with failure to match regulation with environmental conditions leading to the result that commitment can yield lower welfare losses than flexible policy.

The case with grandfathered permits under flexible policy and commitment policy (Figure 4) is quite similar to the case with auctioned permits.[12] For $N = 2$, flexibility is preferred to commitment (panels a and b). For $N = 10$, however, commitment may be preferred to flexibility when either the degree of damage uncertainty is small (panel c) or when the degree of technology spillovers are large (panel d).

[12] We assume in Figure 4 that the equilibrium emission is equal to the amount of distributed permits to each firm, so that there is no emissions payment effect.

6 Discussion

In this paper we analyzed regulation of an externality with uncertainty about the severity of environmental damages from emissions, endogenous choice of emissions abatement technology by regulated firms, and technology spillovers across firms. In order to achieve an optimal outcome, the regulator needs to provide firms with correct incentives for both investments and emissions abatement, and this requires having policy flexible enough to address both margins (two-dimensional policy approaches). We showed that several two-dimensional policy approaches that are slight modifications of common approaches to environmental regulation (emissions taxes, cap-and-trade with freely distributed permits, and cap-and-trade with auctioned permits) are each capable of supporting the optimal outcome in equilibrium. Under the optimal cap-and-trade policy with freely distributed permits, the regulator sets the total emissions cap at the ex-post optimal level given learning about damages and firms' investment levels, and the regulator rewards a firm that invests more with a larger number of emission permits. By rewarding investment in technology, the regulator can internalize the externality due to technology spillovers, thereby providing incentives to firms to choose the optimal investment level. The optimal outcome for investment and emissions can also be achieved with an emissions tax coupled with an inframarginal subsidy, where the amount of the subsidy depends on how much the firm has invested. Similarly, a permit auction can achieve the optimal outcome by allowing firms access to a preferential price for a block of permits where the number of preferred permits a firm can purchase depends on the firm's investment level. Correctly specified, each of these two-dimensional policies can support the optimal emission and investment allocations.

Implementing these optimal policies requires that the regulator observe both investment and emissions. Observing emissions is a requirement for implementation of emissions taxes or cap-and-trade approaches. Investment of firms, however, may not be readily observable, in which case it may not be possible to implement the optimal policies outlined in this paper. The regulator may be constrained to use simpler policies that are not contingent on investment. Using a linear-quadratic specification, we show that simple emissions tax and permit auction approaches generate higher welfare than cap-and-trade with grandfathered permits, with the minor exception of the case with few firms and low degree of technology spillover where there is a slight advantage for cap-and-trade with grandfathered permits.

Existing environmental regulatory approaches typically do not consider investment in setting policy. Permit distribution rules under cap-and-trade policies have generally been determined by historical emissions (grandfathering) and are independent of the firms' investment behavior. For example, the SO_2 allowances under the Clean Air Act Amendments of 1990 were largely distributed according to historical emissions (Ellerman et al. 2000). However, some SO_2 emissions allowances were allocated to plants that reduced emissions significantly by installing scrubber, using desulfurized diesel fuel or renewable energy, which moves towards an optimal policy that rewards firms for investment.[13]

Part of the inefficiency of simple policies considered is caused by the fact that investment occurs prior to the resolution of uncertainty. If it were possible to reverse the order so that uncertainty was resolved prior to investment, the regulator could make policy dependent on environmental conditions but not dependent upon investment. Doing so would avoid distorting investment incentives while still allowing regulation to reflect environmental conditions. This order is not likely to be reversed in practice because investments for technology adoption tend to be long-lived while new information is learned on a fairly frequent basis.[14] However, not all inefficiency will be corrected by this shift in the order of moves. Technology spillover effects will not be internalized by reversing the timing of learning and investment.

We analyzed changing the timing of the choice of policy stringency relative to investment and resolution of uncertainty. For most of the paper, we considered flexible policy in which the regulator chose ex-post efficient policy updated by learning about environmental conditions and investment. Under commitment policy, the regulator chooses stringency of the policy at the outset prior to investment or learning. Because commitment policies do not incorporate learning about damage uncertainty, flexible policies tend to perform better than commitment policies when the degree of uncertainty is large. Flexible emissions taxes generally outperform emissions taxes under commitments. Under cap-and-trade, commitment policies are preferred to flexible policies under

[13] These bonus allowances were given under special provisions including Phase I Extension, Small Diesel Allowances, and Conservation Allowances (EPA "SO_2 Program and Compliance Results, 1996 Compliance Report"). Ellerman et al. (2000) also discuss the bonus allowances for the plants that installed scrubbers (p. 59). We thank Wayne Gray for suggesting this example.

[14] Amacher and Malik (2002) cite Stavins (1998, 2003) and Sterner (2001) in arguing that investment prior to policy choice better reflects the policy process in cases in the United States and Sweden than does assuming that policy choice is made prior to investment.

some, but not all, circumstances. In particular, commitment policies are preferred to flexible policies even under a large degree of uncertainty when the degree of technology spillovers is large.

There are several potential extensions to the line of analysis we have pursued in this paper. In the model, larger investment leads to lower abatement costs with certainty. In reality, R&D is a stochastic process. Firms choose expenditures on R&D and greater investment results in a larger probability of finding a new technology with lower abatement costs but not the certainty of doing so. Stochastic returns to investment in R&D add another source of uncertainty to the model.[15] In this paper, we focused on the case of symmetric uncertainty about damages from emissions and symmetric information about the costs of abatement. An alternative formulation would be to assume that the results of technology adoption or innovation are private information to the firm, which would then make the model one of regulation under asymmetric information. This is an interesting and important extension, but one that makes the analysis considerably more difficult (see Lewis 1996, Spulber 1988 and Montero 2008 for analysis of asymmetric information in environmental regulation without R&D). We leave the analysis of asymmetric information models with R&D for future research. Finally, though we analyzed strategic behavior in permits markets and the influence of strategy behavior upon regulation, we did not analyze strategic competition in output markets (see Fischer et al. 2003 and Montero 2002 for analyses of environmental regulation with oligopoly in output markets).

Appendix

Proof of Proposition 1

Let $\{k^*, x^*(\cdot)\}$ be the optimal investment and emission plan. Consider a permit distribution rule $q^* \equiv \{q_i^*(k;\varepsilon)\}$ that satisfies

$$q_i^*(k;\varepsilon) = \alpha_i(\varepsilon) + \beta_i(\varepsilon)k_i + \gamma_i(k_{-i}, \varepsilon) \geq 0$$

[15] Requate (2005b) analyzes such cases assuming a single innovator, competitive technology adopters and deterministic damages.

for all k, ε such that $\sum_j q_j^*(k;\varepsilon) = X^*(k;\varepsilon)$ for all k, ε and

$$\beta_i(\varepsilon) = \frac{D_{xx}(X^*(\varepsilon);\varepsilon)\frac{\partial X^*(\varepsilon,k)}{\partial k_i}[x_i^*() - q_i^*()] - \lambda \sum_{j\neq i} C_z^j(x_j^*(\varepsilon), z_j(k^*))}{D_x(X^*(\varepsilon);\varepsilon)}.$$

Given q^*, the optimal plan $\{k^*, x^*(\cdot)\}$ satisfies the first-order condition of each firm's payoff maximization:

$$C_x^i(x_i(\varepsilon), z_i(k^*) = p(z(k^*), \varepsilon),$$

$$G_i'(k_i^*) + E\left[C_z^i(x_i^*(\varepsilon), z_i(k^*)) - p(z(k^*), \varepsilon) \cdot \frac{\lambda \sum_{j\neq i} \left|C_z^j(x_j^*(\varepsilon), z_j(k^*))\right|}{D_x(X^*;\varepsilon)}\right] = 0 \quad \text{for all } i,$$

where $p(z(k^*), \varepsilon) = D_x(X^*(\varepsilon);\varepsilon)$. Given that each firm's objective function given q^* is convex, the above condition is necessary and sufficient for firms' cost minimization. Convexity of the firms' cost functions follows under a mild assumption. The second order derivative of firm i's objective function is

$$G_i'' + E\left[C_{zx}^i \frac{\partial x_i}{\partial k_i} + C_{zz}^i - \frac{\partial p}{\partial k_i}\frac{\partial q_i^*}{\partial k_i} + \frac{\partial p_i}{\partial k_i}\left\{\frac{\partial x_i}{\partial k_i} - \frac{\partial q_i^*}{\partial k_i}\right\} + \frac{\partial^2 p_i}{\partial k_i^2}\left\{x_i - q_i^*\right\}\right]$$

$$= G_i'' + E\left[\frac{C_{zz}^i C_{xx}^i - (C_{zx}^i)^2 - C_{zx}^i \frac{\partial p}{\partial k_i}}{C_{xx}^i} + \left(-\frac{\partial p}{\partial k_i}\frac{\partial q_i^*}{\partial k_i}\right) + \frac{\partial p_i}{\partial k_i}\left\{\frac{\partial x_i}{\partial k_i} - \frac{\partial q_i^*}{\partial k_i}\right\} + \frac{\partial^2 p_i}{\partial k_i^2}\left\{x_i - q_i^*\right\}\right].$$

The first three terms are positive because G_i and C^i are convex, $\frac{\partial p_i}{\partial k_i} < 0$, $C_{zx}^i > 0$, and $\frac{\partial q_i^*}{\partial k_i} > 0$. If $\frac{\partial x_i}{\partial k_i} < 0$ and if $\frac{\partial^2 p_i}{\partial k_i^2}$ is sufficiently small, then the whole expression is positive. ∎

Proof of Proposition 2

Recall that the optimal investments satisfy condition (1):

$$G_i'(k_i^*) + E\left[C_z^i(x_i^*(\varepsilon), z_i(k^*)) + \textstyle\sum_{j\neq i}\lambda C_z^j(x_j^*(\varepsilon), z_j(k^*))\right] = 0,$$

for all i. The first-order condition of firm i's cost minimization is given by

$$C_x^i(x_i(k,\varepsilon), z_i(k)) = p(z(k), \varepsilon) \quad \text{where } p(z(k), \varepsilon) = D_x(X(k,\varepsilon);\varepsilon),$$

$$G_i'(k_i) + E\left[C_z^i(x_i(\varepsilon), z_i(k)) + \frac{\partial p(k,\varepsilon)}{\partial k_i} x_i(k,\varepsilon) - \frac{\partial R_i(k,\varepsilon)}{\partial k_i}\right] = 0,$$

for all i. Given that firm i's objective function given R_i is convex, the above condition is necessary and sufficient for the firm's cost minimization. The optimal plan $\{k^*, x^*(\cdot)\}$ satisfies these conditions if

$$\frac{\partial p(k^*;\varepsilon)}{\partial k_i} x_i(k^*,\varepsilon) - \frac{\partial R_i(k^*,\varepsilon)}{\partial k_i} = \lambda \sum_{j \neq i} C_{jz}\left(x_j^*(\varepsilon), z_j(k^*)\right),$$

i.e. if condition (5) holds. ∎

Proof of Proposition 4

Part (i): Under cap-and-trade with linera auction, given (z,ε) the total permits $Q(z,\varepsilon)$ satisfies

$$C_x^i(x_i(z,\varepsilon), z_i(k)) + D_x(Q(z,\varepsilon);\varepsilon) = 0, \quad i = 1, \ldots, N,$$

where $\sum_i x_i(z,\varepsilon) = Q(z,\varepsilon)$ and the equilibrium permit price $p(z,\varepsilon)$ satisfies $p(z,\varepsilon) = D_x(Q(z,\varepsilon);\varepsilon)$. In the investment stage, firm i solves

$$\min_{k_i} G(k_i) + E[C^i(x_i(z,\varepsilon), z_i(k)) + p(z,\varepsilon)x_i(z,\varepsilon)].$$

The first order condition is

$$G_i'(k_i) + E\left[C_z^i + \left\{\frac{\partial p}{\partial z_i} + \lambda \sum_{j \neq i} \frac{\partial p}{\partial z_j}\right\} x_i(z,\varepsilon)\right] = 0. \qquad (10)$$

Under linear emissions taxes, the tax rate (z,ε) satisfies $\tau(z,\varepsilon) = D_x(Q(z,\varepsilon);\varepsilon) = 0$. In the investment stage, firm i chooses k_i to minimize $G_i(k_i) + E[C^i(x_i(z,\varepsilon), z_i(k)) + \tau(z,\varepsilon)x_i(z,\varepsilon)]$. The first order condition is

$$G_i'(k_i) + E\left[C_z^i + \left\{\frac{\partial \tau}{\partial z_i} + \lambda \sum_{j \neq i} \frac{\partial \tau}{\partial z_j}\right\} x_i(z,\varepsilon)\right] = 0. \qquad (11)$$

Because $\tau(z,\varepsilon) = p(z,\varepsilon)$ for all (z,ε), conditions (10) and (11) are equivalent. Therefore, the equilibrium outcomes under linear auction and linear taxes are the same.

Parts (ii) and (iii): The following lemma proves useful for the succeeding analysis.

Lemma 1 *Let $x^*(z(k),\varepsilon)$ be the optimal emission profile given $z(k)$ and ε and $X^* \equiv \sum_i x_i^*$. Let $p(z(k),\varepsilon)$ be the equilibrium permit price given $z(k)$ and ε. Then $\frac{\partial X^*(z(k),\varepsilon)}{\partial k_i} < 0$ for all k, s and all i given any $\lambda \geq 0$ and $\frac{\partial p(z(k),\varepsilon)}{\partial k_i} < 0$ for all i.*

Proof. Suppose not. Suppose k and k' differ only in the ith element where $k_i < k_i'$. Let $z = (z_1(k), \ldots, z_N(k))$ and $z' = (z_1(k'), \ldots, z_N(k'))$. Note that $z_j(k) \leq z_j(k')$ for all j. Suppose $X^*(z,\varepsilon) \leq X^*(z',\varepsilon)$. Because D_x is increasing in emissions, we have

$$D_x(X^*(z,\varepsilon);\varepsilon) \leq D_x(X^*(z',\varepsilon);\varepsilon). \tag{12}$$

By the definition of x^*, we have

$$C_x^i(x_i^*(z,\varepsilon), z_i(k)) + D_x(X^*(z,\varepsilon);\varepsilon) = 0 \text{ and } C_x^i(x_i^*(z',\varepsilon), z_i(k')) + D_x(X^*(z',\varepsilon);\varepsilon) = 0. \tag{13}$$

(12) and (13) implies $C_x^i(x_i^*(z,\varepsilon), z_i(k)) \geq C_x^i(x_i^*(z',\varepsilon), z_i(k'))$. Because C_x^i is increasing in investment, we also have $C_x^i(x_i^*(z,\varepsilon), z_i(k)) < C_x^i(x_i^*(z,\varepsilon), z_i(k'))$. The last two inequalities imply $C_x^i(x_i^*(z',\varepsilon), z_i(k')) < C_x^i(x_i^*(z,\varepsilon), z_i(k'))$, i.e. $x_i(z',\varepsilon) < x_i(z,\varepsilon)$ because $C_{xx}^i > 0$. So $x_j^*(z',\varepsilon) > x_j^*(z,\varepsilon)$ must hold for some $j \neq i$ in order for $X^*(z,\varepsilon) \leq X^*(z',\varepsilon)$ to hold. Hence,

$$C_x^j(x_j^*(z,\varepsilon), z_j(k)) < C_x^j(x_j^*(z',\varepsilon), z_j(k)) \leq C_x^j(x_j^*(z',\varepsilon), z_j(k')),$$

where the last inequality follows from $C_{ez}^j > 0$. However, this implies

$$0 = C_x^j(x_j^*(z,\varepsilon), z_j(k)) + D_x(X^*(z,\varepsilon);\varepsilon) < C_x^j(x_j^*(z',\varepsilon), z_j(k')) + D_x(X^*(z',\varepsilon);\varepsilon) = 0,$$

a contradiction. Hence, $\frac{\partial X^*(k,\varepsilon)}{\partial k_i} < 0$ for all k, s and all i. Because p satisfies $p(z,\varepsilon) = D_x(X^*(z,\varepsilon);\varepsilon)$, we have $\frac{\partial p}{\partial z_i} = D_{xx} \cdot \frac{\partial X^*(z,\varepsilon)}{\partial k_i} < 0$. for all k, ε, i. (End of the proof of Lemma 1)

Lemma 2 *If $\lambda = 0$, then $\frac{\partial x_i^*(k,\varepsilon)}{\partial k_i} < 0$ for all k, ε and all i.*

Proof. The optimal emissions x^* satisfy the first equation of (13). Totally differentiate both sides with respect to x and k_i and obtain

$$\begin{cases} C^i_{xx}dx_j + C^i_{xz}dk_j + D_{xx}\sum_l dx_l = 0 & \text{for } j = i; \\ C^i_{xx}dx_j + D_{xx}\sum_l dx_l = 0 & \text{for } j \neq i. \end{cases}$$

Rewrite the system:

$$\begin{pmatrix} C^1_{xx} + D_{xx} & D_{xx} & \cdots & D_{xx} \\ D_{xx} & C^2_{xx} + D_{xx} & \ddots & \vdots \\ \vdots & \ddots & \ddots & D_{xx} \\ D_{xx} & \cdots & D_{xx} & C^N_{xx} + D_{xx} \end{pmatrix} \begin{pmatrix} dx_1 \\ \vdots \\ dx_N \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ -C^i_{ez}dk_i \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

By Cramer's Rule we have

$$\frac{\partial x_1}{\partial k_1} = -C^1_{xz} \begin{vmatrix} C^2_{xx}+D_{xx} & D_{xx} & \cdots & D_{xx} \\ D_{xx} & C^3_{xx}+D_{xx} & \ddots & \vdots \\ \vdots & \ddots & \ddots & D_{xx} \\ D_{xx} & \cdots & D_{xx} & C^N_{xx}+D_{xx} \end{vmatrix} \cdot \begin{vmatrix} C^1_{xx}+D_{xx} & D_{xx} & \cdots & D_{xx} \\ D_{xx} & C^2_{xx}+D_{xx} & \ddots & \vdots \\ \vdots & \ddots & \ddots & D_{xx} \\ D_{xx} & \cdots & D_{xx} & C^N_{xx}+D_{xx} \end{vmatrix}^{-1}.$$

Because $C + D$ is convex in emissions, the determinants in the denominator and the numerator are both positive. By assumption we have $C^1_{xz} > 0$, and hence $\frac{\partial x_1}{\partial k_1} < 0$. Similarly, we have $\frac{\partial x_i}{\partial k_i} < 0$ for all i. (End of the proof of Lemma 2)

Now we use Lemmas 1 and 2 to prove part (ii). The optimal investment profile k^* satisfies

$$G'_i(k^*_i) + E\left[C^i_z(x^*_i(z(k^*), \varepsilon), z_i(k^*)) + \lambda \sum_{j \neq i} C^j_z(x^*_j(z(k^*), \varepsilon), z_j(k^*))\right] = 0, \quad i = 1, \ldots, N.$$

With $\lambda = 0$, we have $z_i(k) = k_i$ for all i, the last term diminishes to zero, and the equilibrium investment profiles under linear taxes satisfy

$$G'_i(k^T_i) + E\left[C^i_z(x^*_i(k^T, \varepsilon), k^T_i) + \frac{\partial p}{\partial k_i} x^*_i(k^T, \varepsilon)\right] = 0, \quad i = 1, \ldots, N,$$

where $\frac{\partial p}{\partial z_i} < 0$ by Lemma 1. Hence, $k_i^* < k_i^T$ for all i.

The above proof works provided $\lambda = 0$. A positive λ induces firms to under-invest relative to the optimal level because of positive externalities of investments. The net effects of over-investment incentives described above and under-investment incentives due to technology spillovers may or many not be positive (part iii).

Part (iv): Under any policy, each firm takes the policy as given when N is large. The necessary and sufficient condition for the equilibrium allocation coincides with the condition for optimality if and only if there is no technology spillovers. ∎

Proof of Proposition 5

Parts (i), (ii)

If $\lambda = 0$, we have $z_i(k) = k_i$ for all i, k. The second best allocation given state-independent emissions (i.e. emissions are the same under any damage realization) solves

$$\min_{x,k} \sum_i [G_i(k_i) + C^i(x_i, k_i)] + ED(X; \varepsilon).$$

The solution x^c, k^c satisfies

$$G_i'(k_i^c) + C_z^i(x_i^c, k_i^c) = 0, \quad C_x^i(x_i^c, k_i^c) + ED_x(X^c; \varepsilon) = 0, \quad i = 1, \ldots, N.$$

In the case of tax, each firm facing emissions tax τ will choose emissions level and investment satisfying the following equations:

$$G_i'(k_i) + C_z^i(x_i, k_i) = 0, \quad C_x^i(x_i, k_i) + \tau = 0, \quad i = 1, \ldots, N. \tag{14}$$

With $\tau = ED_x(X^c; \varepsilon)$, the equilibrium allocation under tax supports the second best outcome.

Under cap-and-trade, suppose that a permit distribution rule α is given: $q_i = \alpha_i Q$ for all i. As in (6), firm i's equilibrium investment satisfies

$$G_i'(k_i) + C_z^i + \frac{\partial p}{\partial k_i}[x_i(Q, k) - q_i] = 0, \quad i = 1, \ldots, N.$$

The solution satisfies $(x_i(Q,k),k_i) = (x_i^c, k_i^c)$ if and only if $\alpha_i Q = x_i^c$ for all i. With permit auction, the above condition reduces to

$$G_i'(k_i) + C_z^i + \frac{\partial p}{\partial k_i} x_i(Q,k) = 0,, \quad i = 1, \dots, N,$$

which never supports the second-best solution because $\frac{\partial p}{\partial k_i} \neq 0$. When there is no damage uncertainty, the second best allocation is optimal. These arguments prove parts (i) and (ii).

Part (iii)

This result holds because condition (6) for cap-and-trade with free permit distribution is equivalent to the conditions (14) for tax when no permit trading occurs in equilibrium. ∎

Proof of Proposition 6

Part (i): In the last stage of the game, given k and ε, the regulator sets the total cap $Q(k,\varepsilon)$ equal to the optimal level given (k,ε):

$$Q(k,\varepsilon) = X^*(k,\varepsilon) = (\bar{E} - af(\lambda)K)/(1 + N\theta\varepsilon).$$

Hence, $\frac{\partial Q(k,\varepsilon)}{\partial k_i} = -af(\lambda)/(1+N\theta\varepsilon)$ for all k_i and all i where $f(\lambda) \equiv 1 + \lambda(N-1)$. The equilibrium permit price satisfies $p(k,\varepsilon) = \varepsilon X^*(k,\varepsilon)$. The equilibrium investment under grandfathering satisfies condition (3) where $\frac{\partial q_i}{\partial k_i} = \alpha_i \frac{\partial Q}{\partial k_i}$:

$$rk_i + E\left[-\frac{a(\bar{e}_i - az_i(k) - x_i(k,\varepsilon))}{\theta} + \frac{\partial p(z,\varepsilon)}{\partial k_i}(x_i(k,\varepsilon) - q_i(k,\varepsilon)) - \alpha_i p(z,\varepsilon)\frac{\partial Q(k,\varepsilon)}{\partial k_i}\right] = 0. \quad (15)$$

By adding up the left-hand side of equation (15) for all firms, we can cancel out the emissions payment effect (note that $\partial p/\partial k_i = \partial p/\partial k_j$ for all i,j):

$$rK^G - E\left[\frac{a}{\theta}(\bar{E} - af(\lambda)K^G - X(K^G,\varepsilon)) - \frac{\varepsilon(\bar{E} - af(\lambda)K^G)}{1+N\theta\varepsilon}\frac{af(\lambda)}{1+N\theta\varepsilon}\right] = 0.$$

Hence, the equilibrium total investment under grandfathering, K^G, satisfies

$$K^G = \frac{\phi Na\bar{E} - \mu af(\lambda)\bar{E}}{r + \phi Na^2 f(\lambda) - \mu(af(\lambda))^2} < \frac{\phi Naf(\lambda)\bar{E}}{r + \phi N(af(\lambda))^2} = K^*,$$

where $\phi \equiv E\left[\frac{\varepsilon}{1+N\theta\varepsilon}\right]$ and $\mu \equiv E\left[\frac{\varepsilon}{(1+N\theta\varepsilon)^2}\right]$.

The equilibrium total investment under linear tax and linear permit auction satisfies

$$K^T = K^{PA} = \frac{\phi N a \bar{E} + \mu a f(\lambda)\bar{E}}{r + \phi N a^2 f(\lambda) + \mu(af(\lambda))^2}.$$

The difference between K^T and K^* is given by

$$K^T - K^* = \frac{ar\bar{E}[\phi N(1-f(\lambda)) + \mu f(\lambda)]]}{(r + \phi N a^2 f(\lambda) + \mu(af(\lambda))^2)(r + \phi N(af(\lambda))^2))}.$$

It follows that $\partial\{K^T - K^*\}/\partial\lambda < 0$ and $K^T - K^* < (>)0$ if and only if $\lambda > (<)\hat{\lambda} \equiv \frac{\mu}{(N\phi-\mu)(N-1)}$.

As for tax versus cap-and-trade with grandfathering, we have

$$K^G - K^T = -2(r + \phi N a^2 f(\lambda))r\mu a f(\lambda)\bar{E}H < 0 \quad \text{for all } \lambda \geq 0,$$

where $H \equiv 1/[(r + \phi N a^2 f(\lambda) + \mu(af(\lambda))^2)(r + \phi N a^2 f(\lambda) - \mu(af(\lambda))^2)(r + \phi N a^2 f(\lambda))] > 0$. Therefore, $K^T > K^G$.

Part (ii): Under each policy, we can express the equilibrium social cost as a function of the equilibrium total investment. Under any policy instrument, the equilibrium social cost is given by

$$TC = \sum_i \frac{r}{2}k_i^2 + E\sum_i \frac{1}{2\theta}\left(\bar{e}_i - x_i(k;\varepsilon) - az_i(k)\right)^2 + E\frac{\varepsilon}{2}\left(\frac{\bar{E} - af(\lambda)K}{1+N\theta\varepsilon}\right)^2.$$

The equilibrium emissions and investments equate the firms' marginal abatement costs and the marginal damages under each state:

$$\frac{\bar{e}_i - x_i(z;\varepsilon) - az_i(k)}{\theta} = \varepsilon X(K;\varepsilon)$$

for all i, K, ε where $X(K;\varepsilon)$ is the equilibrium total emissions given $(K;\varepsilon)$. Hence,

$$TC = \frac{Nr}{2}\left(\frac{K}{N}\right)^2 + \frac{N}{2\theta}E\left(\theta\varepsilon\frac{\bar{E} - af(\lambda)K}{1+N\theta\varepsilon}\right)^2 + E\frac{\varepsilon}{2}\left(\frac{\bar{E} - af(\lambda)K}{1+N\theta\varepsilon}\right)^2$$

$$= \frac{r}{2N}K^2 + \frac{N\theta}{2}C\left(\bar{E} - af(\lambda)K\right)^2 + \frac{\mu}{2}\left(\bar{E} - af(\lambda)K\right)^2$$

$$= \frac{r}{2N} K^2 + \frac{N\theta C + \mu}{2} \left(\bar{E} - af(\lambda)K\right)^2 = \frac{r}{2N} K^2 + \frac{\phi}{2} \left(\bar{E} - af(\lambda)K\right)^2,$$

where $C \equiv E\left[\frac{\varepsilon^2}{(1+N\theta\varepsilon)^2}\right]$ and the last equality follows from

$$N\theta C + \mu = E\frac{N\theta\varepsilon^2 + \varepsilon}{(1+N\theta\varepsilon)^2} = E\frac{\varepsilon(1+N\theta\varepsilon)}{(1+N\theta\varepsilon)^2} = E\frac{\varepsilon}{1+N\theta\varepsilon} \equiv \phi.$$

Hence, the difference between the total costs under two policies I and J is

$$TC^J - TC^I = \frac{r}{2N}\left((K^J)^2 - (K^I)^2\right) + \frac{\phi}{2}\left[\left(\bar{E} - af(\lambda)K^J\right)^2 - \left(\bar{E} - af(\lambda)K^I\right)^2\right]$$

$$= \frac{r}{2N}\left(K^J + K^I\right)\left(K^J - K^I\right) + \frac{\phi}{2}\left[-2\bar{E}af(\lambda)(K^J - K^I) + (af(\lambda))^2\left(K^J + K^I\right)\left(K^J - K^I\right)\right]$$

$$= \frac{a\bar{E}(K^J - K^I)}{2N} \cdot F(K^J, K^I),$$

where $F(K^J, K^I) \equiv \left(r + N\phi(af(\lambda))^2\right)\left(\frac{K^J + K^I}{a\bar{E}}\right) - 2N\phi f(\lambda)$. Therefore, we have $TC^J > TC^I$ if $K^J < K^I$ and $F(K^J, K^I) < 0$ (or if $K^J > K^I$ and $F(K^J, K^I) > 0$). With linear emissions tax and cap-and-trade with grandfathering, we have

$$F(K^G, K^T) = \frac{(r + \phi N a^2 f(\lambda))2\phi N r(1 - f(\lambda)) - 2r\mu^2 f(\lambda)(af(\lambda))^2}{(r + \phi N a^2 f(\lambda) - \mu(af(\lambda))^2)(r + \phi N a^2 f(\lambda) + \mu(af(\lambda))^2)} < 0$$

for all $\lambda \geq 0$. It follows from $K^G < K^T$ that $TC^G > TC^T$. ∎

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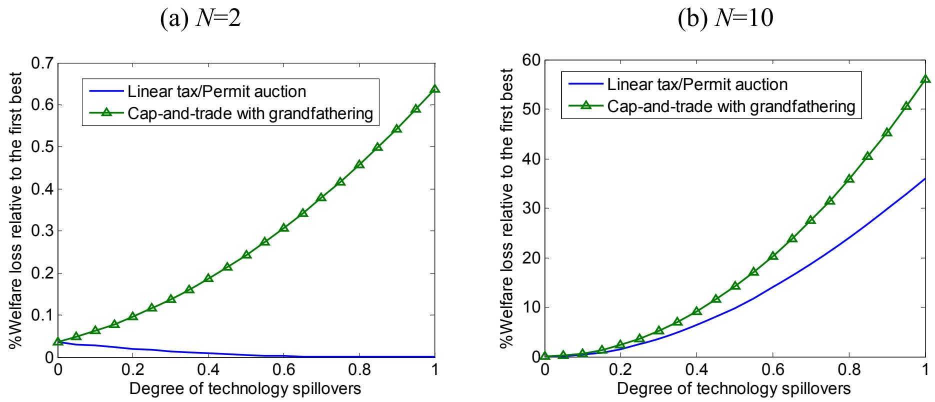

Notes: Based on the linear-quadratic model with a=1, r=100,000, P(ε=50)=P(ε=150) = 0.5, and $\bar{e}_i$ = 500 for all i.

Figure 1: Welfare losses in the linear-quadratic model under linear emissions taxes and cap-and-trade with grandfathering relative to the optimal solution. This example assumes the following parameter values: $a = 1, r = 100,000, \Pr(\varepsilon = 50) = \Pr(\varepsilon = 150) = 1/2$, $\theta = 0.0005$, and $\bar{e}_i = 5,000/N$ for all i.

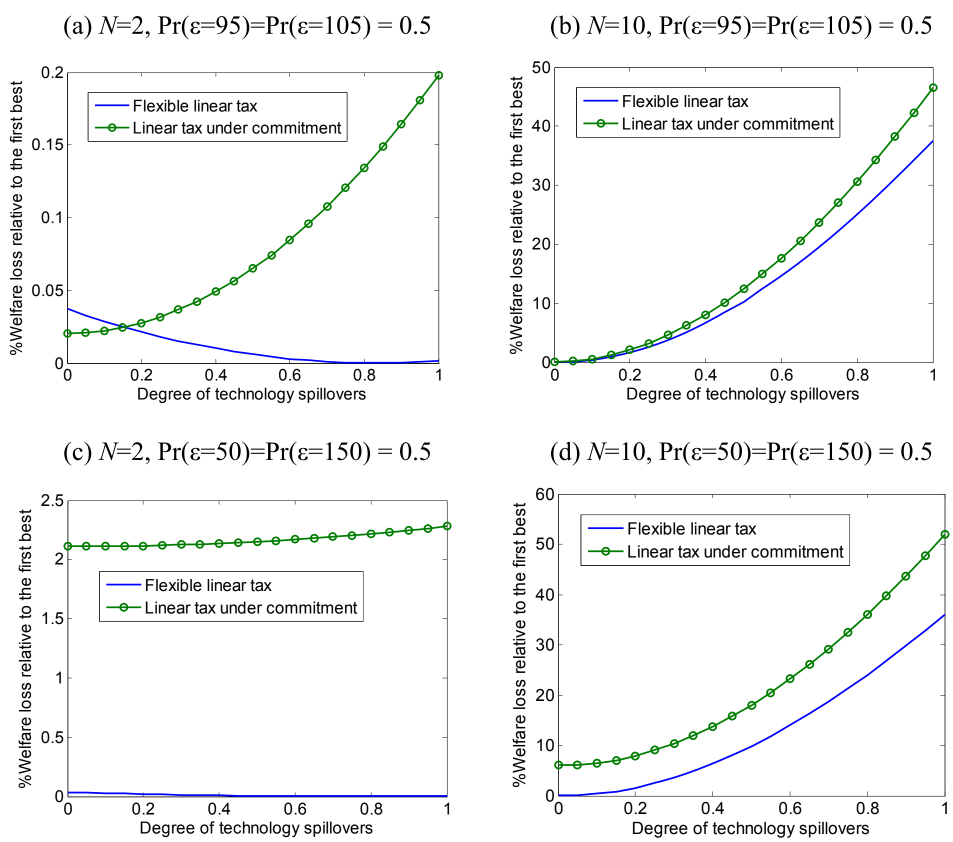

Figure 2: Welfare losses in the linear-quadratic model with linear emissions taxes under flexible and commitment policies. This example assumes the following parameter values: $a = 1$, $r = 100,000$, $\theta = 0.0005$, and $\bar{e}_i = 5,000/N$ for all i.

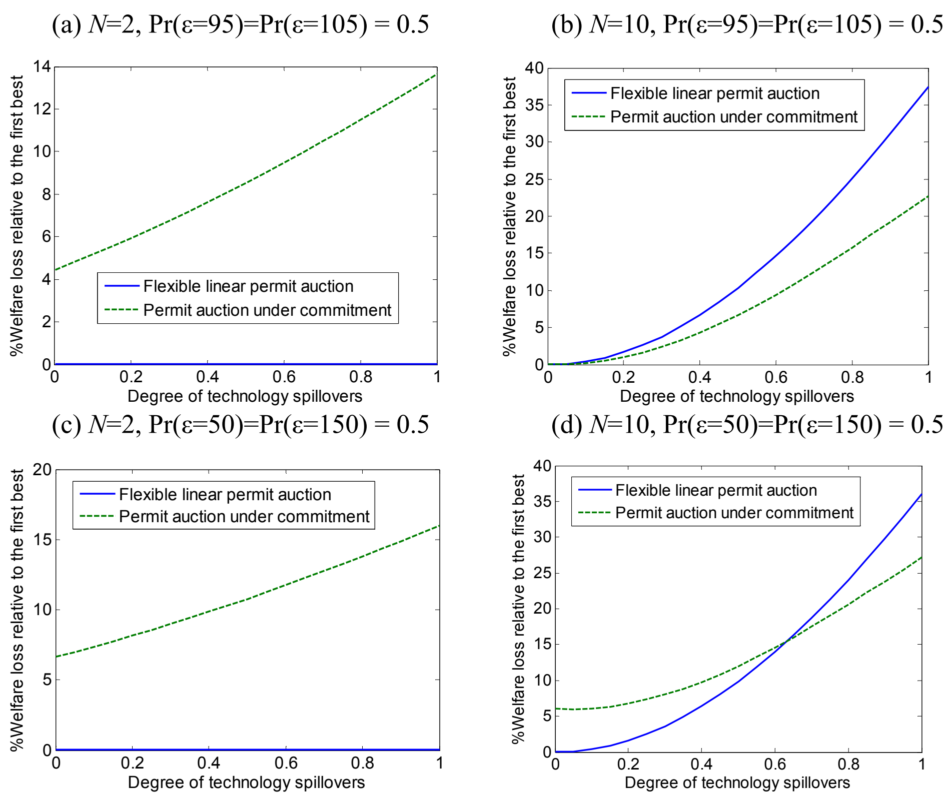

Figure 3: Welfare losses in the linear-quadratic model with a linear permit auction under flexible and commitment policies. This example assumes the following parameter values: $a = 1$, $r = 100,000$, $\theta = 0.0005$, and $\bar{e}_i = 5,000/N$ for all i.

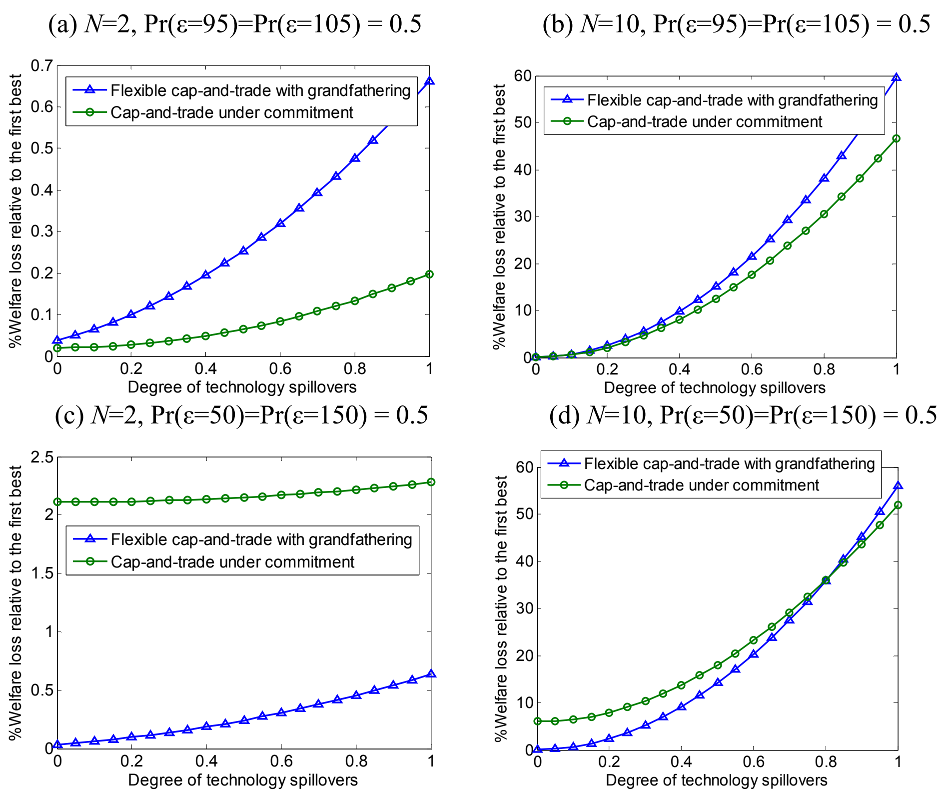

Figure 4: Welfare losses in the linear-quadratic model with free distribution of permits under flexible and commitment cap-and-trade policies. This example assumes the following parameter values: $a = 1$, $r = 100,000$, $\theta = 0.0005$, and $\bar{e}_i = 5,000/N$ for all i.