ECON607 Fall 2010
University of Hawaii
Professor Hui He
Assignment 4

The due date for this assignment is Tuesday, November 30.

1. (*Productivity and Employment*) Consider a neoclassical growth model with leisure in the utility function

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$$
$$s.t.$$
$$c_t + i_t + g_t \leq z f(k_t, n_t)$$
$$k_{t+1} \leq (1-\delta)k_t + i_t$$

where g_t is the government's expenditure per capita. The functions u and f are assumed to be strictly increasing in each argument, concave, and twice differentiable. In addition, we have $\lim_{k_t \to \infty} \frac{\partial f(k_t,n_t)}{\partial k_t} \to 0$.

   (a) Describe the steady state of this economy. If necessary, make additional assumptions to guarantee that it exists and is unique. If you make additional assumptions, go as far as you can giving an economic interpretation of them.

   (b) Assume that $u(c, 1-n) = \frac{[c^{\mu}(1-n)^{1-\mu}]^{1-\sigma}}{1-\sigma}$ and $f(k,n) = k^{\alpha}n^{1-\alpha}$. What is the effect of changes in the technology (say increases in z) on employment and output per capita?

   (c) Consider next an increase in g. Are there conditions under which an increase in g will result in an increase in the steady-state $\frac{k}{n}$ ratio? How about an increase in the steady-state level of output per capita? Go as far as you can giving an economic interpretation of these conditions. (Try to do this for general $f(k,n)$ functions with the appropriate convexity assumptions, but if this proves too hard, use the Cobb-Douglas specification.)

2. (*H-P filter*) Choose the quarterly real GDP data from NIPA. Write a Matlab program to use the H-P filter to detrend the GDP time series and obtain the cyclical part of the GDP ($\{y_t^c\}_{t=1}^T$). Plot the original GDP series, the growth part and the cyclical part. Calculate the standard deviation of the GDP series, the growth part and the cyclical part respectively.

3. (*Replicating RBC exercise*) Download Dynare Version 3.065 from the URL

   http://www.cepremap.cnrs.fr/juillard/mambo/

   Spending some time with Dynare and read the user's guide. Then use it to replicate the results shown in Table 8.6 on the lecture notes. Plot the impulse response of the shock to consumption c, capital stock k, working hours h, and output y. Explain your results.

4. (*Calibrating a RBC model with a labor indivisibility*) Consider a stochastic growth model with preferences

$$E\sum_{t=0}^{\infty}\beta^t\{\log c_t+\alpha\log(1-n_t)\}$$

where $n_t \in \{0, \frac{1}{2}\}$. In other words, labor supply is subject to a indivisibility constraint. A person either does not work or work to a certain amount of time. The production function is

$$y_t = k_t^{\theta} n_t^{1-\theta}.$$

The resource constraint is given by

$$c_t + i_t = y_t.$$

The law of motion for the capital stock is

$$k_{t+1} = (1-\delta)k_t + i_t.$$

Calibrating this model economy (i.e., choose the parameter values for $\beta, \delta, \alpha, \theta$) so that its competitive equilibrium allocation matches the long-run observations in the US economy $c/y = 0.85$, $k/y = 3.00$, labor income share in GDP is 70%, and the fraction of working time is 80%. Due to the labor indivisibility, the model has to be convexified. Richard Rogerson (1988) shows that introducing a lottery technology can solve this issue. In particular, let π_t be the probability of working $n_t = \frac{1}{2}$, and $1-\pi_t$ be the probability of working $n_t = 0$. Then the period utility function becomes

$$\log c_t + \alpha\pi_t\log(1-\frac{1}{2}) + \alpha(1-\pi_t)\log(1-0).$$

The HHs now maximize utility function over the sequences of $\{c_t, i_t, \pi_t\}$.

5. (*Calibrating a RBC model with home production*) Consider an economy with infinitely lived individuals. Individuals maximize

$$\max E_0\sum_{t=0}^{\infty}\beta^t\{b\ln c_t + (1-b)\ln(1-l_t)\}$$

where consumption is a aggregator defined by

$$c_t = [\alpha c_{m,t}^{e} + (1-\alpha)c_{h,t}^{e}]^{\frac{1}{e}}$$

$c_{m,t}$ is the consumption of market-produced goods, $c_{h,t}$ is consumption of home-produced goods.

The model has two technologies, one for market production and one for home production:

$$\begin{aligned} f(z_{m,t}, k_{m,t}, l_{m,t}) &= e^{z_{m,t}} k_{m,t}^{\theta} l_{m,t}^{1-\theta} \\ g(z_{h,t}, k_{h,t}, l_{h,t}) &= e^{z_{h,t}} k_{h,t}^{\eta} l_{h,t}^{1-\eta}. \end{aligned}$$

where θ and η are the capital share parameters for the two technologies, respectively. The two technology shocks follow the processes:

$$\begin{aligned} z_{m,t} &= \rho z_{m,t-1} + \varepsilon_{m,t} \\ z_{h,t} &= \rho z_{h,t-1} + \varepsilon_{h,t} \end{aligned}$$

where the two innovations $\varepsilon_{m,t}$ and $\varepsilon_{h,t}$ are normally distributed with zero means and standard deviations σ_m and σ_h; have a contemporaneous correlation $\varphi = cov(\sigma_m, \sigma_h)$; and are independent over time. In each period, a capital constraint holds

$$k_t = k_{m,t} + k_{h,t}.$$

The law of motion for the aggregate capital stock is

$$k_{t+1} = (1 - \delta_m)k_{m,t} + (1 - \delta_h)k_{h,t} + i_t.$$

We also have following time constraint

$$l_{m,t} + l_{h,t} = l_t.$$

And the resource constraints for two sectors

$$\begin{aligned} c_{m,t} + i_t + G_t &= f(z_{m,t}, k_{m,t}, l_{m,t}) \\ c_{h,t} &= g(z_{h,t}, k_{h,t}, l_{h,t}). \end{aligned}$$

We also assume that there is a government imposes proportional taxes each period on labor and capital income (net of depreciation) at the constant rates τ_l and τ_k, transfers the lump-sum T_t back to individuals, and consumes the surplus in terms of government expenditure G_t. Government balances its budget period by period. Calibrate this model economy to match long-run average ratio in the US economy: $k_m/y = 4$, $k_h/y = 5$, $i_m/y = 0.118$, $i_h/y = 0.135$, $l_m = 0.25$, $l_h = 0.33$, $\tau_k = 0.70$, $\tau_l = 0.25$, quarterly growth rate of output is 0.5%, and annual real rate of return on capital is 6%. How many parameter values can be determined by these moments? What are these values?

6. Consider the planning problem for a neoclassical growth model with logarithmic utility, full depreciation of the capital stock in one period, and a production function of the form $y = zk^\alpha$, where z is a random shock to productivity. The shock z is observed before making the current-period savings decision. Assume that the capital stock can take on only two values: i.e., k is restricted to the set $\{\bar{k}_1, \bar{k}_2\}$. In addition, assume that z takes on values in the set $\{\bar{z}_1, \bar{z}_2\}$ and that z follows a Markov chain with transition probabilities $p_{ij} = \Pr(z' = \bar{z}_j \mid z = \bar{z}_i)$.

   (a) Let $\bar{z}_1 = 0.9$, $\bar{z}_2 = 1.1$, $p_{11} = 0.95$, $p_{22} = 0.9$. Find the invariant distribution associated with the Markov chain for z. Use the invariant distribution to compute the long-run (or unconditional) expected value of z; that is, compute $E(z) = \pi_1\bar{z}_1 + \pi_2\bar{z}_2$, where π_1 and π_2 determines the invariant distribution.

(b) Let $\beta = 0.9$, $\alpha = 0.36$, $\bar{k}_1 = 0.95k_{ss}$, $\bar{k}_2 = 1.05k_{ss}$, where k_{ss} is the steady-state capital stock in a version of this model without shocks and with no restrictions on capital. Show that $k_{ss} = (\alpha\beta)^{\frac{1}{1-\alpha}}$. Let $g(k, z)$ denote the planner's optimal decision rule. Prove that $g(k_i, z_j) = k_j$ for all i and j.

(c) The decision rule from part (b) and the law of motion for z jointly determine an invariant distribution over (k, z)-pairs. Find this distribution. (That is, find probabilities $\pi_{ij} = \Pr(k = k_i, z = z_j)$ that "reproduce" themselves: if π_{ij} is the unconditional probability that the economy is in state (k_i, z_j) today, then it is also the unconditional probability that the economy is in this state tomorrow.) Use your answer to compute the long-run (or unconditional) expected values of the capital stock and of output.

(d) In Matlab, use the optimal decision rule, the law of motion for z, and a random number generator to create a simulated time series $\{k_t, y_t\}_{t=0}^T$ given an initial condition (k_0, z_0). Compute $\frac{\sum_{t=0}^T k_t}{T}$ and $\frac{\sum_{t=0}^T y_t}{T}$ for a suitably large value of T and confirm that these sample means are close to the corresponding population means that you computed in part (c).