Two Primary Phenomena that Macroeconomists study are:

• Economic Growth
• Business Cycle
Economic Growth is Important!

• If business cycles could be completely eliminated, the worst events we would able to avoid would be deviation from the trend of GDP by 5%.

• If changes in economic policy could cause the growth rate of real GDP to increase by 1% per year to 100 years, the GDP would be 2.7 times higher than it would otherwise have been.
Economic Growth Facts

• Pre-1800 (Industrial Revolution): constant per capita income across time and space, no improvement in standards of living.

• Post-1800: Sustained Growth in the Rich Countries. In the US, average growth rate of GDP per capita has been about 2% since 1869.
Figure 6.1 Natural Log of Real per Capita Income in the United States, 1869–2002
Economic Growth Facts Con’d

• High Investment ↔ High Standard of Living

• High Population Growth ↔ Low Standard of Living

Figure 6.2  Output per Worker vs. Investment Rate

The scatter plot shows the relationship between output per worker and investment rate, with data points indicating a positive correlation.
Figure 6.3  Output per Worker vs. the Population Growth Rate
Economic Growth Facts Con’d

• No conditional Convergence amongst all Countries
• (Weakly) Conditional Convergence amongst the Rich Countries
• No Conditional Convergence amongst the Poorest Countries
Figure 6.4 No Convergence Among All Countries
Figure 6.5 Convergence Among the Richest Countries
Figure 6.6  No Convergence Among the Poorest Countries
The Malthusian Model

• Idea was provided by Thomas Malthus in his highly influential book *An Essay on the Principle of Population* in 1798.

• He argued technological change ⇒ improvement in standard living ⇒ population growth ⇒ reduce the average person to the subsistence level again.
• In the long run there would be no increase in the standard of living unless there were some limits on population growth.

• It is a pessimistic theory!
The Malthusian Economy

- Production technology

\[ Y = zF(L, N) \]

- \( L \) is the fixed amount of land, \( N \) is the labor input. \( F \) has all the nice properties (Recall Chapter 4)
- No investment technology (no refrigerator, food perish)
- No government
• No leisure in the utility function.

\[ U(C) = C \]

• We normalize the labor endowment of each person to be 1, so \( N \) is both the population size and the labor input
• Assume the population growth depends on the quantity of consumption per worker (standard of living)

\[
\frac{N'}{N} = g \left( \frac{C}{N} \right)
\]

\(g\) is a increasing function
Figure 6.7  Population Growth Depends on Consumption per Worker in the Malthusian Model
• In equilibrium, we have

\[ C = Y \]

Hence

\[ C = zF(L, N) \]
\[
\frac{N'}{N} = g\left(\frac{zF(L,N)}{N}\right) = g\left(zF\left(\frac{L}{N},1\right)\right)
\]
Steady State

- When $N' = N$, we say the economy reaches the **steady state** (SS).
- In SS, $N = N^*$, $C^* = zF(L, N^*)$.
- Define variable in terms of per capita, for example, $y = Y/N$, $c = C/N$, $l = L/N$. We have

$$y = f(l) \quad (f(l) = F(l, 1))$$
• In equilibrium, $c=y$. Hence we have
  \[ c = zf(l) \]  \hspace{1cm} (1)
• Law of motion of population
  \[ \frac{N'}{N} = g(c) \]  \hspace{1cm} (2)
• (1) + (2) consist the dynamic economic system for this economy
• In SS, \( N'/N=1 \), this determines the SS value of consumption per capita \( c \)

\[ 1 = g(c^*) \]

• Then in equation (1), \( c^* \) in turn determines \( l^* \) through

\[ c^* = zf(l^*) \]

• Finally, the SS population size \( N^* \) is determined by

\[ N^* = L/l^* \]
Figure 6.8  Determination of the Population in the Steady State
Figure 6.9 The Per-Worker Production Function

Output Per Worker, $y$

Land Per Worker, $l$

$zf(l)$
Figure 6.10 Determination of the Steady State in the Malthusian Model
The Effect of TFP↑ on the SS

• Do not improve the standard of living $c^*$ in the long run ( $c^*$ is determined by $1 = g(c^*)$ )

• Only increases the population ($l^*\downarrow$, $N^*\uparrow$)
Figure 6.11 The Effect of an Increase in $z$ in the Malthusian Model
Figure 6.12
Adjustment to the Steady State in the Malthusian Model When \( z \) Increases
Policy Implication: Population Control

- Government directly controls the population growth: $g(c) \downarrow$
- In SS, $c_1^* \rightarrow c_2^*$. Standard of living increases.
- The quantity of land per worker increases too, $l_1^* \rightarrow l_2^*$. That leads to the SS population size decreases $N_1^* \rightarrow N_2^*$.
- Theoretical foundation of Chinese “One Child” policy.
Figure 6.13  Population Control in the Malthusian Model

(a) Consumption Per Worker, $c$

(b) Population Growth, $N/N$

Land Per Worker, $l$

Consumption Per Worker, $c$

$z(l)$
Evaluation of Malthusian Model

- Consistent with the growth facts before 1800: production was mainly agricultural, population grew over time, but no significant improvements in the average standard of living
- What did happen after 1800?
  - Sustained growth in standards of living in the richest countries
  - The richest countries also have experienced a large drop in birth rates
• Malthus was wrong on these two dimensions
  – He did not allow for the effect of increases in $K$ on production. Capital can produce itself.
  – He did not account for all of the effects of economic forces on population growth. As economy develops, the opportunity cost of raising a large family becomes large. Fertility rate decreases.

• We need a GROWTH theory!
Source: Fernandez-Villaverde (2001)
Source: Fernandez-Villaverde (2001)
Source: Bar and Leukhina (2005)
The Solow Model: Exogenous Growth

• Consumers
  – Utility function: $U(C) = C$
  – Budget Constraint: $C + S = Y$ (Why?)
  – Consumers have to make consumption-saving decisions
  – We assume the consumers consume a constant fraction of income in each period

$$C = (1-s)Y, \quad S = sY$$
• Firm
  – Production function $Y = zF(K, N)$
  – It has all of the properties we discussed in Chapter 4 (CRS, increasing, concave, …)

• We can rewrite everything in terms of per capita variables

$$\frac{Y}{N} = z F \left( \frac{K}{N}, 1 \right)$$

$$y = z f \left( k \right)$$
• The capital stock evolves according to

\[ K' = (1-d)K + I \]

\( I \) is the investment. \( 0 < d < 1 \) is the depreciation rate.
Figure 6.14 The Per-Worker Production Function

\[ y = \text{Output Per Worker} \]

\[ k = \text{Capital Per Worker} \]

Slope = \( MP_K \)

\[ y = zf(k) \]
Competitive Equilibrium

- In equilibrium, \( S=I \)
- So we have \( Y=C+I \)
- \( Y=(1-s)Y+K'-(1-d)K \)
- \( K'=sY+(1-d)K \)
- \( K'/N=szF(K,N)/N+(1-d)K/N \)
- \( (K'/N')(N'/N)=szF(K/N,1)+(1-d)K/N \)
- Assume the population growth rate is \( n \). We have \( N'=(1+n)N \)
\[ k' = \frac{szf(k)}{1+n} + \frac{(1-d)k}{1+n} \]

• When \( k' = k \), we reach the steady state (SS).

• Solow model predicts that eventually \( k \) will converge to SS value \( k^* \)
Figure 6.15 Determination of the Steady State Quantity of Capital per Worker

\[
\begin{align*}
\frac{szf(k)}{1 + n} + \frac{(1 - d)k}{1 + n}
\end{align*}
\]
Model Prediction

• There is no long run economic growth in per capita variables.
• But there is a long run economic growth rate in aggregate variables. (If $n,s,z$ are constant.)

\[
K = k\cdot N, \quad K' = k\cdot N' \implies \frac{K'}{K} = \frac{N'}{N} = 1 + n, \quad \text{so}
\]
\[
\frac{(K' - K)}{K} = n
\]

\[
Y = y\cdot N = zf(k^*)N \implies \frac{Y'}{Y} = 1 + n
\]

Since $S = I = s\cdot Y \implies \frac{S'}{S} = 1 + n$

Same as $C$
• All aggregate variables grow at the rate $n$!

• This is the reason why Solow model is an *exogenous* growth model. The long-run growth is determined by exogenous labor force growth.
Analysis of the Steady State

• In SS, \( k' = k = k^* \). So we have

\[
k^* = \frac{szf(k^*)}{1 + n} + \frac{(1 - d)k^*}{1 + n}
\]

or

\[
szf(k^*) = (n + d)k^*
\]
Figure 6.16 Determination of the Steady State Quantity of Capital per Worker
Experiment: The Effect of $s \uparrow$ in SS

- The SS level of per capita capital stock $k^*$ will increase. Hence $c^*, y^*$ also increase.

- It predicts a positive relation b/w $s$ (investment rate) and $y$ (GDP per capita). $\Rightarrow$ Confirmed by data!

- But there is no change in the growth rates of the aggregate variables (still $n$).
Figure 6.17 Effect of an Increase in the Savings Rate on the Steady State Quantity of Capital per Worker
**Figure 6.18** Effect of an Increase in the Savings Rate at Time T
Consumption per Worker and Golden Rule

• In SS, the consumption per worker is
  \[ c = (1-s)zf(k^*) = zf(k^*) - (n+d)k^* \]

• The golden rule quantity of capital per worker \( k^*_{gr} \) is \( k \) such that \( c \) is maximized
  \[ MP_k = n + d \]
Figure 6.19 Steady State Consumption per Worker

c = (1 - s)f(k^*)

(n + d)k^*

zf(k^*)

szf(k^*)

k^*

k_1^*
Figure 6.20
The Golden Rule Quantity of Capital per Worker

\[ \text{Slope} = n + d \]

\[ (n + d)k^* \]

\[ zf(k^*) \]

\[ s_y zf(k^*) \]

\[ k_{gr}^* \]

\[ k^* \]

\[ c = \text{Consumption Per Worker} \]

\[ c^* \]

\[ k_{gr}^* \]

\[ k^* = \text{Steady State Capital Per Worker} \]
Experiment: The Effect of $n \uparrow$ in SS

- The SS quantity of capital per worker ($k^*$) decreases.
- $y^*$ and $c^*$ also decrease. Hence $n$ (population growth rate) is negatively correlated with $y$. $\Rightarrow$ Confirmed by data
- But the aggregate variables $Y, K, C$ all grow at higher rate
Figure 6.21  Steady State Effects of an Increase in the Labor Force Growth Rate
The prediction of Solow Model

- Solow model predicts saving rate (investment rate) $\uparrow \Rightarrow y \uparrow$, and $n \uparrow \Rightarrow y \downarrow$
- It is consistent with the data (recall it)
Experiment: The Effects of TFP↑

• To make \( y \uparrow \) continuously, we need \( s \uparrow \) and \( n \downarrow \) continuously. But sooner or later, they will hit the boundary.

• To make an unbounded long run growth, we need TFP (or \( z \) \( \uparrow \))

• TFP \( \uparrow \) \( \Rightarrow \) \( k \uparrow \), hence \( y, c \uparrow \)

• Now recall what Malthus model says about the TFP\( \uparrow \), we can have long-run growth now with Solow model
Figure 6.22 Increases in Total Factor Productivity in the Solow Growth Model
Growth Accounting

• Typically, growing economies are experiencing growth in factors of production and in TFP.

• A natural question is can we measure how much of the growth in $Y$ is accounted for by growth in each of the inputs to production and by increases in TFP.
• We call this exercise is **Growth Accounting**.

• Start from aggregate production function

\[ Y = zK^\alpha (N)^{1-\alpha} \]

• Profit maximization implies

\[ MP_N = (1-\alpha)zK^\alpha (N)^{-\alpha} = w \]

\[ wN = (1-\alpha)zK^\alpha (N)^{1-\alpha} = (1-\alpha)Y \]
• (1-\(a\)) is the share of labor incomes in GDP. In postwar US data, it is 0.64.
• Similarly, \(a=0.36\) is the capital share in national income.
• Hence the production function is

\[ Y = zK^{0.36} (N)^{0.64} \]
The \( z \), called Solow residual, is measured from the production

\[
Z = \frac{Y}{K^{0.36} (N)^{0.64}}
\]
Table 6.1  Average Annual Growth Rates in the Solow Residual

<table>
<thead>
<tr>
<th>Years</th>
<th>Average Annual Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950–1960</td>
<td>1.45</td>
</tr>
<tr>
<td>1960–1970</td>
<td>1.60</td>
</tr>
<tr>
<td>1970–1980</td>
<td>0.50</td>
</tr>
<tr>
<td>1980–1990</td>
<td>1.02</td>
</tr>
<tr>
<td>1990–2000</td>
<td>1.33</td>
</tr>
</tbody>
</table>
Figure 6.23  Natural Log of the Solow Residual, 1948–2001
Figure 6.24 Percentage Deviations from Trend in Real GDP (black line) and the Solow Residual (colored line), 1948–2001
Growth Accounting Decomposition

• Take a natural log on aggregate production function

\[
\ln Y = \ln z + 0.36 \ln K + 0.64 \ln N
\]

• Take first order derivatives w.r.t. time \( t \) on both sides

\[
\frac{\dot{Y}}{Y} = \frac{\dot{z}}{z} + 0.36 \frac{\dot{K}}{K} + 0.64 \frac{\dot{N}}{N}
\]
• Growth rate of output =
  Growth rate of TFP
  + 0.36 * Growth rate of capital
  + 0.64 * Growth rate of labor
Table 6.2  Measured GDP, Capital Stock, Employment, and Solow Residual

<table>
<thead>
<tr>
<th>Year</th>
<th>( \hat{Y} ) (billions of 1996 dollars)</th>
<th>( \hat{K} ) (billions of 1996 dollars)</th>
<th>( \hat{N} ) (millions)</th>
<th>( \hat{z} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>1686.6</td>
<td>5553.1</td>
<td>58.89</td>
<td>5.574</td>
</tr>
<tr>
<td>1960</td>
<td>2376.7</td>
<td>7920.9</td>
<td>65.78</td>
<td>6.439</td>
</tr>
<tr>
<td>1970</td>
<td>3578.0</td>
<td>11547.1</td>
<td>78.67</td>
<td>7.548</td>
</tr>
<tr>
<td>1980</td>
<td>4900.9</td>
<td>15922.3</td>
<td>99.30</td>
<td>7.934</td>
</tr>
<tr>
<td>1990</td>
<td>6707.9</td>
<td>20871.1</td>
<td>118.80</td>
<td>8.784</td>
</tr>
<tr>
<td>2000</td>
<td>9191.4</td>
<td>26993.8</td>
<td>136.90</td>
<td>10.027</td>
</tr>
</tbody>
</table>
## Table 6.3 Average Annual Growth Rates

<table>
<thead>
<tr>
<th>Years</th>
<th>$\hat{Y}$</th>
<th>$\hat{K}$</th>
<th>$\hat{N}$</th>
<th>$\hat{Z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950–1960</td>
<td>3.49</td>
<td>3.62</td>
<td>1.11</td>
<td>1.45</td>
</tr>
<tr>
<td>1960–1970</td>
<td>4.18</td>
<td>3.84</td>
<td>1.81</td>
<td>1.60</td>
</tr>
<tr>
<td>1970–1980</td>
<td>3.20</td>
<td>3.27</td>
<td>2.36</td>
<td>0.50</td>
</tr>
<tr>
<td>1980–1990</td>
<td>3.19</td>
<td>2.74</td>
<td>1.81</td>
<td>1.02</td>
</tr>
<tr>
<td>1990–2000</td>
<td>3.20</td>
<td>2.61</td>
<td>1.43</td>
<td>1.33</td>
</tr>
</tbody>
</table>
An Example: East Asian Miracles

• Alwyn Young did a growth accounting exercise for “Four Little Dragons”
• Found high rates of GDP growth in these countries were mainly due to high growth rates in factor inputs.
• Implication: East Asian Miracle is probably not sustainable over a longer period. (Japan recession in 1990s, South Korea Financial Crisis…)
Table 6.4 East Asian Growth Miracles (Average Annual Growth Rates)

<table>
<thead>
<tr>
<th>Country</th>
<th>Output</th>
<th>Capital</th>
<th>Labor</th>
<th>Total Factor Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hong Kong (1966–1991)</td>
<td>7.3%</td>
<td>7.7%</td>
<td>2.6%</td>
<td>2.3%</td>
</tr>
<tr>
<td>Singapore (1966–1990)</td>
<td>8.7%</td>
<td>10.8%</td>
<td>4.5%</td>
<td>0.2%</td>
</tr>
<tr>
<td>South Korea (1966–1990)*</td>
<td>10.3%</td>
<td>12.9%</td>
<td>5.4%</td>
<td>1.7%</td>
</tr>
<tr>
<td>Taiwan (1966–1990)*</td>
<td>9.4%</td>
<td>11.8%</td>
<td>4.6%</td>
<td>2.6%</td>
</tr>
<tr>
<td>United States (1966–1990)</td>
<td>3.0%</td>
<td>3.2%</td>
<td>2.0%</td>
<td>0.6%</td>
</tr>
</tbody>
</table>