Chapter 10

A Monetary Intertemporal Model: The Neutrality of Money
Outline of this Chapter

• Introduce the money into the real intertemporal model in Chapter 9
• **Neutrality of Money**: a one-time change in the money supply has no real consequences for the economy.
• The Quantity Theory of Money and Monetarism.
Functions of Money

- **Medium of Exchange**
- **Store of Value**
- **Unit of Account**

- In this chapter, we will focus on the distinguishing feature of money: its medium-of-exchange role.
Measuring the Money Supply

• The Monetary Base (M0)
  – Currency Outside the Fed
  – Depository Institution Deposits at the Fed
  – It includes entirely the liabilities of the Federal Reserve System.
• M1
  = M0 +
  – Traveler’s Checks
  – Demand Deposits
  – Other Checkable Deposits
• M2
  = M1 +
  – Savings Deposits
  – Small-Denomination Time Deposits
  – Retail Money Market Mutual Funds
• M3
  \[ = M2 + \]
  – Large-Denomination Time Deposits
  – Institutional Money Market Mutual Funds
  – Repurchase Agreements
  – Eurodollars
Table 10.1 Monetary Aggregates, September 2003 (in $billions)

<p>| | |</p>
<table>
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<tr>
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<tr>
<td>M0</td>
<td>720.7</td>
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<tr>
<td>M1</td>
<td>1275.6</td>
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<td>6083.9</td>
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<td>M3</td>
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A Monetary Intertemporal Model

• Why we need money?
• To overcome **double coincidence of wants** caused by barter exchange
• To avoid the difficulties arose by the credit transactions
Real and Nominal Interest Rates

- Focus on two periods: current and future
- Focus on two assets: money and nominal bonds
- Use money as the *numeraire*
- $P$ is the current price level, $P'$ is the future price level
• Inflation rate $i$

$$i = \frac{P' - P}{P}$$
The real interest rate $r$ is determined by the **Fisher Equation**

$$1 + r = \frac{1 + R}{1 + i}$$

$R$ is the nominal interest rate
• The real rate of interest on money $r^m$ is

\[
1 + r^m = \frac{1 + 0}{1 + i} = \frac{1}{1 + i}
\]

• As long as $R > 0$, we have $r^m < r$. 
• Approximate the Fisher Equation

\[ r \approx R - i \]
Figure 10.1  Real and Nominal Interest Rates, 1948–2003
Representative Consumer

• At the beginning of current period, she has two available assets: nominal money $M^-$ and nominal bonds $B^-$
• Then she pays a lump-sum tax $PT$
• After that, she chooses to rearrange her asset portfolio by choosing $B^d$
• After leaving the credit market, she goes to work, supplies $h-l$ unit of labor, earns nominal wage $P_w$

• After finishing work, she goes to goods market to purchase consumption goods. We assume that all consumption goods must be purchased with money on hand.
• Cash-in-advance constraint (CIA)

\[ PC \leq M^- + B^- (1 + R^-) - PT - B^d \]
• Finally at the end of the period, the consumer’s BC is

\[ PC + B^d + M^d \leq M^- + B^- (1 + R^-) + Pw(h-l) + P\pi - PT \]
Figure 10.2 The Sequence of Transactions During a Period in the Monetary Intertemporal Model

(a) Tax Collection

(b) Credit Market

(c) Goods Market

(d) Wage and Dividend Payments
• Representative Consumer’s problem:
Choose $C, I, B^d$ and $M^d$ to max present value of discounted utility
subject to
1. CIA constraint
2. BC
• A key feature of the consumer’s problem is that wage and dividend income cannot be spent on consumption goods today. They must be held as money until the future period.
• Since $R > 0$, the rate of return on bonds is greater than that on money, this implies CIA will be binding (is satisfied with equality)

• Hence we have

$$M^d \leq Pw(h-l) + P\pi \leq PY$$
• $M^d$ is also affected by the nominal interest rate $R$ because $R$ represents the opportunity cost of holding money.

• So we have the demand for money in real terms as

$$\frac{M^d}{P} = L(Y, R), \frac{\partial L}{\partial Y} > 0, \frac{\partial L}{\partial R} < 0$$
• Recall Fisher Equation

\[ M^d = PL(Y, r + i) \]

• If inflation is constant

\[ M^d = PL(Y, r) \]
Figure 10.3 The Nominal Money Demand Curve in the Monetary Intertemporal Model

\[ M^d = PL(Y, r + i) \]
Figure 10.4 The Effect of an Increase in Current Real Income on the Nominal Money Demand Curve

\[ M_1^d = PL(Y_1, r + i) \]

\[ M_2^d = PL(Y_2, r + i) \]
Government

• Government is responsible for both fiscal and monetary policy. Think about government in this model is the combination of Federal Reserve System (issue money) and U.S. Treasury (collect tax).
• Government’s BC

\[ PG + (1 + R^-)B^- = PT + B + M - M^- \]

• \( M^- - M^- \) represents the money creation via monetary policy
Representative Firm

• The firm wants to maximize the profits

\[ P\pi = PF(N) - P_wN = PY - P_wN \]
Competitive Equilibrium in Intertemporal Monetary Model

• Three markets in this economy
  – Labor market: labor supply = labor demand
  – Goods market: goods supply = goods demand
  – Money market: \[ M^s = M^d = PL(Y, r) \]
Definition of CE

A CE in the intertemporal monetary economy is an allocation \( \{C, I, B^d, M^d\} \) for each period \( t \) for the consumers, an allocation \( \{N^d\} \) for each period \( t \) for the firm, a combination of policies \( \{T, G, M, B\} \) for the Gov, and a price system \( \{P, w, r\} \) for each period \( t \) such that
• \( \{C, l, B^d, M^d\}_t \) are the solutions to the consumer’s problem.

• \( \{N^d\}_t \) are the solutions to the firm’s problem.

• Markets clear.

\[
N^d = h - l = N^s, \quad C + G = Y, \\
M^s = M^d, \quad B^d = B
\]
FOCs

• Consumer’s problem

\[
\max \sum_{t=0}^{\infty} U(C_t, l_t) \\
\text{s.t.} \\
PC + B^d + M^d \leq \\
M^- + B^- (1 + R^-) + P\omega (h - l) + P\pi - PT, \forall t
\]
• FOCs

\[ C_t : \frac{\partial U}{\partial C} = \lambda_t P_t \]

\[ C_{t+1} : \frac{\partial U}{\partial C'} = \lambda_{t+1} P_{t+1} \]

\[ B^d_t : \lambda_{t+1} (1 + R) = \lambda_t \]

\[ \Rightarrow \text{MRS}_{C',C} = \frac{P_{t+1}}{1 + R} = \frac{1 + i}{1 + R} = \frac{1}{1 + r} \]
\[ l_t : \frac{\partial U}{\partial l} = \lambda_t P_t w_t \]

\[ \Rightarrow M R S_{l.C} = w_t \]
• FOCs for the consumers are the same as those in the real intertemporal model!

• FOC for the firm is same too!

\[ N^d : P \frac{\partial F}{\partial N^d} = Pw \]

\[ \Rightarrow M P_L = w \]
Neutrality of Money

- The equilibrium conditions for the labor and goods market are as same as in the real model.
- Except that now we have additional money market. And the money market equilibrium is determined by the real variables.
Figure 10.5 The Current Money Market in the Monetary Intertemporal Model

\[ M^d = PL(Y, r + i) \]
Figure 10.6 The Complete Monetary Intertemporal Model
Experiment: An Once-for-all Increase in $M^s$

• Suppose $M_{t+1} - M_t > 0, M_{t+2} - M_{t+1} = 0$

• Sources of Changes in the Money Supply
  – Helicopter Drops (lump-sum tax $T \downarrow$, everyone has more money)
  – Open-Market Operations (Fed buys bonds $B$ to release $M$ to the banks)
  – Seigniorage (printing out $M$ to finance $G$)
• Since \( M \) (or \( P \)) never enters into the equilibrium conditions of the current labor and goods market, so it will not affect the real variables in this economy.

• In the money market. Since we have

\[
\frac{M}{P} = L (Y, r)
\]

So price level must increase in proportion to \( M \) so that real money stock \( M/P \) remains unchanged.
• An 10% increase in money supply will only induce a 10% inflation rate. The real economy will be unaffected. This result is called **Neutrality of Money**.
Figure 10.7 A Level Increase in the Money Supply in the Current Period
Figure 10.8  The Effects of a Level Increase in M—The Neutrality of Money
Experiment: A Decrease in Current $z$

- We already knew that $z \downarrow$ (in the real economy) will cause $Y \downarrow$, $r \uparrow$, $w \downarrow$, $N \downarrow$, $C \downarrow$, $I \downarrow$

- In the money market, $Y \downarrow$, $r \uparrow$ will induce $M^d \downarrow$, given the money supply unchanged, will see $P \uparrow$.

- Recall the business cycle facts, price level is counter-cyclical.
Figure 10.9 Short-Run Analysis of a Temporary Decrease in Total Factor Productivity
Figure 10.10 Relative Price of Energy
Figure 10.11 Percentage Deviations from Trend in the Price Level
Shifts in Money Demand

• So far we assume the money demand function $L(Y,R)$ is fixed.

• Let’s relax this assumption. Shifts in the real demand for the money could due to
  – Costs of Using Alternatives to Money
  – Costs of Converting Other Assets into Money
  – Government Regulations
  – Inflation Risk
  – Riskiness of Alternative Assets
• Neutrality vis-a-vis Real Variables, $Y$ and $r$ are unaffected.

• Price-Level Effects
Figure 10.12 A Shift in the Demand for Money
The Velocity of Money

• The velocity of an asset is a measure of how fast that asset circulates.

• The most common measure of the velocity of money is income velocity.

\[ V = \frac{P \cdot Y}{M} \]
• Substitute into the money demand function

\[ V = \frac{P \cdot Y}{M} = \frac{Y}{L(Y, R)} \]

• If we assume \( L(Y, R) = aYH(R), H'(R) < 0 \)

We will have

\[ V = \frac{1}{a \cdot H(R)} \]
• Hence we should observe a positive relation b/w $R$ and $V$ in the data.
Figure 10.13  M1
Figure 10.14  Velocity of M1
Figure 10.15  Scatter Plot of the Velocity of M1 vs. the Nominal Interest Rate, 1959–2003
Quantity Theory of Money and Monetarism

• Rewrite the velocity formula

\[ M = \frac{1}{V} P Y \]

• If \( V \) is constant, or money demand function is stable, we will have

\[ M = \frac{1}{\bar{V}} P Y \]
Two key elements of monetarism:

1. The money supply is the key measure of the level of aggregate economic activity, in that there is a systematic relationship b/w the money supply and aggregate nominal income.

2. The money supply is the key indicator of monetary policy.
Further, $Y$ is determined by the technology, so it is pretty stable in short-run. This leaves

$$M = \frac{1}{P \bar{Y} \bar{V}}$$
• Hence we have

\[ \frac{\Delta M}{M} = \frac{\Delta P}{P} \]

• Inflation is always a monetary phenomenon!
Policy Implication of Monetarism

• In order to control inflation, we need to control the growth in money supply.
• Motivate Friedman Rule.
Figure 10.16 Central Bank Response Stabilizes Price Level
• But problem occurs when there is unpredictable shifts in money demand functions.
Figure 10.17 Central Bank Does Not Observe the Price Level Response to a Shift in Demand for Money
Monetary Policy Rules

• Facing incomplete information, monetary policy decision rule is costly and time-consuming

• To simplify policy decisions, Fed focuses on a small set of simple monetary policy rules
Money Supply Targeting

- **Friedman Rule**: Central Bank should set a target for the growth rate in some monetary aggregate and then stuck to it forever
- Fed adopted in 1970s and 1980s
- Not very effective in price stability
Nominal Interest Rate Targeting

- Fisher Equation: \[ R = r + i \]
- set a target for the nominal interest rate \[ R^* = r \Rightarrow i = 0 \]
- Mechanism: money demand curve ↑, price level \( P \downarrow \), inflation \( i \downarrow \), nominal interest rate \( R \downarrow \), to keep the target, money supply needs to increase to get \( P \uparrow \)
Nominal Interest Rate Targeting

• Fed is currently using
• FOMC meets every six weeks to set a new target for the nominal Federal Funds Rate (FFR)
Taylor Rule

• Stanford economist John B. Taylor proposed a policy rule to suggest that Fed should set its nominal interest rate target based on the observed behavior of the inflation rate and aggregate output relative to an inflation target and “potential” output, respectively.
Taylor Rule

\[ R_t^* = 2\% + \frac{1}{2} (y_t - y_t^*) + \frac{1}{2} (i_t - 2\%) \]

• “Tight” monetary policy when the economy is steaming and inflation is above the target.