Normal Forms (ii)

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## Redundancies & Decompositions

<table>
<thead>
<tr>
<th>SSN</th>
<th>Name</th>
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<th>Rating</th>
<th>Hourly_wages</th>
<th>Hours_worked</th>
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### Hourly_Emps

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### RatingWages

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Decompositions

• Reduces redundancies and anomalies, but could have the following potential problems:
  1. Some queries become more expensive.
  2. Given instances of the decomposed relations, we may not be able to reconstruct the corresponding instance of the original relation!
  3. Checking some dependencies may require joining the instances of the decomposed relations.

• Two desirable properties:
  – Lossless-join decomposition
  – Dependency-preserving decomposition
Lossless-join Decomposition

- Decomposition of R into X and Y is **lossless-join** w.r.t. a set of FDs F if, for every instance r that satisfies F:

  \[ \pi_X(r) \text{ join } \pi_Y(r) = r \]

- In general one direction \( \pi_X(r) \text{ join } \pi_Y(r) \supseteq r \) is always true, but the other may not hold.

- Definition extended to decomposition into 3 or more relations in a straightforward way.

- *It is essential that all decompositions used to deal with redundancy be lossless!* (Avoids Problem (2).)
Conditions for Lossless Join

• The decomposition of \( R \) into \( X \) and \( Y \) is lossless-join wrt \( F \) if and only if the closure of \( F \) contains:
  – \( X \cap Y \rightarrow X \), or
  – \( X \cap Y \rightarrow Y \)

• In particular, the decomposition of \( R \) into \( UV \) and \( R - V \) is lossless-join if \( U \rightarrow V \) holds over \( R \).
Chase Test for Lossless Join

- \( R(A,B,C,D) \) is decomposed into \( S_1=\{A,D\}, S_2=\{A,C\}, S_3=\{B,C,D\} \)

- Construct a Tableau
  - One row for each decomposed relation
  - For each row \( i \), subscript an attribute with \( i \) if it does not occur in \( S_i \).

- FDs: \( A \rightarrow B, B \rightarrow C, CD \rightarrow A \)

- Rules for “equating two rows” using FDs:
  - If one is unsubscripted, make the other the same
  - If both are subscripted, make the subscripts the same

- Goal: one unsubscripted row

\[
\begin{array}{cccc}
<table>
<thead>
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<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>\text{b}_1</td>
<td>\text{c}_1</td>
<td>d</td>
</tr>
<tr>
<td>a</td>
<td>\text{b}_2</td>
<td>c</td>
<td>\text{d}_2</td>
</tr>
<tr>
<td>a_3</td>
<td>b</td>
<td>c</td>
<td>d</td>
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one unsubscripted row imply lossless join
Dependency-preserving Decomposition

• **Dependency preserving decomposition (Intuitive):**
  – If $R$ is decomposed into $X$, $Y$ and $Z$, and we enforce the FDs that hold on $X$, on $Y$ and on $Z$, then all FDs that were given to hold on $R$ must also hold. *(Avoids Problem (3)).*

• **Projection of set of FDs $F$:** If $R$ is decomposed into $X$, ... projection of $F$ onto $X$ (denoted $F_X$) is the set of FDs $U \rightarrow V$ in $F^+$ *(closure of $F$)* such that $U, V$ are in $X$. 

F = \{ SC \rightarrow I, I \rightarrow C \}
Dependency-preserving Decomp. (Cont)

Decomposition of R into X and Y is **dependency preserving** if \((F_X \cup F_Y)^+ = F^+\) if we consider only dependencies in the closure \(F^+\) that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in \(F^+\).

- Example: ABC decomposed into AB and BC.
  - \(F=\{A \rightarrow B, \ B \rightarrow C, \ C \rightarrow A\}\).
  - Is this dependency preserving?

- Dependency preserving does not imply lossless join:
  - Eg. ABC,  \(A \rightarrow B\), decomposed into AB and BC.
  - And vice-versa! (Example?)
Decomposition into BCNF

• Consider relation $R$ with FDs $F$. How do we decompose $R$ into a set of small relations that are in BCNF?

• Algorithm:
  – If $X \rightarrow Y$ violates BCNF, decompose $R$ into $R-Y$ and $XY$
  – Repeat until all relations are in BCNF.

• Example: $ABCD$, $B \rightarrow C$, $C \rightarrow D$, $C \rightarrow A$.

• Order in which we deal with the violating FD can lead to different relations!
BCNF Decomposition Algorithm (3.20)

- **Input**: $R_0$, set of FDs $S_0$
- **Output**: A decomposition of $R_0$ into a collection of relations, all of which are in BCNF
- Initially $R = R_0$, $S = S_0$

1. If $R$ is in BCNF, return \{R\}
2. Let $X \rightarrow Y$ be a violation.
   a. Compute $X^+$. 
   b. Choose $R_1 = X^+$
   c. Let $R_2 = X \cup (R - X^+)$
3. Compute FD projections $S_1$ and $S_2$ for $R_1$ and $R_2$
4. Recursively decompose $R_1$ and $R_2$ and return the union of the results
BCNF & Dependency Preservation

• BCNF decomposition using Algo 3.20 is lossless join

• BUT in general there may not be a dependency preserving decomposition into BCNF
  
  – Example: Bookings\( (\text{Title}, \text{City}, \text{Theater}) \), with FD’s: \( \text{Th} \rightarrow \text{C} \), \( \text{TiC} \rightarrow \text{Th} \)
  
  – Not in BCNF.
  
  – Can’t decompose while preserving 2nd FD;

• This is the motivation for 3NF.
Decomposition into 3NF

• Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier).

• How can we ensure dependency preservation?
  – If $X \rightarrow Y$ is not preserved, add relation $XY$.
  – Problem is that $XY$ may violate 3NF!
  – Example: $JP \rightarrow C$ is not preserved, so add $JPC$. What if FDs also include $J \rightarrow C$?

• Refinement: Instead of the given set of FDs $F$, use a minimal cover for $F$. 
Minimum Cover for a Set of FDs

- **Minimal cover or basis** $G$ for a set of FDs $F$:
  - Closure of $F = \text{closure of } G$.
  - Right hand side of each FD in $G$ is a single attribute.
  - If we modify $G$ by deleting an FD or by deleting attributes from an FD in $G$, the closure changes.

- Intuitively, every FD in $G$ is needed, and "as small as possible" in order to get the same closure as $F$.

- e.g., $A \rightarrow B$, $ABCD \rightarrow E$, $EF \rightarrow GH$, $ACDF \rightarrow EG$ has the following minimal cover:
  - $A \rightarrow B$, $ACD \rightarrow E$, $EF \rightarrow G$ and $EF \rightarrow H$
Computing the Minimal Cover

• Algorithm
  1. **Put the FDs into standard form X → A.** RHS is a single attribute.
  2. **Minimize the LHS of each FD.** For each FD, check if we can delete an attribute from LHS while preserving $F^+$. 
  3. **Delete redundant FDs.**

• Minimal covers are not unique. Different order of computation can give different covers.
• e.g., $A → B, ABCD → E, EF → GH, ACDF → EG$ has the following minimal cover:
  – $A → B, ACD → E, EF → G$ and $EF → H$
3NF Decomposition Algorithm (3.26)

• **Input**: R, set of FDs F

• **Output**: A decomposition of R into a collection of relations, all of which are in BCNF

1. Find a minimal basis/cover for F, say G

2. For each FD $X \rightarrow A$ in G, use $XA$ as one of the decomposed relations.

3. If none of the relations from Step 2 is a superkey for R, add another relation whose schema is a key for R.
Summary of Schema Refinement

• If a relation is in BCNF, it is free of redundancies that can be detected using FDs. Thus, trying to ensure that all relations are in BCNF is a good heuristic.

• If a relation is not in BCNF, we can try to decompose it into a collection of BCNF relations.
  – Must consider whether all FDs are preserved.
  – If a lossless-join, dependency preserving decomposition into BCNF is not possible (or unsuitable, given typical queries), should consider decomposition into 3NF.
  – Decompositions should be carried out and/or re-examined while keeping performance requirements in mind.