Chapter 13
Production and Cost Functions

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Total Revenue, Total Cost, Profit

- We assume that the firm's goal is to maximize profit.

Profit = **Total revenue** – **Total cost**

- **Total revenue**: the amount a firm receives from the sale of its output
- **Total cost**: the market value of the inputs a firm uses in production

Costs: Explicit vs. Implicit

- **Explicit costs** require an outlay of money, *e.g.*, paying wages to workers.
- **Implicit costs** do not require a cash outlay, *e.g.*, the opportunity cost of the owner's time.
- ***The cost of something is the highest value opportunity foregone.***
- This is true whether the costs are implicit or explicit. Both matter for firms' decisions.

Explicit vs. Implicit Costs: An Example

You need $100,000 to start your business.
The interest rate is 5%.

- Case 1: borrow $100,000
  - explicit cost = $5000 interest on loan
- Case 2: use $40,000 of your savings, borrow the other $60,000
  - explicit cost = $3000 (5%) interest on the loan
  - implicit cost = $2000 (5%) *foregone* interest you could have earned on your $40,000.

In both cases, total (exp + imp) costs are $5000.

Economic Profit vs. Accounting Profit

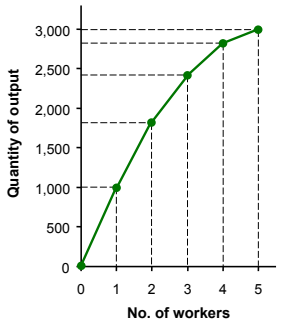
- **Accounting profit**
  = total revenue minus total explicit costs
- **Economic profit**
  = total revenue minus total costs (including explicit and implicit costs)
- Accounting profit ignores implicit costs, so it's higher than economic profit.

The Production Function

- A **production function** shows the relationship between the quantity of inputs used to produce a good and the quantity of output of that good.
- It can be represented by a table, equation, or graph.
- Example 1:
  - Farmer Akira grows wheat.
  - He has 5 acres of land.
  - He can hire as many workers as he wants.

Ex 1: Farmer Akira's Production Function

L (no. of workers)	*Q* (bushels of wheat)
0	0
1	1000
2	1800
3	2400
4	2800
5	3000



Marginal Product

- If Akira hires one more worker, his output rises by the *marginal product of labor*.
- The **marginal product** of any input is the increase in output arising from an additional unit of that input, holding all other inputs constant.
- Notation:
  Δ (delta) = "change in…"
  Examples:
  Δ***Q*** = change in output, Δ***L*** = change in labor
- Marginal product of labor (*MPL*) = $\frac{\Delta Q}{\Delta L}$

Why MPL Is Important

- Because rational owners of firms think at the margin!
- When Farmer Akira hires an extra worker,
  - his costs rise by the wage he pays the worker
  - his output rises by *MPL*
- Comparing them helps Akira decide whether he would benefit from hiring the worker.

Why MPL Diminishes

- Farmer Akira's output rises by a smaller and smaller amount for each additional worker. Why?
- As Akira adds workers, the average worker has less land to work with and will be less productive.
- In general, *MPL* diminishes as ***L*** rises whether the fixed input is land or capital (equipment, machines, etc.).
- **Diminishing marginal product**:
  the marginal product of an input declines as the quantity of the input increases (other things equal)

Marginal Cost

- **Marginal Cost** (*MC*)
  is the increase in Total Cost from producing one more unit:

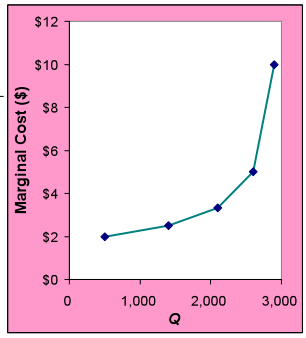
$$MC = \frac{\Delta TC}{\Delta Q}$$

EXAMPLE 1: Total and Marginal Cost

	Q (bushels of wheat)	Total Cost		Marginal Cost (*MC*)
	0	$1,000		
Δ***Q*** = 1000			Δ**TC** = $2000	$2.00
	1000	$3,000		
Δ***Q*** = 800			Δ**TC** = $2000	$2.50
	1800	$5,000		
Δ***Q*** = 600			Δ**TC** = $2000	$3.33
	2400	$7,000		
Δ***Q*** = 400			Δ**TC** = $2000	$5.00
	2800	$9,000		
Δ***Q*** = 200			Δ**TC** = $2000	$10.00
	3000	$11,000		

EXAMPLE 1: The Marginal Cost Curve

Q (bushels of wheat)	*TC*	*MC*
0	$1,000	
		$2.00
1000	$3,000	
		$2.50
1800	$5,000	
		$3.33
2400	$7,000	
		$5.00
2800	$9,000	
		$10.00
3000	$11,000	



Why MC Is Important

- Farmer Akira is rational and wants to maximize his profit. To increase profit, should he produce more or less wheat?
- To find the answer, Farmer Akira needs to "think at the margin."
- If the cost of additional wheat (*MC*) is less than the revenue he would get from selling it, then Akira's profits rise if he produces more.

Fixed and Variable Costs

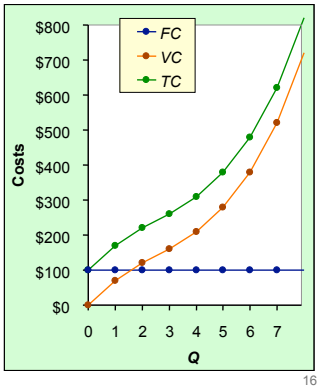
- **Fixed costs (*FC*)** do not vary with the quantity of output produced.
  - For Farmer Akira, *FC* = $1000 for his land
  - Other examples: cost of equipment, loan payments, rent
- **Variable costs (*VC*)** vary with the quantity produced.
  - For Farmer Akira, *VC* = wages he pays workers
  - Other example: cost of materials
- **Total cost (*TC*)** = *FC* + *VC*

EXAMPLE 2

- Our second example is more general, applies to any type of firm producing any good with any types of inputs.

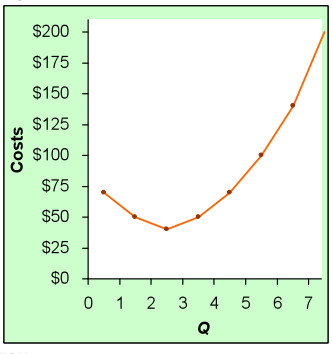
EXAMPLE 2: Costs

Q	*FC*	*VC*	*TC*
0	$100	$0	$100
1	100	70	170
2	100	120	220
3	100	160	260
4	100	210	310
5	100	280	380
6	100	380	480
7	100	520	620



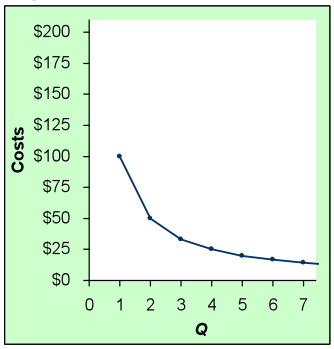
EXAMPLE 2: Marginal Cost

Q	*TC*	*MC*
0	$100	
		$70
1	170	
		50
2	220	
		40
3	260	
		50
4	310	
		70
5	380	
		100
6	480	
		140
7	620	



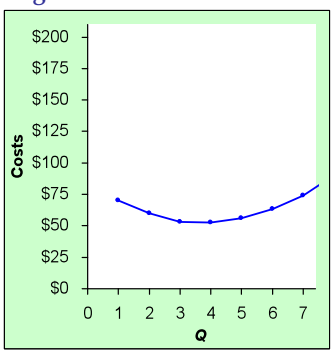
EXAMPLE 2: Average Fixed Cost

Q	FC	AFC
0	$100	n/a
1	100	$100
2	100	50
3	100	33.33
4	100	25
5	100	20
6	100	16.67
7	100	14.29



EXAMPLE 2: Average Variable Cost

Q	VC	AVC
0	$0	n/a
1	70	$70
2	120	60
3	160	53.33
4	210	52.50
5	280	56.00
6	380	63.33
7	520	74.29



EXAMPLE 2: Average Total Cost

Q	TC	ATC	AFC	AVC
0	$100	n/a	n/a	n/a
1	170	$170	$100	$70
2	220	110	50	60
3	260	86.67	33.33	53.33
4	310	77.50	25	52.50
5	380	76	20	56.00
6	480	80	16.67	63.33
7	620	88.57	14.29	74.29

Average total cost (*ATC*) equals total cost divided by the quantity of output:

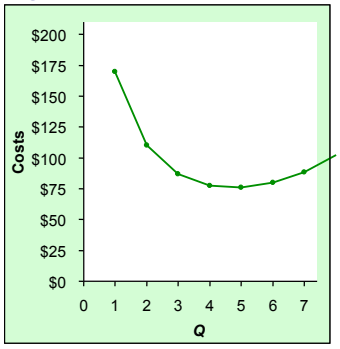
ATC = *TC*/***Q***

Also,

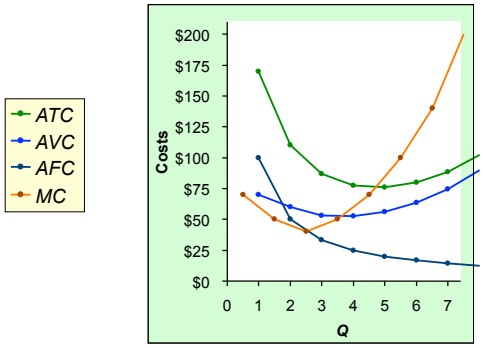
ATC = *AFC* + *AVC*

EXAMPLE 2: Average Total Cost

Q	TC	ATC
0	$100	n/a
1	170	$170
2	220	110
3	260	86.67
4	310	77.50
5	380	76
6	480	80
7	620	88.57



EXAMPLE 2: The Various Cost Curves Together



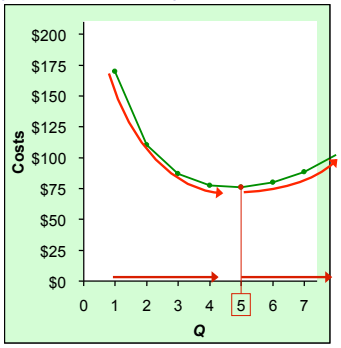
EXAMPLE 2: Why ATC Is Usually U-Shaped

As ***Q*** rises:

Initially, falling *AFC* pulls *ATC* down.

Eventually, rising *AVC* pulls *ATC* up.

Efficient scale: The quantity that minimizes ATC.

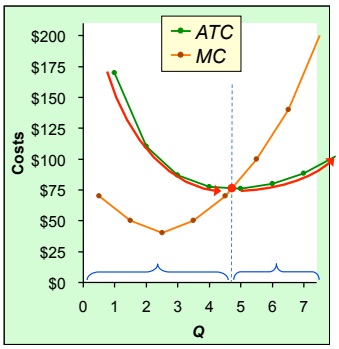


EXAMPLE 2: ATC and MC

When *MC* < *ATC*, *ATC* is falling.

When *MC* > *ATC*, *ATC* is rising.

The *MC* curve crosses the *ATC* curve at the *ATC* curve's minimum.



Costs in the Short Run & Long Run

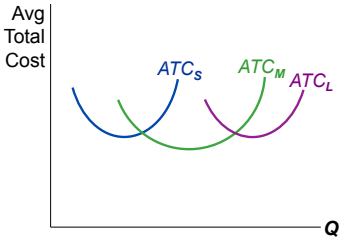
- Short run:
  Some inputs are fixed (*e.g.,* factories, land).
  The costs of these inputs are *FC*.
- Long run:
  All inputs are variable
  (*e.g.,* firms can build more factories,
  or sell existing ones).
- In the long run, *ATC* at any ***Q*** is cost per unit using the most efficient mix of inputs for that ***Q*** (*e.g.*, the factory size with the lowest *ATC*).

EXAMPLE 3: LRATC with 3 factory Sizes

Firm can choose from 3 factory sizes: **S**, **M**, **L**.

Each size has its own *SRATC* curve.

The firm can change to a different factory size in the long run, but not in the short run.

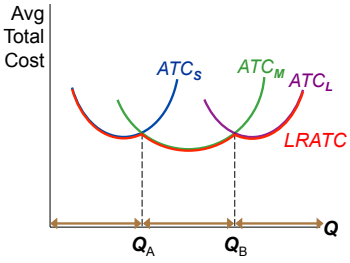


EXAMPLE 3: LRATC with 3 factory Sizes

To produce less than Q_A, firm will choose size **S** in the long run.

To produce between Q_A and Q_B, firm will choose size **M** in the long run.

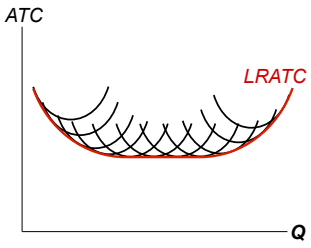
To produce more than Q_B, firm will choose size **L** in the long run.



A Typical LRATC Curve

In the real world, factories come in many sizes, each with its own *SRATC* curve.

So a typical *LRATC* curve looks like this:

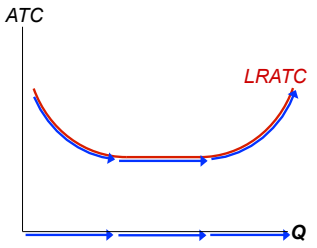


How ATC Changes as the Scale of Production Changes

Economies of scale: *ATC* falls as ***Q*** increases.

Constant returns to scale: *ATC* stays the same as ***Q*** increases.

Diseconomies of scale: *ATC* rises as ***Q*** increases.



How ATC Changes as the Scale of Production Changes

- Economies of scale occur when increasing production allows greater specialization: workers more efficient when focusing on a narrow task.
  - More common when ***Q*** is low.
- Diseconomies of scale are due to coordination problems in large organizations. *E.g.*, management becomes stretched, can't control costs.
  - More common when ***Q*** is high.