ECON607 Fall 2010
University of Hawaii
Professor Hui He
Assignment 1

The due date for this assignment is Thursday, Sep. 23.

1. Consider an stochastic optimal growth model as in the lecture with utility function $u(c) = \ln c$, production function $f(k) = k^{\alpha}$, and resource constraint $c_t + k_{t+1} \leq e^{z_t} f(k_t)$, where z_t is the i.i.d. productivity shock belongs to a lognormal distribution.
   (a) Write the Bellman equation for the social planner's problem.
   (b) Use "Guess and Verify" method to slove the Bellman equation. (Hint: Guess $v(k, z) = H + F \ln k + Gz$.)

2. Prove that the budget constraint in sequential market equilibrium (SME) as in the lecture is as same as the one in ADE, i.e., BC in SME $\Rightarrow$ BC in ADE.

3. (*Two-period Emdowment Economy*) Consider following two-period pure exchange economy. There is a single consumption good in each period. There are two HHs who have identical preferences over consumption given by

$$u(c_1, c_2) = \ln c_1 + \beta \ln c_2$$

   HH 1 has endowments given by $(w_1, 0)$ and HH 2 has endowments $(0, w_2)$.
   (a) Define an ADE for this economy.
   (b) Suppose HHs have incentive to smooth their consumption over time by borrowing and lending. Define a SME for this economy. Show that the ADE and SME give identical allocations.
   (c) Suppose $w_1 = w_2 = 1$ and $\beta = 1$, calculate the competitive equilibrium.
   (d) Suppose HHs cannot borrow or lend from each other. Try to define a competitive equilibrium in this case. Show that a policy which transfers goods from the rich to the poor in each period in Pareto-improving.

4. (*When Robinson Crusoe meets Arrow-Debreu*) Consider a Robinson-Crusoe economy. Crusoe as the single HH on the island has a utility function over consumption of fish c_t and leisure $(1 - l_t)$ as following

$$\sum_{t=0}^{T} \beta^t [\ln c_t + \gamma \ln(1 - l_t)]$$

   where $\beta \in (0, 1)$ and $\gamma > 0$. Crusoe has a technology transforming labor into output of fish by

$$y_t = A_t l_t^{\alpha}$$

   where $\alpha \in (0, 1)$, $\{A_t\}_{t=0}^{T}$ is a sequence of numbers which measures Crusoe's state of productivity. The fish can only be used for consumption (Robinson does not have a refrigerator).

(a) Set up the social planner's problem for this economy.

(b) One interpretation of the production technology is there is a fixed supply of one boat (or catamaran, denoted by k) so that the technology is

$$y_t = A_t k_t^{1-\alpha} l_t^{\alpha}.$$

Assume that the boat is owned by Crusoe and the technology is operated by a firm (Dole Fishing Co.) which rents capital k_t and labor l_t in a competitive market at factor prices r_t and w_t. Let p_t denote the Arrow-Debreu price of one unit of consumption at time t.

1. Set up Crusoe's decison problem as a HH problem.
2. Set up the firm's problem.
3. What are the resource constraint?
4. Define a competitive equilibrium.
5. Compare a CE to the planner's problem above.
6. Solve for the CE allocations.
7. How does the CE change when A_0 rises? When A_1 rises?

5. (*Irrelevance of capital ownership*) Exercise 2.9 in SLP.

6. *(Continuity of contraction mapping)* Exercise 3.8 in SLP.

7. *(An example of contraction mapping)* Exercise 3.10 in SLP.

8. (*No Ponzi game*) Consider a consumer with the following optimization problem

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$\begin{aligned} c_t + q b_{t+1} &= b_t + w, \forall t \\ & b_0 \text{ is given} \end{aligned}$$

and a no Ponzi Game (NPG) condition

$$\lim_{t\to\infty} q^t b_t = 0$$

Assume that the bond price q is less than one, that the wage w is positive, and that the discount factor $\beta \in (0, 1)$. The period utility function u is strictly increasing, strictly concave, and twice continuously differentiable.

(a) Derive the consumer's consolidated (or lifetime) budget constraint. (Hint: you will need to use NPG condition.)

(b) Find the transversality condition (TVC) for this problem. Show that the NPG condition is met if the TVC and the Euler equation are both satisfied.

(c) Prove that a sequence $\{b_t^*\}_{t=0}^{\infty}$ that satisfies the transversality condition and the Euler equation maximizes the consumer's objective, subject to the sequence of budget constraints and the NPG condition.

9. Consider an exchange economy with two types of consumers. Type-A consumers comprise fraction λ of the economy's population and type-B consumers comprise fraction $1-\lambda$ of the economy's population. Each consumer has constant endowment ϖ in each period. A consumer of type i has preferences over consumption streams of the form $\sum_{t=0}^{\infty} \beta_i^t \log(c_{it})$. Assume that $1 > \beta_A > \beta_B > 0$: type-A consumers are more patient than type-B consumers. Consumers trade a risk-free bond in each period. There is no restriction on borrowing except for a no-Ponzi-game (NPG) condition. Each consumer has zero assets in period 0.

   (a) Carefully define a sequential competitive equilibrium (SME) for this economy.

   (b) Carefully define a recursive competitive equilibrium (RCE) for this economy.

   (c) Show that this economy has no steady state: in particular, show that the type-B agents become poorer and poorer over time and consume zero in the limit.