Chapter 9

A Real Intertemporal Model with Investment
Outline

- In this chapter, we will complete a model of real side of the economy.
- We will show how real aggregate output, real consumption, real investment, employment, the real wage, and the real interest rate are determined in the macroeconomy.
Outline

• In this model we will bring together the work-leisure choice from Chapter 4 with the intertemporal consumption behavior (consumption-saving decision) from Chapter 8.
The Representative Consumer

• Utility function

\[ U(C, C', l, l') \]

• Current BC

\[ C + S^p = w(h - l) + \pi - T \]
The Representative Consumer

- Future BC

\[ C' = w'(h-l') + \pi' - T' + (1+r)S^p \]

- Lifetime BC

\[ C + \frac{C'}{1+r} = w(h-l) + \pi - T + \frac{w'(h-l') + \pi' - T'}{1+r} \]
• Representative consumer’s problem

$$\max_{C,C',l,l'} U(C,C',l,l')$$

Subject to lifetime BC
• FOCs

\[
\frac{\partial U}{\partial C} = \lambda, \quad \frac{\partial U}{\partial C'} = \frac{\lambda}{1 + r} \\
\frac{\partial U}{\partial l} = \lambda w, \quad \frac{\partial U}{\partial l'} = \frac{\lambda w'}{1 + r}
\]
The Optimization Conditions

\[
MRS_{l,c} = \frac{\partial U}{\partial l} / \frac{\partial U}{\partial C} = w
\]

\[
MRS_{l',c'} = \frac{\partial U}{\partial l'} / \frac{\partial U}{\partial C'} = w'
\]

\[
MRS_{c,c'} = \frac{\partial U}{\partial C} / \frac{\partial U}{\partial C'} = 1 + r
\]
The Determinants of Current Labor Supply

• In Chapter 4, we learn $N=h-l$ increases when $w$ increases when we assume the substitution effect > income effect.

• $N$ increases when the real interest rate $r$ increases.

Intuition: $w(1+r)/w'$ is the current price of leisure relative to the future price of leisure. $r \uparrow \Leftrightarrow l$ is more expensive, people shift from $l$ to $l'$. (inter temporal substitution of leisure)
• $N$ decreases when the lifetime wealth increases.

Intuition: because leisure is the normal good.
Figure 9.1 The Representative Consumer's Current Labor Supply Curve
Figure 9.2 An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right
Figure 9.3 Effects of an Increase in Lifetime Wealth
The Current Demand for the Consumption Goods

• In Chapter 8, we learned that when current income $Y$ increases, current consumption increases. But MPC is less than one.

• Also in Chapter 8, we knew current consumption $c$ will decrease if real interest rate $r$ increases by assuming the substitution effect dominates in lender’s case.
• Holding constant \( Y \) and \( r \), if lifetime wealth increases, current consumption increases by income effect.
Figure 9.4 The Representative Consumer's Current Demand for Consumption Goods Increases with Income
Figure 9.5 An Increase in the Real Interest Rate from $r_1$ to $r_2$ Shifts the Demand for Consumption Goods Down
Figure 9.6  An Increase in Lifetime Wealth for the Consumer Shifts Up the Demand for Consumption Goods.
The Representative Firm

• Current Production Function

\[ Y = zF(K, N) \]

• Future Production Function

\[ Y' = z'F(K', N') \]
• Model investment goods as being produced from output on a one-to-one basis

\[ K' = (1 - d)K + I \]
• Current profits

\[ \pi = Y - \omega N - I \]

• Future profits

\[ \pi' = Y' - \omega' N' + (1 - d)K' \]
• The Representative Firm’s problem

$$\max_{N,N',I} V = \pi + \frac{\pi'}{1+r}$$
The Optimization Conditions

• Employment decisions

\[ MP_N = z \frac{\partial F}{\partial N} = w \]

\[ MP_N' = z' \frac{\partial F}{\partial N'} = w' \]
Figure 9.7 The Demand Curve for Current Labor Is the Representative Firm's Marginal Product of Labor Schedule
Figure 9.8  The Current Demand Curve for Labor Shifts Due to Changes in Current Total Factor Productivity \( z \) and in the Current Capital Stock \( K \)
The investment decision

- The marginal cost of investment

\[ MC(I) = 1 \]

Because when \( I \) increases one unit, the present value of profits \( V \) decreases one unit accordingly. Hence 1 is the marginal cost of investment.
• The marginal benefit from investment

\[ MB(I) = \frac{MP'_K + 1 - d}{1 + r} \]
• Firm will equate the marginal benefit and marginal cost of investment

\[
\frac{MP'_K + 1 - d}{1 + r} = 1
\]

i.e.

\[
MP'_K - d = r
\]
• Firm will invest until the net marginal product of capital is equal to the real interest rate. This is the **optimal investment rule**.
• Two types of shifts in the optimal investment schedule
  – When \( z' \) increases, optimal investment schedule shifts to the right.
  – When \( K \) is higher, then optimal investment schedule shifts to the left.
Figure 9.9  Optimal Investment Schedule for the Representative Firm

The diagram illustrates the relationship between the real interest rate ($r$) and the demand for investment goods ($I^d$). The marginal product of capital ($MP_K$) minus depreciation ($d$) is depicted as a downward-sloping curve. The optimal investment schedule is shown at points $I_1$ and $I_2$ on the $I^d$ axis, corresponding to real interest rates $r_1$ and $r_2$, respectively.
Figure 9.10 The Optimal Investment Schedule Shifts to the Right if Current Capital Decreases or Future Total Factor Productivity Is Expected to Increase
Optimal Investment: A Numerical Example

• Paula, a small-scale farmer, has an apple orchard. $K=10$.

• 10 trees can produce 100 bushels of apples. $Y=100$.

• At the end of each period, 20% of trees die. $d=0.2$.

• The liquidation price is one tree for one bushel of apples.

• Real interest rate = 5%.
### Table 9.1 Data for Paula’s Orchard

<table>
<thead>
<tr>
<th>$K' = \text{trees in future}$</th>
<th>I</th>
<th>$Y'$</th>
<th>V</th>
<th>$\text{MP}'_k - d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0</td>
<td>95</td>
<td>196.6</td>
<td>—</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>98</td>
<td>199.2</td>
<td>2.8</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>100</td>
<td>200.9</td>
<td>1.8</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>101</td>
<td>201.6</td>
<td>0.8</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>101.5</td>
<td>201.8</td>
<td>0.3</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>101.65</td>
<td>201.7</td>
<td>−0.05</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>101.75</td>
<td>201.6</td>
<td>−0.1</td>
</tr>
<tr>
<td>15</td>
<td>7</td>
<td>101.77</td>
<td>201.4</td>
<td>−0.18</td>
</tr>
</tbody>
</table>
Government

- Government has present-value BC

\[ G + \frac{G'}{1+r} = T + \frac{T'}{1+r} \]
Competitive Equilibrium

In this two-period economy, an CE is an allocation \((c,c',s,l,l')\) for the representative consumer, an allocation \((N,N',I)\) for the representative firm, a policy \((G,G',T,T')\) for the government, and a price system \((w,w',r)\) such that:

- The consumer chooses \(c,c',s,l,l'\) optimally given \(r,w,w'\)
Competitive Equilibrium

• The firm chooses $N, N', I$ to maximize the lifetime profit $V$
• Gov balances its lifetime BC
• Labor market clears: $h-l=N, h-l'=N'$
• Good market clears:
  
  $$C+I+G=Y, C'+G'=Y'$$
Competitive Equilibrium

• We will show how a CE, where supply equals demand in the current-period labor and goods markets, can be expressed in terms of diagrams.
The Current Labor Market and the Output Supply Curve

- Given real interest rate $r$, current labor market clears at the current wage $w^*$ and the current equilibrium employment $N^*$. Hence the equilibrium current output $Y^*$ is determined.

- When $r$ changes, $Y^*$ will change accordingly. We show the relationship in the output supply curve $Y^s$. 
Figure 9.11
Determination of Equilibrium in the Labor Market Given the Real Interest Rate $r$
Figure 9.12 Construction of the Output Supply Curve
• Slope of Output Supply—Real Interest Rate Effects

• Shifts in Output Supply
  – Lifetime Wealth, e.g., an increase in \( G \) or \( G' \) will shift labor supply curve to the right and shifts the output supply curve to the right due to the income effect on labor supply. (less income, less leisure)
– TFP. Increase in z will shift the output supply curve to the right.
– Capital Stock $K$. Same as $z$. 
Figure 9.13 An Increase in Current or Future Government Spending Shifts the $Y^s$ Curve
Figure 9.14 An Increase in Current Total Factor Productivity Shifts the Ys Curve
The Current Goods Market and the Output Demand Curve

- Current good market clearing condition

\[ Y = C^d(r) + I^d(r) + G \]
• From this condition, we can trace out the output demand curve $Y^d$ as a function of real interest rate $r$.
• The slope of $Y^d$ is negative.
Figure 9.15 The Demand for Current Goods

The diagram illustrates the demand for current goods with the following elements:

- The 45° line represents equality between income and consumption.
- The demand curve $C^d(r)$ intersects the 45° line at $Y_1$.
- The demand is given by $C^d(r) + I^d(r) + G$,
  where $I^d(r)$ represents the demand for investment goods.
- The slope of the demand curve is equal to the marginal propensity to consume ($MPC$).

In the context of the diagram, $Y = \text{Current Income}$, and the demand curve shows how consumption varies with income.
Figure 9.16  Construction of the Output Demand Curve

(a) Demand for Current Goods

- $Y_1$
- $Y_2$

45° line

$C^d(r_1) + I^d(r_1) + G$

$C^d(r_2) + I^d(r_2) + G$

$Y = \text{Current Income}$

(b) $r = \text{Real Interest Rate}$

- $r_1$
- $r_2$

$Y^d$

$Y_2$

$Y_1$

$Y = \text{Current Income}$
Shifts in Output Demand

• Current Government Spending ↑
• Present value of taxes ↓
• Future Income ↑
• Future Total Factor Productivity ↑
• Current Capital Stock ↓

Will shift the output demand curve to the right.
Figure 9.17  The Output Demand Curve Shifts to the Right if Current Government Spending Increases
The Complete Real Intertemporal Model

- Equilibrium in the Labor Market

\[ N^d = N^s (r^*) \Rightarrow w^*, N^* \]

- Equilibrium in the Goods Market

\[ Y^d (r) = Y^s (r) \Rightarrow r^*, Y^* \]
Figure 9.18 The Complete Real Intertemporal Model

(a) \( N = \text{Current Employment} \)

(b) \( Y = \text{Current Output} \)
Working with the Model

Two key messages in this chapter

• It does matter that whether the shock is temporary or permanent

• The effects of a shock to the economy expected in the future will have important macroeconomic effects in the current period.

We will use several experiments to show them.
A Temporary Increase in Government Purchases

- Impact Effects ($G \uparrow$, $G'$ unchanged)
  - Labor supply: PV of taxes increases $\Rightarrow$ lifetime wealth decreases $\Rightarrow$ labor supply curve shifts to the right
  - Output supply: shifts to the right.
  - Output demand: drops due to reduction in the lifetime wealth, increases due to $G \uparrow$. But since $MPC < 1$, net effect is increase.
• Equilibrium Effects
  – Goods Market: $Y \uparrow, \ r \uparrow$
  – Labor Market: $w \downarrow, \ N \uparrow$
  – $C \downarrow$ ($\Delta G > \Delta Y$, also $r \uparrow$), $I \downarrow$ ($r \uparrow$), increases in $G$ **crowds out** both the consumption and investment.
  – Recall static model in Chapter 5: $G \uparrow$ leads to $Y \uparrow, \ w \downarrow, \ C \downarrow, \ N \uparrow$
  – Key message: Increase in $G$ comes at a cost!
Figure 9.19 A Temporary Increase in Government Purchases

(a) $w = \text{Current Real Wage}$

(b) $r = \text{Real Interest Rate}$

$N = \text{Current Employment}$

$Y = \text{Current Output}$
Figure 5.6 Equilibrium Effects of an Increase in Government Spending
A Permanent Increase in Government Purchases

- **Impact Effects**
  - Labor Supply: lifetime wealth decreases $\Rightarrow$ labor supply curve shifts to the right
  - Output demand: the increase due to $G \uparrow$ exactly offset the decrease in demand due to the permanent decrease in lifetime income. There is no change on the output demand curve.
  - Another possibility is the increase in output demand curve is exactly offsetting the supply curve to leave $r$ unchanged.
• Equilibrium Effects
  – Goods Market: $Y \uparrow, r \downarrow$
  – Labor Market: $w \downarrow, N \uparrow$
  – $C$? ($r \downarrow$ makes $C \uparrow$, but it might be counted by the decrease in life time wealth), $I \uparrow$ (since real interest rate decreases). Increases in $G$ does not crowd out both the consumption and investment.
Figure 9.20  A Permanent Increase in Government Purchases

(a) \( N = \text{Current Employment} \)

(b) \( Y = \text{Current Output} \)
Test the Theory: WWII

• $G \uparrow$
• Previously we knew during this period
  $Y \uparrow$ and $C \downarrow$
• We can also see $I \downarrow$
• But real interest rate $r$ was quite low, no increase, contradict to the theory.
Figure 5.7  GDP, Consumption, and Government Expenditures

[Graph showing the relationship between GDP, Consumption, and Government Expenditures from 1920 to 2010.]

Copyright © 2005 Pearson Addison-Wesley. All rights reserved.
Figure 9.21  Natural Log of Real Investment, 1929–2002
A Reduction in the Current Capital Stock

• Impact Effects
  – Labor demand: MPL decreases induces the labor demand ↓
  – Output supply: shifts to the left since $N^d$ decreases.
  – Output demand: investment increases since future MPK will increase. Output demand shifts to the right.
• Equilibrium Effects
  – Goods Market: \( r \uparrow, Y? \)
  – Labor Market: \( N?, w\downarrow \)
  – \( C\downarrow \) since \( r \) increases. But \( I? \) because MPK’ increases will induce more investment, while increases in \( r \) will decrease the investment.
  – Think about: Japan and Germany after WWII, Hurricane Katrina
Figure 9.22 The Equilibrium Effects of a Decrease in the Current Capital Stock
An Increase in Current TFP

• Impact Effects
  – Labor demand: MPL increases, shifts the curve to the right
  – Output supply: shifts to the right
• General equilibrium effect
  – Goods market: $Y \uparrow$, $r \downarrow$
  – Labor market: $w \uparrow$, $N$? (likely increase)
  – $C \uparrow$ (both $Y \uparrow$ and $r \downarrow$ increase $C$), $I \uparrow$

• Recall the business cycle facts, the model predicts that consumption, employment (likely), investment, and the real wage are all procyclical, just as in the data.
Figure 9.23 The Equilibrium Effects of an Increase in Current Total Factor Productivity
• Recall static model in Chapter 5, $z \uparrow$
  – Increased Consumption
  – Leisure and Hours Worked may Rise or Fall
  – Increased Output
  – Higher Real Wage

• Chapter 9 adds in *intertemporal* choice, hence we can see the effects on $r$ and $l$
Figure 5.9 Competitive Equilibrium Effects of an Increase in Total Factor Productivity

The diagram illustrates the effects of an increase in total factor productivity on the competitive equilibrium. The graph shows the consumption function, with consumption increasing as total factor productivity increases. The shaded areas represent the new equilibrium positions after the productivity change, compared to the original equilibrium at point A. The labels B, C1, C2, D, H, F, I1, I2, and G indicate key points on the graph, with G representing the original productivity level and I1 and I2 indicating investment levels. The change in productivity shifts the indifference curve, leading to a new equilibrium at point H.
An Increase in Future TFP

• Impact Effects
  – Output demand: \( z' \uparrow \) will increase MPK’, so firm wants to invest more
  – Labor supply: \( r \uparrow \) will make it shift to the right
• Equilibrium Effects
  – Goods market: $Y \uparrow$, $r \uparrow$
  – Labor market: $w \downarrow$, $N \uparrow$
  – $C\,?,\, l \uparrow$. $C$ is ambiguous since $r \uparrow$ will cause $C$ to fall, but $Y \uparrow$ will increase current consumption.
Figure 9.24 The Equilibrium Effects of an Increase in Future Total Factor Productivity

(a) \( N = \text{Current Employment} \)

(b) \( Y = \text{Current Output} \)
The “New Economy” and Stock Market Bust

• Increasing optimism about future TFP leads to increases in $I$ and $Y$, is also reflected in increases in stock prices.

• Increasing pessimism exactly does the opposite.
Figure 9.25  Percentage Deviations From Trend in GDP and Investment, 1990–2003
Figure 9.26  Investment as a Percentage of GDP, 1990--2003
Figure 9.27  Standard and Poor's 500 Stock Price Index, 1990--2003
Sectoral Shocks

- A sectoral shock is a shift in production from one sector of the economy to another, caused by a shift in demand or a change in relative productivities between sectors.

- Impact effect: Labor supply curve will shift to the left due to the temporary dislocation of labor from the declining sector. But there is no effect on the aggregate demand for the consumption and investment.
• General equilibrium effect
  – Goods market: $Y \downarrow$, $r \uparrow$
  – Labor market: $w \uparrow$, $N \downarrow$
  – The shock is temporary. Once the workers are reallocated, the economy returns to its initial equilibrium.
Figure 9.28 The Effects of a Sectoral Shock

(a) $w_1$ to $w_2$ changes in the current real wage
(b) $r_1$ to $r_2$ changes in the real interest rate

$N = \text{Current Employment}$

$Y = \text{Current Output}$
Discussion

• What is the macro effect of the failure of GM?
Figure 9.29 Percentage Deviations From Trend in GDP and Employment, 1990–2003