Chapter 8

A Two-Period Model: The Consumption-Savings Decision and Ricardian Equivalence
In this chapter we will introduce the **intertemporal choice**. More specifically, consumption-saving decision over time.

Solow model has done it in an arbitrary way.

While we will solve it as an optimization results.
• We will also talk about an important implication of this model—Ricardian Equivalence Theorem.
• We start from the simplest framework: a two-period model
The Two-period Model

- First period: current
- Second period: future
- No production and investment (no $K$)
Consumer Behavior

- $N$ consumers
- Receive $y$ in the current period, $y'$ in the future period. Paid tax $t$ and $t'$ in two periods respectively.
Budget Constraint

• Current period budget constraint

\[ c + s = y - t \]

\[ c \geq 0, \text{ } s \text{ can } >, \text{ or } < 0 \]
• Second period budget constraint
  \[ c' = y' - t' + (1+r)s \]

• The life time budget constraint
  \[ c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \left( t + \frac{t'}{1+r} \right) \]
• It says
  present value of lifetime consumption
  $\bullet$ present value of lifetime income –
  present value of lifetime tax
• Call the RHS the lifetime wealth we
  \[ c + \frac{c'}{1+r} = we \]
Figure 8.1 Consumer's Lifetime Budget Constraint
Preferences

• Still keep the same properties as before, in stead that now is a trade-off b/w current and future consumption.
  – More is Preferred to Less (increasing)
  – Consumers Value Diversity (concave)
  – Current and Future Consumption are Normal Goods
Figure 8.2 A Consumer's Indifference Curves
• Concavity of utility function (or convexity of the indifference curve) determines consumer’s desire to smooth consumption over time
Table 8.1 Sara’s Desire for Consumption Smoothing

<table>
<thead>
<tr>
<th></th>
<th>Week 1 Coconuts</th>
<th>Week 2 Coconuts</th>
<th>Total Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bundle 1</td>
<td>5</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Bundle 2</td>
<td>17</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>Preferred Bundle</td>
<td>11</td>
<td>9</td>
<td>20</td>
</tr>
</tbody>
</table>
Consumer Optimization

The optimization condition

\[ MRS = \frac{MU_c}{MU_{c'}} = 1 + r \]
Figure 8.3  A Consumer Who Is a Lender

\[ \text{we}(1 + r) \]

\[ c^* \]

\[ y' - t' \]

\[ c' \]

\[ c = \text{Current Consumption} \]

\[ c^* \]

\[ y - t \]

\[ \text{we} \]
Figure 8.4 A Consumer Who Is a Borrower
An Increase in $y$

- The effect of $y \uparrow$ on $c$, $c'$ and $s$
  - Current consumption $c$ increases
  - Future consumption $c'$ increases
  - Saving $s$ increases

\[ \Delta s = \Delta y - \Delta t - \Delta c \]

\[ \Delta y > \Delta c > 0, \Delta s > 0 \]
• Why $s \uparrow$?
  – Consumer wants to smooth the consumption over time
• The theory predicts aggregate consumption is less volatile than aggregate output.
• Recall the business cycle facts, it is true.
• But the theory also predicts that if people are rise averse enough, they should fully smooth consumption, \( c = c' \).

• But in the data, the relative volatility is 0.67, not zero, i.e. consumption is not quite smooth enough to tightly match the theory. Why?
• Two possible explanations
  – Imperfections in credit market, for example, borrowing constraints.
  – The effect on the market prices when all consumers want to smooth consumption simultaneously.
Figure 8.5 The Effects of an Increase in Current Income for a Lender
Figure 8.6 Percentage Deviations from Trend in GDP and Consumption, 1947–2003
An Increase in $y'$

- The effect of $y' \uparrow$ on $c$, $c'$ and $s$
  - Current consumption $c$ increases
  - Future consumption $c'$ increases
  - Saving $s$ decreases

\[
\Delta s = \Delta y - \Delta t - \Delta c
\]
\[
\Delta y = \Delta t = 0, \Delta c > 0, \Delta s < 0
\]
Figure 8.7 An Increase in Future Income
Permanent Income Hypothesis

- Consumer’s response will be different when they face temporary or permanent changes in income.
- Think about the example. You win lottery $1000 only this year, or you win lottery $1000 every year. Does this make difference in your consumption behavior?
• Milton Friedman claims a primary determinant of a consumer’s current consumption is his or her permanent income (we in our model). Temporary changes in income yield small changes in permanent income, hence will have small effects on current consumption. While permanent changes in income will have large effects on we and current consumption.
Figure 8.8  Temporary Versus Permanent Increases in Income
• In our two-period setting, permanent increases in income will have a larger effect on current consumption.

• If the increase is temporary, we see saving increases; while if the increase is permanent, there need not be an increase in saving.
Figure 8.9 Stock Prices and Consumption of Nondurables and Services, 1985–2003
Figure 8.10 Scatter Plot of Percentage Deviations from Trend in Consumption of Nondurables and Services Versus Percentage Deviations from Trend in a Stock Price Index
An Increase in $r$

- Budget constraint pivots around the endowment point.
- Income effects vs. substitution effects
• \( r \uparrow \) for lender
• Substitution effect: \( c \downarrow, c' \uparrow \) (recall the relative price of future consumption is \( 1/(1+r) \))
• Income effect: \( c \uparrow, c' \uparrow \)
• Total effect: \( c?, c' \uparrow, s? \)
Figure 8.11  An Increase in the Real Interest Rate
Figure 8.12 An Increase in the Real Interest Rate for a Lender
• $r \uparrow$ for borrower
• Substitution effect: $c \downarrow$, $c' \uparrow$ (recall the relative price of future consumption is $1/(1+r)$)
• Income effect: $c \downarrow$, $c' \downarrow$
• Total effect: $c \downarrow$, $c' \uparrow$, $s \uparrow$
Figure 8.13  An Increase in the Real Interest Rate for a Borrower
An Example: Perfect Complements

\[ c' = ac \]

\[ c + \frac{c'}{1 + r} = we \]

\[ c = \frac{we(1 + r)}{1 + r + a} , c' = \frac{awe(1 + r)}{1 + r + a} \]
Figure 8.14 Example with Perfect Complements Preferences
The Demand Function of $c$

- When $y \uparrow$, $c \uparrow$. So we can express $c^d$ as a function of current income $y$.
- But we have $\Delta y > \Delta c > 0$, which means Marginal Propensity to Consume (MPC) $< 1$.
- Change in $r$ affects the shifts of the demand curve.
Figure 8.15 A Consumer's Demand for Current Consumption Goods, $c^d$, as a Function of Current Income
Figure 8.16  A Shift in a Consumer's Demand for Current Consumption

c^d = Demand for Current Consumption

y = Current Income
Add in Government

- Government collect current taxes
  \[ T = N_t \]
- Future taxes
  \[ T' = N_{t'} \]
- Government can also issue bonds \( B \), bearing the interest rate \( r \)
• Government current period budget constraint

\[ G = T + B \]

• Future period budget constraint

\[ G' + (1+r)B = T' \]
• Hence government has present-value budget constraint

\[ G + \frac{G'}{1+r} = T + \frac{T'}{1+r} \]
Competitive Equilibrium

In this two-period economy, an CE is an consumption allocation \((c, c')\), an saving decision \(s\) for \(N\) consumers, and a real interest rate \(r\) such that

- Each consumer chooses \(c, c'\) and \(s\) optimally given \(r\)
• The government present-value BC holds.
• The credit market clears.

\[ S_p^p = NS = B = -S^g \]

\[ S_p^p + S^g = S \]
• National income identity

\[ Y = N_y = C + G \]
The Ricardian Equivalence Theorem

- If current and future government spending are held constant, then a change in current taxes with an equal and opposite change in the present value of future taxes leaves the equilibrium real interest rate $r$ and the consumption of individuals unchanged.
• Put in this way: a change in the timing of taxes by the government is neutral.
• It is a very strong result!
• Why? Because a tax cut is not a free lunch.
• Proof

Gov’s present-value BC

\[
G + \frac{G'}{1 + r} = Nt + \frac{Nt'}{1 + r}
\]

\[\Rightarrow\]

\[
t + \frac{t'}{1 + r} = \frac{1}{N} \left( G + \frac{G'}{1 + r} \right)
\]
• Substitute into the consumer’s life-time BC

\[ c + \frac{c'}{1+r} = y + \frac{y'}{1+r} - \frac{1}{N} \left( G + \frac{G'}{1+r} \right) \]
• Now consider the experiment as following

\[ \Delta t < 0, \Delta t' = - \frac{\Delta t}{1 + r} > 0 \]

• There is no change in Gov’s BC, so does the consumer’s life-time BC.
• Hence \( c, c' \) and \( r \) stay the same.
• But private saving \( s \) increases.

\[
\Delta s = \Delta y - \Delta t - \Delta c = -\Delta t > 0
\]

• Consumers anticipate the increases in future taxes, so they increase their savings by the amount of the tax cut.
Figure 8.17 Ricardian Equivalence with a Cut in Current Taxes for a Borrower
• But in the reality, the government deficits (tax cut) do matter because
  – The Distribution of Taxes across Different Individuals
  – The Distribution of Taxes across Different Generations
  – Distorting Taxes
  – Credit-market Imperfections
What Happens When RET Does Not Hold?

• We will relax the assumptions of RET, especially focus on intergenerational redistribution (Social Security System) and the credit market imperfections.
Social Security: Pay-As-You-Go System

• Taxes on the young are used to finance social security transfers to old.
• Assume each consumer lives for two periods: young and old age.
• So in any period there are $N$ old consumers and $N'$ young consumers alive.
• We have
  \[ N' = (1+n)N \]
• No PAYG
  \[ T = T' = G = G' = 0 \]
• With PAYG, old receive transfer \( b \).
  \[ t' = -b \]
The benefits for old consumers must be financed by the taxes on the young

\[ Nb = N' t \]

\[ t = \frac{b}{N' / N} = \frac{b}{1 + n} \]
By introducing PAYG, olds are clearly better off

\[ we = y + \frac{y'+b}{1+r} > y + \frac{y'}{1+r} \]

But the welfare effects on the young is ambiguous. If \( n>r \), better off. Vise Versa.

\[ we = y - \frac{b}{1+n} + \frac{y'+b}{1+r} = y + \frac{y'}{1+r} + \frac{b(n-r)}{(1+r)(1+n)} \]

PAYG can be Pareto improvement because it facilitates the intergenerational transfer.
Figure 8.18 Pay-As-You-Go Social Security for Consumers Who Are Old in Period T
Figure 8.19  Pay-As-You-Go Social Security for Consumers Born in Period T and Later
Baby Boom and PAYG

• When “Baby Boomers” are at working age, more likely $n > r$, so PAYG is Pareto improvement.

• But when “Baby Boomers” enter into retirements, roughly between 2010 and 2030, then working generation at that time (which is us!) needs to pay higher social security tax, they will be worse off.
Fully Funded Social Security

• FFSS is a forced savings program.
• Government invests the proceeds from social security taxes in the private credit market, the social security benefits are determined by the payoff received.
• Government will also allow the consumer to choose in which assets to invest his or her social security savings.
• If this forced saving is below the saving without any social security, then FF has no effects.

• FFSS only matters when the social security system mandates a higher level of saving than the consumer would choose in the absence of the program.
Figure 8.20 Fully Funded Social Security When Mandated Retirement Saving Is Binding
Credit Market Imperfections

- For a consumer, the real interest rate of lending \((r_1)\) might be less than the rate of borrowing \((r_2)\).
- The difference b/w the borrowing and lending rates of interest leads to a more complicated lifetime budget constraint.
• For a lender \((s>0)\), the future period BC
\[c' = y' - t' + s(1 + r_1)\]
• For a borrower \((s<0)\), the future period BC
\[c' = y' - t' + s(1 + r_2)\]
• For a lender, the lifetime BC
\[
c' + \frac{c'}{1+r_1} = y + \frac{y'}{1+r_1} - \frac{t}{1+r_1} - \frac{t'}{1+r_1} = w_1
\]
• For a borrower, the lifetime BC
\[
c' + \frac{c'}{1+r_2} = y + \frac{y'}{1+r_2} - \frac{t}{1+r_2} - \frac{t'}{1+r_2} = w_2
\]
Figure 8.21 A Consumer Facing Different Lending and Borrowing Rates
• For this credit-constrained economy, there is a significant number of consumers whose optimal consumption bundle is the endowment point. (Why?)

• Now the tax cut today will make them increase the consumption by the exact amount. RET fails.
• Because tax cut acts as having the gov. loan the consumer $-\Delta t$ at the interest rate $r_1$.

• If credit market imperfections matter significantly, then tax policy can be used to increase the economic welfare.
Figure 8.22 Effects of a Tax Cut for a Consumer with Different Borrowing and Lending Rates
George H. W. Bush and the Withholding Reduction

• On Jan. 28, 1992, President Bush proposed tax withholding reduction.
• But RET works, there is no sharp increase or decrease in consumptions.
Figure 8.23  Real Consumption of Durables, 1991–1993
Figure 8.24  Real Consumption of Nondurables, 1991–1993
Figure 8.25  Real Consumption of Services, 1991–1993