1 Matrix multiplication

Write out the pseudocode for an $O(\log n)$-time, $O(n^3)$-work matrix multiplication algorithm of multiplying two $n \times n$ matrices. Analyze your algorithm and prove it to be correct.

2 Half-plane intersections (Exercise 6.31)

Let $P$ be a simple polygon with $n$ vertices. The kernel of $P$, denoted by $K(P)$, is the set of points $q$ in $P$ such that, for any point $z$ on the boundary of $P$, the segment $zq$ lies entirely in $P$. (See Figure 6.21 in the textbook for an example of a polygon and its kernel).

1. Show that $K(P)$ is the intersection of $n$ half-planes defined by the edges of $P$.

2. Deduce an $O(\log n)$ time algorithm to compute $K(P)$. What is the total number of operations used?

3 Segmented Prefix Sums (Exercise 6.23)

Suppose that you are given an array $A$ of length $n$ with some of its elements marked, and you want to compute the prefix sums of each subarray consisting of the elements of $A$ between two consecutive marked elements (the marked elements themselves are assigned arbitrarily to the subarrays). Show how to perform this task in $O(\log n)$ time using a total of $O(n)$ operations.

4 Convex hull intersections

Let $U$ and $L$ be a sequence of $n$ points each of two convex polygonal chains. $U$ and $L$ are given in sorted order by increasing $x$-coordinates. Write out the pseudocode for a parallel algorithm that finds where the two polygonal chains intersect. Analyze the time and work complexity of your algorithm and prove its correctness.