ECON607 Fall 2010
University of Hawaii
Professor Hui He
Assignment 5

The due date for this assignment is Friday, December 10.

1. Consider a two-period overlapping generations model without production. At each time period $t = 0, 1, 2, ...$ a new generation of measure N_t is born. Population grows at a constant rate $n = N_{t+1}/N_t - 1 \geq 0$. The endowment when young and when old are $\{\omega_1 > 0, \omega_2 > 0\}$ with $\omega_1 > \omega_2$. The preference and endowment structures of successive generations are identical. The preference of the representative member of generation t are given by
$$\frac{(c_t^t)^{1-\gamma}}{1-\gamma} + \frac{(c_{t+1}^t)^{1-\gamma}}{1-\gamma}, \gamma > 0$$
There is an initial old generation with endowment $\omega_2 > 0$ and consume c_1^0.

   (a) Define a Pareto Optimal allocation for this economy.

   (b) Define an Arrow-Debreu competitive equilibrium.

   (c) Obtain the offer curve, and characterize the equilibrium defined in part (b). Is it unique?

   (d) Now assume that $\gamma = 1$. Assume that there is a pay-as-you-go (PAYG) social security system in the economy. The young pay lump-sum taxes in the amount τ each period, and when they get old, they receive benefits in the amount b. Assume that the system is self-financing each period. Compute the competitive equilibrium with social security.

   (e) Compute the optimal level of the PAYG social security system; that is find the optimal tax rate that takes the economy to the Golden Rule.

2. Consider the economy in question 1 with $n = 0$. But now the initial old are also endowed with m units of unbacked fiat money. The stock of currency is constant over time.

   (a) Find a saving function of a young agent, i.e., find savings as a function of the interest rate. Characterize the saving function.

   (b) Define an Arrow-Debreu competitive equilibrium with fiat money.

   (c) Define an stationary Arrow-Debreu competitive equilibrium with fiat money.

   (d) Describe how many equilibria with valued fiat currency there are. (You are not being asked to compute them.)

3. Consider an economy with overlapping generations of a constant population of an even number N of two-period-lived agents. New young agents are born at each date $t \geq 1$. Half of the young agents are endowed with $\omega_1 > 0$ when young and 0 when old. The other half are endowed with 0 when young and $\omega_2 > 0$ when old. Assume that $\omega_1 > \omega_2$.

Preferences of all young agents are as in question 1, with $\gamma = 1$. Half of the N initial old are endowed with ω_2 units of the consumption good and half are endowed with nothing. Each old person at $t = 1$ is endowed with m units of unbacked fiat currency. No other generation is endowed with fiat currency. The stock of fiat currency is fixed over time.

(a) Find the saving function of each of the two types of young person for $t \geq 1$.

(b) Define an equilibrium without valued fiat currency. Compute all such equilibria.

(c) Define an equilibrium with valued fiat currency.

(d) Compute all the (nonstochastic) equilibria with valued fiat currency.

(e) Argue that there is a unique stationary equilibrium with valued fiat currency.

(f) How are the various equilibria with valued fiat currency ranked by the Pareto criterion?