Health and Portfolio Choice in Retirement: The Impact of Ambiguity

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Abstract

This paper first documents that in the U.S., the stock market participation rate over the life-cycle decreases as people age. This fact, however, can not be captured by standard model where smooth expected utility function predicts the decision maker stay in the stock market, given positive equity premium and independence between older people's non-asset income and stock return. This paper successfully replicates this fact by introducing Knightian uncertainty into a dynamic model with choices on medical care and consumption, as well as saving and portfolio. We assume the agents in the model have ambiguity towards the correlations between risky stock return and risky medical price inflation. In this environment, retirees quit the stock market under reasonable range of ambiguity towards the correlation. The key mechanism is that: The agents do not hold positive amount of stocks since they worry stocks co-vary positively with the non-asset income minus the health expenses. Similarly, they do not short stocks as they also worry that stocks and their de facto non-asset income may co-move negatively.

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1 Introduction

The older people hold disproportional amount of wealth. And health risk is the primary non-financial risk that older people are facing. In public policy, the fresh Health Care Bill can change the distribution of health expenditure that older people are exposed to and might so change their financial asset demand. Despite the grave importance of health risk’s impact on older people’s financial decision, relatively little has been done to help us understand on these questions: how does health affect people’s financial decision, and through what channel?

This paper answers the above questions by first documenting evidence on older people’s portfolio choice and health status. We find evidence that suggests that relatively more healthy people are more likely to take financial risks by investing in stocks, and that relatively more healthy people are more likely to invest large share of their wealth into stocks than unhealthy people. If we focus on the participation margin, we observe that even among households with significant amount of financial wealth, the participation rate in stock market starts to drop at age 60. This fact suggests that some older people are divesting their stock holdings completely to safe assets. And the dropout happens mostly to unhealthy and not wealthy group.

Previous literature on life cycle portfolio choice, however, failed to reconcile this dropout. Not only in the environment without health risk, such as Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2006), and Davis, Kubler, and Willen (2006), but also in the environment with health risk, such as Yogo (2009), the literature has predicted 100% participation rate for retirees. The reason for the failure is based on the assumption that health expenditure risk is orthogonal to financial risk. With this assumption, and with positive equity premium, the decision maker in those economy will always have incentives to invest in stocks when their non asset income risk has been resolved. Intuitively, given the fact that stocks are good deals, and with background risk orthogonal to stock risk, the first dollar they save should actually go to stock account. With such strong incentive to participate in stock market as retirees, the frictions introduced to these models can not help to explain dropout either. More specifically, These literature fails to explain, 1) Why health matters in portfolio choice problem; and 2) Why older people with significant wealth are not investing stocks. For instance, Vissing-Jorgensen tries to introduce different cost structures such as on-going participation cost or transaction cost to account for nonparticipation. However, there isn’t a sound cost structure yet to explain why some people with positive wealth are not investing in stocks. And this cost structures have no potential to account for different portfolio composition by people with different health status.

This paper develops a theoretical model to explain the drop out in the stock market among older people. We use a model of ambiguity with maxmin utility, where the ambiguity is towards the correlation between risky stock return and risky relative medical price inflation. Due to the ambiguity, agent is not sure the exact correlation between his de facto income and stock return. Because of maxmin utility, agent might worry that the income is very much like stock, and thus will not long in stocks. In his mind, the following scenario is his nightmare: that when he gets sick, is also the same time when stock return is low. And this
worry can prevent him from longing stocks. Also, the agent is worried that after he shorts in stock, if he gets sick, needing money to hospital bills, is also the time the stock price is high and have to purchase stocks back to clear the short position. The worries from both extreme cases thus prevent the agent from either longing in stocks or shorting. And thus we have non-participants when agent is aged.

This paper conducts quantitative exercise by using the CRRA utility function and using historical correlations between stock return and medical inflation as the set of agents’ belie. The quantitative exercise predicts a reasonable threshold on real medical expenditure, above which the households might choose to quit the stock market completely. This quantitative exercise can be extended to conduct welfare analysis given the content of recent Health Care Bill.

This paper finds a novel connection between health risk and financial choice. And the model can be addressed to different policy evaluations and predictions. As baby boomers are getting retired, the age structure in the U.S. will be changed significantly. According to National Center for Health Statistics, the current share of people who are older than 65 is around 13.0%, and it will be increased to 20.0% in 2030. With the share of people older than 75 increased from 6.1% to 9.2% in 2030, and to 11.4% in 2040. In the meantime, the older people hold unproportionate amount of wealth. On the other hand, as to current debate in health reform, it’s interesting to ask the following question, as change in public health policy, such as medicare happens, how much will that affect risky asset holding in senior people? And the answers to those questions can be found by applying this paper to accommodate those policy or demographic changes.

2 Facts

The participation pattern is observed from SCF data. The descriptions of the data set is in the Appendix. We restrict our samples to those without privately held business, farms, professional practices, limited partnerships or any other types of partnerships. This sub-sample as described in the appendix, accounts for a large and stable size of U.S. population, which is around 86%. The reason to exclude private business owner is that as the nature of riskiness of their business is unknown, it’s then hard to distinguish the contribution of their background risk to participation pattern.

Follow Campbell (2006), among others, we first assume there is no cohort effect, and then construct the participation pattern over the life cycle by control for time effect. The figure 2 shows several cross sectional views for different waves, where we further control the sample such that the household wealth level is greater than $30,000 (2007 U.S. dollars).

While figure 2 illustrates the regressions of participation after grouping all six waves from 1989-2004 and uses another threshold (wealth greater than $10,000), it shows the same pattern, which is the participation stays high and relatively stable, and decreases around age 60.

The wealth threshold is robust. We conduct robustness check by using other levels such as $30,000 and $20,000, and the decrease pattern and the age at which participation rate
drops don’t change. We conduct still another robustness check by looking into participation over age and relative wealth level within that age group. Figure 2 illustrates the regressions of participation rates against age group and wealth groups of different quartiles within that age.

The figure suggests that the drop in participation mainly happens within asset level in the middle (quartile 2 and 3). Interestingly, the average portfolio share for each group drop as people aged, but the average portfolio share conditional on people are stock market participants don’t drop. These patterns can be shown in figure 2 and 2

This implies that there might exist some factor that hits some group of people most, and affect their portfolio choice so much that those agents decrease their portfolio share to zero. While this factor seems to not affect on-going participants that much that their portfolio share is not affected.

And health is a good candidate for this factor. As figure 2 suggests, people with bad health are more likely to drop the stock market. And health status affects most the relative poor people (wealth quartile group 2).
3 The model

In this section, we will first present a model that is with medical care service, consumption and portfolio choices and that the agent has expected utility. We will show that this model can’t robustly deliver nonparticipation in stock market, this conclusion holds no matter what assumptions we have towards the distribution of medical care inflation and stock return.

And then we will present a model deviating from the previous one by assuming the agent has ambiguity towards the distribution of medical care inflation and stock return. More specifically, the ambiguity is towards the correlation with medical care inflation and stock return. Or in the other way, the set of priors that agent has towards the distributions of joint distributions of medical inflation and real stock return is isomorphic to a set of value of correlations between risky medical care inflation and stock return. In this environment, we present a simplest example where medical expenditure is collapsed so as to illustrate the intuition that how non-participation can take place.

The key intuition for non-participation is summarized as the following. With multi-prior and maxmin utility function, for each action the agent chooses, he picks the distribution associated with that action so that it gives the lowest expected utility. If he contemplates
to invest positive amount in stock market, the distribution used to calculate the marginal benefit is the one that medical care inflation and real stock return have the most positive correlation. And his marginal benefit might be less than his opportunity cost, namely, investing in safe assets. On the other hand, if he contemplates to invest negative amount in stock market, or to short the stock market, the distribution used to calculate the marginal benefit is the one that medical care inflation and real stock return have the most negative correlation. If the set of priors that the agent bear in mind include both strong negative and positive correlations between medical price inflation and real stock return, the agent will find neither invest in stocks or short stocks are beneficial and thus stay out of the stocks market.

The analysis focuses on senior people who have retired already. The labor supply decision is thus absent from current analysis. And the households are thus not subject to non-asset income risk, as social security income is adjusted annually according to CPI-U of U.S. city average and all items.
3.1 The model without ambiguity

In the model, time is discrete, the agent starts at retirement age, which is normalized as 0, and the age can go from 0 up to \( T \). After \( T \) the survived agent will die for sure. The agent seeks to maximize his or her expected lifetime utility. The individual’s flow utility depends on current consumption, \( C \), health status, \( h \), and medical care service, \( m \). The flow utility is given by \( u(C, h, m) \).

Both the consumption and medical expenditure spaces are \([0, \infty)\). And space of health status are normalized to be \([h, 1]\), where \(0 < h < 1\). And the higher \( h \) indicates higher better status.

And the utility has the property that, \( u \) is increasing in all arguments. That is to say, with same level of health status and consumption level, the more the medical service an agent receives, the betteroff is he. Besides, we have: \( u_{CC}, u_{hh}, u_{mm} < 0 \)

The individual faces several sources of exogenous uncertainties.

1) Survival uncertainty. The survival probability \( s_{t+1}^{t} \) from period \( t \) to \( t+1 \) is determined by function \( s(t, h_t, m_t), \forall t \in T - \{T\}, \) where \( T = \{0, 1, 2, \cdots, T\} \). The survival probability is assumed to have the following property: \( s(t, h_t, m_t) \) is non-increasing in \( t \), and non-decreasing in both \( h \) and \( m \). That’s to say, with the same health status and same level of medical service,
the younger the agent is, the more likely he will survive. Similarly, with same age and health status, the more medical service the agent receives, the more likely he will survive. And last, with same age and enjoy same level of medical service, the healthier the agent is, the more likely he will survive.

2) Stock return uncertainty. The real return on stock investment is I.I.D. over time.

3) Medical price uncertainty. The relative medical care price is risky. We assume also that the medical price is I.I.D. over time.

The joint distribution of \((\tilde{R}_t, \tilde{P}_t) \forall t \in \mathcal{T} - \{0\}\) is \(Q(\tilde{R}_t, \tilde{P}_t)\).

The evolvement of the health status follows that:

\[ h_{t+1} = H(t, h_t, m_t), \quad \forall t \in \mathcal{T} - \{T\} \]  \hspace{1cm} (1)

The timing in each period is that, the survival probability resolves in the beginning of each period, if the agent survives, then agent learns the realization of stock return and medical price inflation for that period, he then receives his non-asset income \(y\). The agent afterwards chooses consumption, medical care service, safe and risky assets to save.
The problem can be summarized by a dynamic programming problem:

\[
V(t, h_t, B_t, S_t; R_t, P_t) = \max_{C_t, m_t, B_{t+1}, S_{t+1}} u(C_t, h_t, m_t) + s_t^{t+1} \beta E_t \left[ V(t + 1, h_{t+1}, B_{t+1}, S_{t+1}; \tilde{R}_{t+1}, \tilde{P}_{t+1}) \right]
\]

s.t.

\[
\begin{align*}
C_t + P_t m_t &= y + R_f B_t + R_t S_t \\
h_{t+1} &= H(t, h_t, m_t) \\
s_{t+1}^{t+1} &= s(t, h_t, m_t) \\
C_t, m_t &\geq 0
\end{align*}
\]

where the expectation is taken with respect to the distribution on the uncertainty of next period stock return \( \tilde{R}_{t+1} \), and medical price inflation \( P_{t+1} \). And in this problem, \( t \) refers to age, \( h_t \) is health status in the beginning of current period, \( B_t \) and \( S_t \) are safe assets and stock holding for the current period, and \( R_t, P_t \) are realizations of current period stock return and medical care price. \( y \) is agent’s real non-asset income, which is constant over time for the same agent. And \( R_f \) refers to risk free rate, and is also constant over time.
This model can endogenize medical expenses\(^1\). The model has the feature of “investment motive” for health expenditure. But unlike Grossman (1972), the investment motive is not through the channel of labor supply, but through the following channels: higher health expenditure delivers 1) higher current utility, 2) higher survival probability, and thus higher expected (on survival probability) continuation value and 3) better next period health and thus higher next period utility.

### 3.2 A two-period model

We present a two-period model to illustrate the difficulty to deliver non-participation in stock market using the above model. In the beginning of each period, the agent learns his health status, realized stock return and medical care price. The agent then chooses consumption, \(C\) and medical expenditure, \(m\); and in the first period, he chooses bond and stock holdings, \(B\) and \(S\). Assume that the stock price in second period is \(\tilde{R} = \bar{R} + \varepsilon_1\), where \(\bar{R}\) refers to gross expected stock of stock, and \(\varepsilon_1\) is a shock with mean zero and variance \(\sigma_1^2\). And second period medical price is \(\tilde{P} = P_0(I_M + \varepsilon_2)\), where \(\varepsilon_1\) is a shock with mean zero and variance \(\sigma_2^2\).

The maximization problem of the agent is:

\[
V(0, h_0, B_0, S_0; R_0, P_0) = \max_{C_0, C_1, B, S, m_0, m_1} u(C_0, h_0, m_0) + s_0^1 \beta E_0 [u(C_1, h_1, m_1)]
\]

\[\text{s.t.}
C_0 + P_0 m_0 + B + S = RB_0 + R_0 S_0 + y
\]

\[
C_1 + \tilde{P} m_1 = R_f B + \tilde{R} S + y
\]

\[
C_0, C_1, m_0, m_1 \geq 0
\]

\[
s_0^1 = s(0, h_0, m_0)
\]

\[
h_1 = H(0, h_0, m_0)
\]

To illustrate the investment incentive, it’s better to look at the first order condition for medical expenditures at the first period. We have:

\[
u_{m_0} + \frac{\partial s_0^1}{\partial m_0} \beta E_0 [u(C_1, h_1, m_1)] + s_0^1 \beta E_0 \left[ u(C_1, h_1, m_1) \frac{\partial h_1}{m_0} \right] = u_{C_0} P_0
\]

\[\text{(4)}\]

The right hand side is the marginal cost of spending in health at period 0, and the left terms are the benefits, which have three components. The first component is the direct benefit from flow utility, which says, with the same health status, extra medical expenditure will deliver marginal utility on medical care. The second term results from higher survival probability by spending more in health in current period. From this term, it is clear that

\(^1\text{which deviates from, say, Palumbo (1999), and De Nardi et al (2006) among others.}\)
not only marginal utility but the level of utility matters. And the last term is the betteroff from realization of next period health status by increasing current health expenditure.

Given such environment, Lemma 1 shed some light on stock market participation for senior people.

**Lemma 1.** Assume that stock return is independent with medical care inflation, and that equity premium is positive, and \( u_{cc} < 0, u_{mm} < 0, u_{mc} \geq 0 \), then \( S > 0 \).

Proof: See Appendix. ■

This intuition can easily extended to multi-period model. Lemma 2 is the extension of Lemma 1 to multi-period case.

**Lemma 2.** Assume that stock return is independent with medical care inflation, and that equity premium is positive, and \( u_{cc} < 0, u_{mm} < 0, u_{mc} \geq 0 \), then \( S_{t+1} > 0, \forall t \in T - \{T\} \).

Proof: See Appendix. ■

Lemma 2 reinforces the prediction from life cycle model with portfolio choice problem but abstract from health service decision, that the participation rate is 100% for retirees whose non-asset income is not co-vary with stock return.

When \( \tilde{R}, \tilde{P} \) are not independent, however, the nonparticipation in stock market can be shown to be a non-robust result. In a two-period model, an agent enters the economy characterized by her initial health status, \( h_0 \), her initial bond holding, \( B_0 \), her initial stock holding, \( S_0 \), and her constant defined benefit non asset income, \( y \). Without loss of generality, we assume that for any distinctive value \((h_0, B_0, S_0, y)\), the “natural” measure of such agent is equal to 0. The following Lemma 3 verifies the non-robustness in a two-period model where \( \tilde{R}, \tilde{P} \) are not independent, but follows some certain distribution.

**Lemma 3.** For an economy where initial \( P_0 \) and \( R_0 \) are realized, assume that any specific type of agent has measure 0, and that for any open set of initial parameters, the measure is positive. We also assume that \((\tilde{R}_1, \tilde{P}_1)\) follows some certain distribution, then the set of \((h_0, B_0, S_0, y)\) such that: \( S(h_0, B_0, S_0, y) = 0 \) has measure 0.

Proof: See Appendix. ■

We next consider a model where health expenditure is instead exogenous. Agent makes consumption, saving and investment decision in this two-period model. The agent can invest in both risk free bonds and stocks. In period zero, the agent holds initial wealth \( W_0 \), a pool of his bond holding, stocks holding and labor income. He decides how much to consume, to invest in bonds and stocks. In period 1, the agent receives risky labor income \( \tilde{Y} \) and returns from his investment. The gross return to the stocks is \( \tilde{R} \), that for bonds is \( R \). Both \( \tilde{R} \) and \( \tilde{Y} \) are random variables defined on probability space \((\Omega, \mathcal{F}, P)\).

The agent has a set of priors \( \mathcal{P} \) that over \((\Omega, \mathcal{F})\). The implicit assumption for multiple priors is that the agent is not able to form a single prior for the joint performance of his de facto income and stock return. We assume that the ambiguity is only towards the correlation
between income and stock return. We assume the reference distribution $P$ for joint performance of stock and income is:

$$\mathbb{E}(\tilde{R}, \tilde{Y}) \sim P \left( \left( \begin{array}{c} \mu_1 \\ \mu_2 \end{array} \right), \left( \begin{array}{cc} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{array} \right) \right)$$

(5)

And the set of priors is

$$\mathbb{P}(P) = \{ Q\mathbb{E}(\tilde{R}, \tilde{Y}) \sim N \left( \left( \begin{array}{c} \mu_1 \\ \mu_2 \end{array} \right), \left( \begin{array}{cc} \sigma_1^2 & \sigma_{12} + v \\ \sigma_{12} + v & \sigma_2^2 \end{array} \right) \right) : v \in [\underline{v}, \bar{v}] \}$$

(6)

And we assume that $0 \in [\underline{v}, \bar{v}]$.

The agent has multi-prior utility as in Gilboa and Schmeidler (1989). We assume the utility function is CARA. Thus the objective function is

$$u(C_0) + \beta \min_{Q \in \mathbb{P}} [u(C_1)]$$

(7)

where $u(c) = -\frac{1}{\theta} \exp(-\theta c)$, where $\theta > 0$. Thus we can summarize the agent’s problem in two-period setting as follows:

$$\max_{B_0, S_0} u(C_0) + \beta \min_{Q \in \mathbb{P}} [u(C_1)]$$

s.t. :

$$C_0 + B_0 + S_0 = W_0$$

(9)

$$C_1 = \tilde{Y} + RB_0 + \tilde{R}S_0$$

(10)

To solve the model, we need first solve the inside minimization problem, and this is equivalent to pick a proper value of disturbance on correlation. Under the assumption that a generic distribution is normal and that utility function is CARA, we can rewrite the objective function depending on the sign of stock holdings and then solve it accordingly.

Case 1: if $S_0 > 0$, the objective function becomes:

$$\max_{B_0, S_0} \frac{1}{\theta} \exp [-\theta(W_0 - B_0 - S_0)] -$$

$$\frac{\beta}{\theta} \exp \left[ -\theta RB_0 - \theta S_0 \mu_1 - \theta \mu_2 + \frac{\theta^2}{2} \left( S_0^2 \sigma_1^2 + 2S_0 (\sigma_{12} + \bar{v}) + \sigma_2^2 \right) \right]$$

(11)

Solving this for $S_0$ we can get

$$S_0 = \frac{\mu_1 - R - \theta (\sigma_{12} + \bar{v})}{\theta \sigma_1^2}$$

(12)

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1Gilboa and Schmeidler’s (1989) work does not impose any restrictions on structure of $\mathcal{P}$. Some work, including Kogan and Wang (2002), Cao, Wang and Zhang (2005), and Miao (2004), among others, use entropy criterion to define $\mathcal{P}$, which can deliver analytical and intuitive results. Their work assume that the ambiguity is only towards the mean of stock return, we will show later that this way of modeling can’t deliver the pattern of quitting from stock market as we observe from data.
where \( \bar{\nu} < \frac{\mu_1 - R}{\theta} - \sigma_{12} \).

The intuition is that, due to the ambiguity towards the correlation, the equity premium is discounted by risk coefficient times the correlation. And the investor longs in stocks only when the discounted equity premium is still positive under the “worst” scenario, where “worst” means when stocks and income are most like each other. If the stocks and income are so much like each other that at some point, the agent will quit the stock market to avoid risks.

Similarly, under case 2, if \( S_0 < 0 \), we get

\[
S_0 = \frac{\mu_1 - R - \theta(\sigma_{12} + \nu)}{\theta \sigma_1^2}
\]  

(13)

where \( \nu > \frac{\mu_1 - R}{\theta} - \sigma_{12} \).

Similar intuition applies here. The investor shorts stocks only when in the “best” scenario the discounted equity premium is still negative. And here “best” scenario is when stocks and income are very dislike each other. If stocks and income are too dislike each other, the investor would not have incentive to hedge against his income uncertainty by short selling stocks.

The last and most important case is when agent is not participating in stocks. We have the necessary condition for non-participation:

\[
\bar{\nu} \leq \frac{\mu_1 - R}{\theta} - \sigma_{12} \leq \nu
\]

(14)

The agent is not longing since he’s worried that his income will be too much like a stock. His nightmare is that the stock return will be low when his non-asset income is also low. Meanwhile, due to the ambiguity, the agent is also worried that his income will be too dislike a stock. He is concerned with the following scenario: he shorts in stocks, and his income hits a low shock. However, at the same time, the stock return is high, meaning he will be hurt by buying stocks with higher prices to pay back his short selling. The multiple prior utility captures these two direction worries at the same time, and therefore deliver the non-participation.

This two period model offers enough intuitions about why we need ambiguity towards the correlation to solve the puzzle. In the following, we will show other alternative specifications do not work.

**No ambiguity** Assume for now there is no ambiguity in the model. We have a single prior of joint distribution between stock return and non-asset income then all agents share the same single belief about the joint distribution of stock return and non-asset income as in equation (5). Then it is easy to show that the stock holding is

\[
S_0 = \frac{\mu_1 - R - \theta \sigma_{12}}{\theta \sigma_1^2}
\]

(15)

Given expected equity premium is positive, we have \( S_0 > 0 \). Nobody will ever quit the stock market.
**Ambiguity towards the mean of non-asset income** Alternatively, if we assume that the ambiguity is towards the mean of the last period non-asset income $\tilde{Y}$, the set of priors changes to

$$
\mathbb{P}(P) = \{ Q(\tilde{R}, \tilde{Y}) \sim \mathcal{N}\left(\begin{pmatrix} \mu_1 \\ \mu_2 - v \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}\right) : v \in [\underline{v}, \overline{v}] \}.
$$

We can show that it would change the consumption-savings (bond holding) decision. However, it would not change the stock participation decision since the mean of non-asset income $\mu_2$ never enters into the equation of stocking holding as in (13).

**Ambiguity towards the mean of stock return** Last, if we assume that the ambiguity is towards the mean of the last period stock return $\tilde{R}$, the set of priors now is as follows

$$
\mathbb{P}(P) = \{ Q(\tilde{R}, \tilde{Y}) \sim \mathcal{N}\left(\begin{pmatrix} \mu_1 - v \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}\right) : v \in [\underline{v}, \overline{v}] \}.
$$

In this case, the optimal stock holding is given by

$$
S_0 = \begin{cases} 
\mu_1 - R - \theta \sigma_{12} 
& \underline{v} < \mu_1 - R - \theta \sigma_{12} \\
0 & \underline{v} \leq \mu_1 - R - \theta \sigma_{12} \leq \overline{v} \\
\mu_1 - R - \theta \sigma_{12} 
& \overline{v} > \mu_1 - R - \theta \sigma_{12}
\end{cases}
$$

We do generate non-participation region depending on the dispersion of ambiguity. However, in order to explain why older people quit the stock market, which is the puzzle we are interested, we have to assume the ambiguity about the mean of risky stock return becomes larger when people are aging. We find it is not convincing. In fact, when we consider people can learn through experience, we would expect the opposite. Besides, it says nothing about the data fact that health status is a significant predictor of stock market participation.

### 4 Dynamic Model with Ambiguity

This section develops a dynamic model with ambiguity about the joint distribution of stock return and medical care inflation. More specifically, as in the previous section, the ambiguity is towards the correlation between stock return and medical care inflation.

To write the model into dynamic programming form, we have:

$$
V(W, h, P_m, t) = u(C, h, m') + \beta s(h, m', t) \min_{Q \in \mathcal{Q}} \mathbb{E}_Q [V(W', h', P'_m, t + 1)]
$$

where $W$ is wealth, and is composed of 1) current non-asset income $y$, for retiree its social security income, 2) financial wealth, including investments in safe assets and risky assets; $h$ denotes health status; $P_m$ is realized medical care inflation; and $t$ refers to age of the agent. And $Q$ is the joint distribution of risky or stock return and medical care inflation, with $\mathcal{Q}$ denoting the set of such distributions, which the agent has ambiguous of the true underlying distribution.

More specifically, we assume that the set of joint (marginal) distributions on next period risky factors are independent. By that we mean, suppose $Q_t$ are generic distributions from $\mathcal{Q}_t$, then $Q_t$ and $Q_{t'}$ are independent as long as $t \neq t'$. 

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4.1 Dynamic consistency

With multi-prior and maxmin utility, the independence axiom breaks down, and generally there is dynamic inconsistency problem. However, with assuming the joint distribution of stock return and medical care inflation is independent over time, and there is no learning from the agent, we can restore dynamic consistency property by applying results from Epstein-Schneider (2003).

Domain

Time is discrete and goes over $T = \{0, 1, \cdots , T\}$. The state space is $\Omega$, where $\Omega = X_1 \cdot X_2 \cdots X_T$. And information structure is represented by the filtration $\{\mathcal{F}_t\}^T_0$ with the following properties:

- $\mathcal{F}_t$ is a $\sigma$-algebra generated by a finite partition of $\Omega$, $\forall t \in T$
- $\mathcal{F}_0 = \sigma(\Omega)$
- $\mathcal{F}_t \subseteq \mathcal{F}_{t+1} \forall t \in T \setminus \{T\}$
- $\mathcal{F}_T = 2^\Omega$, with $\mathcal{F}_t(\omega)$ denoting the partition component containing $\omega$.

$Y$ is a measurable function from measurable space $(\Omega, \mathcal{F}_T)$ to $(X, \mathcal{B})$, where $\mathcal{B}$ is the Borel $\sigma$-algebra on $X$. When a probability measure $P$ is attached to the measurable space $(\Omega, \mathcal{F}_T)$, $Y(P)$ is a random variable from $(\Omega, \mathcal{F}_T, P)$ to $(X, \mathcal{B})$, and $Q$ denotes the distribution of the random variable $Y(P)$, which is a probability measure induced from $Y(P)$. The set of all probability measures on $(\Omega, \mathcal{F}_T)$ is $\mathcal{P}$ and the set of all distributions induced from $\mathcal{P}$ is denoted by $\mathcal{Q}$.

The agent’s belief about riskiness in the economy is characterized by a set of priors, $\mathcal{P}_0$, which is a subset of $\mathcal{P}$. The corresponding set of distribution measures is $\mathcal{Q}_0$, which is induced from $\mathcal{P}_0$, meaning for any $Q \in \mathcal{Q}_0$, $\exists P \in \mathcal{P}_0$ such that $Q$ is the distribution induced from $P$. There is no ambiguity if $\forall P_1, P_2 \in \mathcal{P}_0$, $P_1 = P_2$, a.s.

Given $T$, $\Omega$, $\mathcal{P}_0$, and $\{\mathcal{F}_t\}^T_0$, we can define marginal distribution and one-step-forward distribution as the following. For any $P \in \mathcal{P}_0$, $P_t(\omega) = p(\cdot | \mathcal{F}_t)(\omega)$, and $P_t^{+1}$ is the restriction of $P_t$ to $\mathcal{F}_{t+1}$. The sets of marginal priors and one-step-forward priors are:

$$\mathcal{P}_t(\omega) = \{P_t(\omega) : P \in \mathcal{P}_0\}$$  \hspace{1cm} (17)
$$\mathcal{P}_t^{+1}(\omega) = \{P_t^{+1}(\omega) : P \in \mathcal{P}_0\}$$  \hspace{1cm} (18)

Alternatively we can introduce one-step-forward priors first. For each $t$ and $\omega$, we have set of conditional one-step-forward measures:

$$\tilde{\mathcal{P}}_t^{+1}(\omega) = \{\tilde{P}_t^{+1}(\omega)\}$$  \hspace{1cm} (19)

, where $\tilde{P}_t^{+1}(\omega) : \Omega \rightarrow \Delta(\Omega, \mathcal{F}_{t+1})$ is $\mathcal{F}_t$-measurable, $\Delta(\Omega, \mathcal{F}_{t+1})$ is the set of probability measures on $(\Omega, \mathcal{F}_{t+1})$. 

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Based on one-step-forward set of priors, we can define unconditional set of priors:

\[
\hat{P}_0(\omega) = \left\{ \int_\Omega \int_\Omega \cdots \int_\Omega d\hat{P}_{T-1}^1 \cdots d\hat{P}_1^1 d\hat{P}_0^1(\omega), \text{ for some } \hat{P}_0^1, \cdots, \hat{P}_{T-1}^1 \in \hat{P}_{T-1}^1 \right\}
\]

(20)

**Consistencies**

**Lemma 4.** Suppose \( T = 2, \Omega = X_1 \cdot X_2, \mathcal{F}_1 = \sigma(X_1), \mathcal{F}_2 = \sigma(X_1, X_2), \) and we define \( Q_0 \equiv \{ Q_0 \mid Q_0(x_1, x_2) = Q_0^1(x_1)Q_0^1(x_2|x_1), \text{ for some } Q_1 \in Q_0^1 \text{ and } Q_1^+1 \in Q_1^+1 \} \), and we define:

\[
\hat{Q}_1,_{\{x_1\}} \equiv \left\{ \hat{Q}_1,_{\{x_1\}} | \hat{Q}_1,_{\{x_1\}} = Q_0(\cdot|x_1), \text{ for some } Q_0 \in Q_0 \right\}
\]

(21)

, then we have \( \hat{Q}_1,_{\{x_1\}} = Q_1^+1(\cdot|x_1) \).

Proof: Pick any \( Q_1,_{\{x_1\}} \in \hat{Q}_1,_{\{x_1\}} \), we have \( Q_1,_{\{x_1\}} = Q_0(\cdot|x_1) \) for some \( Q_0 \in \hat{Q}_0 \).

By construction of \( \hat{Q}_0, \), \( Q_0(x_1, x_2) = Q_0^1(x_1)Q_0^1(x_2|x_1) \), for some \( Q_1 \in Q_0^1 \) and \( Q_1^+1 \in Q_1^+1 \), so \( Q_0(\cdot|x_1) = Q_1^+1(\cdot|x_1) \).

Since \( Q_1^+1(\cdot|x_1) \in Q_1^+1(\cdot|x_1) \), we have \( Q_0(\cdot|x_1) \in Q_1^+1(\cdot|x_1) \) as well. Thus \( \hat{Q}_1,_{\{x_1\}} \subseteq Q_1^+1(\cdot|x_1) \).

Pick any \( Q_1^+1(\cdot|x_1) \) \( \in Q_1^+1(\cdot|x_1) \), and pick any \( Q_0^+1 \in Q_0^+1 \), define \( Q_0 = Q_0^+1Q_1^+1 \), by definition of \( Q_0 \), this particular \( Q_0 \) is an element of \( Q_0 \). So \( Q_0(\cdot|x_1) \) is an element of \( \hat{Q}_1,_{\{x_1\}} \) by definition of \( \hat{Q}_1,_{\{x_1\}} \). Q.E.D.

The Lemma can be easily extended to any finite periods. What the Lemma guarantees that the way we construct the set of unconditional priors is probability-measure-wise consistent. While the next Lemma tells that using the same way to construct the set of unconditional priors, we have dynamic consistency.

**Lemma 5.** Suppose \( T = 2, \Omega = X_1 \cdot X_2, \mathcal{F}_1 = \sigma(X_1), \mathcal{F}_2 = \sigma(X_1, X_2), \) and \( Q_0 \equiv \{ Q_0 \mid Q_0(x_1, x_2) = Q_0^1(x_1)Q_0^1(x_2|x_1), \text{ for some } Q_1 \in Q_0^1 \text{ and } Q_1^+1 \in Q_1^+1 \} \), we have \( Q_0 \) satisfy rectangularity condition.

Proof: Notice that originally, the rectangularity conditions says that for any set of unconditional priors, it’s \( \{ \mathcal{F}_t \} \)-rectangular if for all \( t, \omega \), we have:

\[
P_t(\omega) = \left\{ \int_\Omega P_{t+1}(\omega')dm : \text{ for some } P_{t+1}(\omega') \in \mathcal{P}_{t+1}(\omega'), \forall \omega', \text{ and } m \in \mathcal{P}_{t+1}(\omega) \right\}
\]

And when \( T = 2 \), it’s trivial to check that it’s true when \( t = 0 \) and for all \( \omega \) by the definition of \( Q_0 \). And as the \( P_2(\omega) \) are degenerated distribution, it’s easy to prove that equality holds for rectangularity when \( t = 1 \).
4.2 An example in three period model

To qualitatively capture the stock market participation pattern over the life cycle, we extend two period model to a three period model. The three periods are 0, 1, 2.

We assume an agent has income profile \((W, aW, bW)\) over three periods. Parameters \(a, b\) are designed to approximate real life income profile, where \(a > 1\) can approximate the peak of mid-aged income and \(bW\) can capture the social security benefits, which depends on agent’s previous income. The income profile is certain. The agent faces the health expenditure shock at the last period. The expenditure is a random variable denoted by \(\tilde{H}\). In each period, the agent decides how much to consume and to invest in bonds and stocks. The returns on stocks are uncertain. And there is ambiguity towards the joint distribution on last period stock return and health expenditure shock. We assume that the above joint distribution is independent of first period stock return. These assumptions guarantee the regularity condition hold. (See Epstein and Schneider (2003))\(^2\). Henceforth, we can use backward induction to solve the maximization problem.

The three period problem can be summarized as\(^3\):

\[
\begin{align*}
\max_{B_0, S_0, B_1, S_1} & \quad u(C_0) + \beta E_1[u(C_1)] + \beta^2 \min_{Q \in \mathcal{F}} [u(C_2)] \\
\text{s.t.:} & \quad C_0 + B_0 + S_0 = W \\
& \quad C_1 + B_1 + S_1 = aW + RB_0 + \tilde{R}_1 S_0 \\
& \quad C_2 = (bW - \tilde{H}) + RB_1 + \tilde{R}_2 S_1
\end{align*}
\]

\((22)\)

Since we use backward induction to solve this problem, we can use the results from the two period model and determine the stock holding at time 1 as follows:

\[
S_1 = \begin{cases} \\
\frac{\mu_1 - R - \theta(\sigma_{12} + \tau)}{\theta \sigma_1^2} & \bar{v} < \frac{\mu_1 - R}{\theta} - \sigma_{12} \\
0 & \bar{v} \leq \frac{\mu_1 - R}{\theta} - \sigma_{12} \leq \bar{v} \\
\frac{\mu_1 - R - \theta(\sigma_{12} + \tau)}{\theta \sigma_1^2} & \bar{v} > \frac{\mu_1 - R}{\theta} - \sigma_{12}
\end{cases}
\]

\((26)\)

where non-asset income \(\tilde{Y}\) is equal to \(bW - \tilde{H}\). And we assume that the set of joint distributions on \(\tilde{Y}\) and \(\tilde{R}_2\) is similar to that in definition (2).

The bond holding, in case when \(S_1 > 0\), is:

\[
B_1 = \frac{\ln \beta R}{\theta(1 + R)} + \frac{W_0 - \mu_2 - (1 + \mu_1)S_1}{1 + R} + \frac{1}{1 + R} \frac{\theta \sigma_2^2}{2} + \frac{1}{1 + R} \frac{\theta S_1^2 \sigma_2^2}{2} + \frac{\theta S_1 (\sigma_{12} + \bar{v})}{1 + R}
\]

\((27)\)

\(^2\)Here the one-step-ahead belief as defined in Epstein and Schneider (2003) is a collection of joint distributions on stock return and health shock in period 2. This collection is independent of the realization of period 1 stock return due to the assumption of independence, which, in turn, guarantees the regularity condition satisfied. We thus have the legitimacy to use the one-step-ahead belief rather than belief at time 0 when we define the utility function.

\(^3\) The utility function is still CARA.
where the first term is the savings caused by impatience. It’s positive if interest rate dominates time discount factor ($\beta R > 1$). The second term is the difference between tomorrow’s expected income and today’s wealth. Here $W_0 = aW + RB_0 + R_1S_0$ is the total wealth received at the beginning of period 2, where $R_1$ is the realization of period 1 stock return. The third term is the precautionary savings due to uncertainty in income. Similarly, the forth term is the precautionary savings caused by uncertainty in stock return. The last term is precaution saving for the co-movement between stock and income.

For the case when $S_1 < 0$, the $B_1$ formula will be modified accordingly to the last term, which is changed to $\frac{\theta S_1(\sigma_{12} + \nu)}{1 + R}$. When $S_1 = 0$, the bond holding is:

$$B_1 = \frac{\ln \beta R}{\theta(1 + R)} + \frac{W_0 - \mu_2}{1 + R} + \frac{1}{1 + R} \frac{\theta \sigma_2^2}{2} \tag{28}$$

obviously, it is absent of the two precautionary savings terms due to stock holding uncertainty and ambiguity towards correlation between stock return and de facto income.

Using backward induction, in period 0, it’s easy to show that everyone holds stock as follows

$$S_0 = \frac{1 + R \mu_1 - R}{R} \frac{\mu_1 - R}{\theta \sigma_1^2} \tag{29}$$

In fact, the time 0 stock holding is bigger than zero for every agent, therefore, the model predicts that the stock market participation rate in period 0 is 1, however, due to the existence of ambiguity, someone will quit the stock market in period 1. Hence we deliver the decreasing stock market participation for older people in this model. It’s also worth pointing out that $S_0$ is larger than $S_1$, therefore we generate the decreasing stock share for older people over the life cycle.

### 4.3 Who Quit and Who Stay in Stock Market

From the discussion above, we found that in this model the key component to determine non-participation is the range of band on ambiguity towards the correlation between stock return and non-asset income.

The non-participation condition in (14) is that:

$$\nu \leq \frac{\mu_1 - R}{\theta} - \sigma_{12} \leq \bar{\nu}$$

where $\sigma_{12} = \text{Cov}_P(\tilde{Y}, \tilde{R})$. Here $P$ is the reference distribution as defined in (5). We also have $\tilde{Y} = bW - \tilde{H}$. Where $\tilde{H}$ is the random health expenditure. We can rewrite $\tilde{H} = W\tilde{h}$, i.e., $\tilde{h}$ is a share of health expenditure over initial period labor income $W$. Henceforth, the correlation for reference distribution can be rewritten as:

$$\sigma_{12} = W[E(\tilde{h})E(\tilde{R}) - E(\tilde{h}\tilde{R})]$$

and the non-participation condition can be rewritten as:

$$\nu \leq \frac{\mu_1 - R}{\theta} + W[E(\tilde{h}\tilde{R}) - E(\tilde{h})E(\tilde{R})] \leq \bar{\nu} \tag{30}$$
First we assume people have same knowledge of $\mu$ and $R$. When people share same parameters of risk aversion, ambiguity level $E(\tilde{h}\tilde{R}) - E(\tilde{h})E(\tilde{R})$, the higher is $W$, the less likely that people will stop participating. If $E(\tilde{h}\tilde{R}) - E(\tilde{h})E(\tilde{R}) > 0$, the agent will likely to long if $W$ is big enough. And they will short when $W$ is big enough and $E(\tilde{h}\tilde{R}) - E(\tilde{h})E(\tilde{R}) < 0$.

If we assume people have same parameters except for ambiguity level, then those with smaller band of ambiguity will less likely stop participating.

Similarly, if people have same parameters as others except for risk aversion $\theta$, then the less risk averse the agent is, the more likely he will long stocks.

5 Conclusion

This paper documents a life cycle pattern of stock market participation rate. We find that the stock market participation rate decreases dramatically after age 60. Why do older people quit the stock market?

We provide an answer based on the observation that older people face much stronger health risk than young and the empirical evidence that shows the health risk is indeed a powerful predictor of stock participation (Rosen and Wu 2004). In our model, older people face health expenditure shock. In addition, they are uncertain about the correlation between stock return and de facto nonfinancial income which is negatively affected by their health expenditure. Under a multi-prior utility framework, we show that there exists a range of ambiguity level towards this correlation over which investors neither long or short sell a stock. Heterogeneity among older people on the ambiguity towards the correlation between risky stock return and uncertain health expenditure thus guarantees some people will fall into this range. Therefore they will choose not to participate in the stock market.

A. Appendix

A.1: Proof

Proof of Lemma 1: It’s helpful to illustrate it by assuming there are finite states of uncertainty. And the intuition can be easily extended to infinite many states case. Let’s assume there are $M_1$ number of distinctive states for $\varepsilon_1$ and $M_2$ for $\varepsilon_2$. As $\varepsilon_1$ and $\varepsilon_2$ are independent, $\pi(\varepsilon_1, \varepsilon_2) = \pi(\varepsilon_1) \cdot \pi(\varepsilon_2)$ for any values of $\varepsilon_1$ and $\varepsilon_2$, where $\pi(\varepsilon_1, \varepsilon_2)$ denotes the joint probability for $(\varepsilon_1, \varepsilon_2)$ and $\pi(\varepsilon_i)$ denotes the probability for the event that $\varepsilon_i$ happens.

The agent’s problem can be so rewritten as:

$$\max_{C_0, C_1(\varepsilon_1, \varepsilon_2), B, S, m_0, m_1(\varepsilon_1, \varepsilon_2)} u(C_0, h_0, m_0) + s_0(h_0, m_0)\beta \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2)u(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2))$$

$$C_0 + P_0 m_0 + B + S = R_f B_0 + R_0 S_0 + y (\lambda_0) \quad \text{(31)}$$

$$C_1(\varepsilon_1, \varepsilon_2) + \tilde{P}(\varepsilon_2)m_1(\varepsilon_1, \varepsilon_2) = R_f B + \tilde{R}(\varepsilon_1) S + y (\lambda(\varepsilon_1, \varepsilon_2)) \quad \text{(32)}$$

$$C_0, C_1(\varepsilon_1, \varepsilon_2), m_0, m_1(\varepsilon_1, \varepsilon_2) \geq 0$$
\[ u_c(C_0, h_0, m_0) = \lambda_0 \ (C_0) \]  
\[ \beta s(h_0, m_0) \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) = \lambda(\varepsilon_1, \varepsilon_2) \ (C_1(\varepsilon_1, \varepsilon_2)) \]  
\[ \lambda_0 = \sum_{(\varepsilon_1, \varepsilon_2)} R_f \lambda((\varepsilon_1, \varepsilon_2)) \ (B) \]  
or  
\[ u_c(C_0, h_0, m_0) = \beta s(h_0, m_0) \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) R_f \]
\[ \lambda_0 = \sum_{(\varepsilon_1, \varepsilon_2)} \tilde{R}(\varepsilon_1) \lambda((\varepsilon_1, \varepsilon_2)) \ (S) \]  
or  
\[ u_c(C_0, h_0, m_0) = \beta s(h_0, m_0) \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) \tilde{R}(\varepsilon_1) \]
\[ u_m(C_0, h_0, m_0) + \beta \frac{s(h_0, m_0)}{m_0} E[u(C_1, h_1, m_1)] + \beta s(h_0, m_0) E[u_a(C_1, h_1, m_1) \frac{\partial h_1}{\partial m_0}] \]
\[ = P_0 u_c(C_0, h_0, m_0) \ (m_0) \]  
\[ u_m(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) = u_c(C_1, h_1, m_1) \bar{P}(\varepsilon_2) \ (m_1(\varepsilon_1, \varepsilon_2)) \]  

Together with the budget constraints, and they are totalled \(5 + 3M_1 M_2\) equations, which will be solved unknowns: \(C_0, m_0, B, S, C_1(\varepsilon_1, \varepsilon_2), m_1(\varepsilon_1, \varepsilon_2), \lambda_0, \lambda(\varepsilon_1, \varepsilon_2)\), totalled at \(5 + 3M_1 M_2\) as well. Although it’s been assumed that the optimal consumption and medical care are interior solutions, as it turns out below that, it’s not necessarily needed for the proof.

Now suppose at optimal \(S = 0\). And consider a deviation from this plan by moving some marginal safe assets (or borrow if necessary) by amount of \(\Delta\) to stock account, the marginal benefit minus marginal cost by doing so is:

\[-\beta s^1_0 \Delta \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) R_f + \beta s^1_0 \Delta \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) \tilde{R}(\varepsilon_1) \]
\[= \beta s^1_0 \Delta \left[ \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2))(\tilde{R}(\varepsilon_1) - R_f) \right] \]  

where, evaluated at \(S = 0\), \(C_1(\varepsilon_1, \varepsilon_2) = y + R_f B - \bar{P}(\varepsilon_2)m_1(\varepsilon_1, \varepsilon_2)\). From the first order condition for \(m_1(\varepsilon_1, \varepsilon_2)\), or equation (38), we have:

\[ u_m(y + R_f B - \bar{P}(\varepsilon_2)m_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) = u_c(y + R_f B - \bar{P}(\varepsilon_2)m_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) \bar{P}(\varepsilon_2) \]  

We want to show that if \(S = 0\), \(m_1(\varepsilon_1, \varepsilon_2) = m_1(\varepsilon_1^0, \varepsilon_2)\) for any \(\varepsilon_1, \varepsilon_1^0\). Without loss of generality, we will discuss the case for, say, \(\varepsilon_1^H, \varepsilon_1^L\) with \(\varepsilon_1^H > \varepsilon_1^L\), with \(m_1(\varepsilon_1^H, \varepsilon_2) > m_1(\varepsilon_1^L, \varepsilon_2)\).
Apply equation (38), we have conditions:

\[ u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^H_1, \varepsilon_2), h_1, m_1) = u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^H_1, \varepsilon_2), h_1, m_1) \tilde{P}(\varepsilon_2) \]

\[ u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^L_1, \varepsilon_2), h_1, m_1) = u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^L_1, \varepsilon_2), h_1, m_1) \tilde{P}(\varepsilon_2) \]

which implies:

\[
\frac{u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^H_1, \varepsilon_2), h_1, m_1(\varepsilon^H_1, \varepsilon_2))}{u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^H_1, \varepsilon_2), h_1, m_1(\varepsilon^H_1, \varepsilon_2))} = \frac{u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^L_1, \varepsilon_2), h_1, m_1(\varepsilon^L_1, \varepsilon_2))}{u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^L_1, \varepsilon_2), h_1, m_1(\varepsilon^L_1, \varepsilon_2))}
\]

(41)

However, as we also have:

\[ u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^H_1, \varepsilon_2), h_1, m_1(\varepsilon^H_1, \varepsilon_2)) > u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^L_1, \varepsilon_2), h_1, m_1(\varepsilon^L_1, \varepsilon_2)) \geq u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^L_1, \varepsilon_2), h_1, m_1(\varepsilon^L_1, \varepsilon_2)) \]

, where the first inequality comes from \( u_{cc} < 0 \), and the second inequality is justified by \( u_{cm} \geq 0 \), and as:

\[ u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^H_1, \varepsilon_2), h_1, m_1(\varepsilon^H_1, \varepsilon_2)) < u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^L_1, \varepsilon_2), h_1, m_1(\varepsilon^L_1, \varepsilon_2)) \leq u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon^L_1, \varepsilon_2), h_1, m_1(\varepsilon^L_1, \varepsilon_2)) \]

, where the first inequality comes from \( u_{mm} < 0 \), and the second inequality is again based on the fact that \( u_{cm} \geq 0 \). The equality of (41) can never hold.

Henceforth, we have that if \( S = 0 \), then \( m_1(\varepsilon_1, \varepsilon_2) = m_1(\varepsilon'_1, \varepsilon_2) \) for any \( \varepsilon_1, \varepsilon'_1 \).

And from the first order condition of \( m_1 \) or (38), we have also if \( S = 0 \), then \( C_1(\varepsilon_1, \varepsilon_2) \equiv C_1(\varepsilon'_1, \varepsilon_2) \) for any \( \varepsilon_1, \varepsilon'_1 \), which in turn implies that the marginal gain by moving from saving to stocks from (39) is:

\[
\beta s_0^1 \Delta \left[ \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2))(\tilde{R}(\varepsilon_1) - R_f) \right]
\]

\[
= \beta s_0^1 \Delta \left[ \sum_{\varepsilon_1} \sum_{\varepsilon_2} \pi(\varepsilon_1) \pi(\varepsilon_2) u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2))(\tilde{R}(\varepsilon_1) - R_f) \right]
\]

\[
= \beta s_0^1 \Delta \left[ \sum_{\varepsilon_1} \pi(\varepsilon_1)(\tilde{R}(\varepsilon_1) - R_f) \right] \left[ \sum_{\varepsilon_2} \pi(\varepsilon_2) u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) \right]
\]

\[
> 0 \quad (42)
\]

if \( \Delta > 0 \), where the last equation comes from the fact that \( C_1(\varepsilon_1, \varepsilon_2) = C_1(\varepsilon'_1, \varepsilon_2) \) for any \( \varepsilon_1, \varepsilon'_1 \), and that \( m_1(\varepsilon_1, \varepsilon_2) = m_1(\varepsilon'_1, \varepsilon_2) \) for any \( \varepsilon_1, \varepsilon'_1 \). And the last inequality comes from
the facts that: 1) expected equity premium is positive, or \( \sum_{\varepsilon_1} \pi(\varepsilon_1)(\tilde{R}(\varepsilon_1) - R_f) > 0 \) and 2) \( u_c > 0 \).

Based on the above argument, the agent actually has incentive to deviate from the proposed optimal plan where \( S = 0 \) by increasing his investment in stocks. This implies that the stock market participation rate should be 100%. Q.E.D.

**Proof of Lemma 2:** We want to prove in the general setup like in equation 2 there is \( S_{t+1} > 0 \) \( \forall t \in T - \{T\} \). It’s quite obvious to see that, by deviating from \( S_{t+1} = 0 \), the extra benefit subtracted by extra cost is:

\[
\beta s_t^{t+1} \Delta \left[ \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_{t+1}(\varepsilon_1, \varepsilon_2), h_{t+1}, m_{t+1}(\varepsilon_1, \varepsilon_2))(\tilde{R}(\varepsilon_1) - R_f) \right]
\]

(43)

where \( C_{t+1}(\varepsilon_1, \varepsilon_2) = y + R_f B_{t+1} - \tilde{\pi}(\varepsilon_2)m_{t+1}(\varepsilon_1, \varepsilon_2) - B_{t+2}(\varepsilon_1, \varepsilon_2) - S_{t+2}(\varepsilon_1, \varepsilon_2) \) if \( S_{t+1} = 0 \). If we impose that \( B_{t+2}(\varepsilon_1, \varepsilon_2) = B_{t+2}(\varepsilon_1', \varepsilon_2) \) and \( S_{t+2}(\varepsilon_1, \varepsilon_2) = S_{t+2}(\varepsilon_1', \varepsilon_2) \) when \( S_{t+1} = 0 \), then it’s easy to show that: \( m_{t+1}(\varepsilon_1, \varepsilon_2) = m_{t+1}(\varepsilon_1', \varepsilon_2) \) by using the same argument as we used above to prove Lemma 1. And thus the marginal net benefit by deviation from \( S_{t+1} = 0 \) is:

\[
\beta s_t^{t+1} \Delta \left[ \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_{t+1}(\varepsilon_1, \varepsilon_2), h_{t+1}, m_{t+1}(\varepsilon_1, \varepsilon_2))(\tilde{R}(\varepsilon_1) - R_f) \right]
\]

\[
\beta s_t^{t+1} \Delta \left[ \sum_{\varepsilon_1} \pi(\varepsilon_1)(\tilde{R}(\varepsilon_1) - R_f) \right] \left[ \sum_{\varepsilon_2} \pi(\varepsilon_2) u_c(C_{t+1}(\varepsilon_1, \varepsilon_2), h_{t+1}, m_{t+1}(\varepsilon_1, \varepsilon_2)) \right] > 0
\]

And thus it shows that \( S_{t+1} > 0 \) \( \forall t \in T - \{T\} \). Q.E.D.

**Proof of Lemma 3:** [Sketch of the proof]: Suppose not, i.e., if we denote the set of all \((h_0, B_0, S_0, y)\) such that \( S(h_0, B_0, S_0, y) = 0 \) by \( A \), then \( A \) contains non-empty interior. Now if we pick any \((\hat{h}_0, \hat{B}_0, \hat{S}_0, \hat{y})\), we then contemplate some perturbation along the continuum variables. For instance, if non-asset income is the only continuum variable, we consider a perturbation to \((h_0, B_0, S_0, y) + \epsilon\), where \( \epsilon \) is small number such that after the perturbation, the new point is kept in \( A \).

After the

\[
E\left[u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2))|\tilde{R} - R\right] = E\left[u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2))\right] E(\tilde{R} - R) + \text{cov}(u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)), \tilde{R} - R)
\]

(44)

**A.2: SCF data description**

The Survey of Consumer Finances (SCF) data is a triennial survey data designed to provide detailed information on the financial status of the U.S. households. The information include
details of households’ wealth or balance sheet, current income and part of future income, namely, pensions. The survey is sponsored by the Board of governors of the Federal Reserve System in cooperation with Internal Revenue Service of the Department of Treasury. And since 1992, data have been collected by the National Organization for Research at the University of Chicago (NORC).

In terms of its quality and characteristics, the SCF data has the following advantages over other data set:

First, the SCF data has a good coverage of US households. Thanks to a dual-frame sample design, the SCF data has one subset with a good coverage of characteristics such as age, education, income and wealth level (e.g., there are 3,007 cases in the set in 2004 survey); while in the other subset it oversamples relatively wealthy households drawn from a list of records offered by IRS (there are 1,515 cases in 2004 survey). By doing so it helps to enhance accuracy of statistics with respect to asset allocation, since wealthy people tend to more likely to hold and to hold more financial assets. Proper weights are then applied to both samples in order to make estimations right for the whole population.

Second, the SCF data covers a complete breakdown of all categories for household wealth. It covers both liquid and illiquid assets. And for both categories, it further breaks down with more details.

Third, for some assets or pensions, he SCF data disentangle the asset holding to individual levels. Since the survey asks questions to individuals of household, such as head, spouse, and other member of the household, they survey can potentially shed light on intra-household diversification problem.

Besides, SCF has high accuracy. The survey applies computer assisted interviewing program to enhance quality of survey data. There usually exists high frequency of ”don’t know” responses to questions asking about value of assets. The computer program, however, uses a host of ways to extract information and to avoid ”don’t know” responses. For example, it offers range cards to those who are uncomfortable saying exact number to specify range. And it also uses series of questions in a decision tree to clarify the range more specifically.

The above advantages make SCF a unique and good data set to studies on household portfolio decisions.

Next we will describe some key variables used in this paper, the questions and variables will refer to 2004 survey. Throughout the paper age refers to that of the head of a primary economic unit (PEU). Where PEU is the household unit in the SCF data and it’s comprised of a core single individual or couple in a household and all others who are financially dependent on that individual or couple. And head refers to the single core in individual in a PEU without a core couple or the male in a PEU with mixed-sex couple or the older individual in a PEU with same-sex couple.

The measure of stock investments includes:

1. Full investments in directly held stocks. At question Q872, it asks “What is the total

As time progresses, the survey asks more detailed questions on stock investment through different financial vehicles, and thus the measurement covers more different categories. The following list is the measurement from 2004 survey.
market value of this (publicly traded) stock?”, and the value is stored as variable X3915.

2. Full investments in stock mutual funds. At question Q834A1, it asks “What is the total market value of all of the stock mutual funds that you have?”, and the value is stored as variable X3822.

3. Half investments in combination funds. At question Q834A5, it asks “What is the total market value of all of the combination funds that you have?”, and the value is stored as variable X3830, half of which is counted for stock investment.

4. Full investments in other mutual funds. At question Q834A6, it asks “What is the total market value of all of these other funds that you have?”, and the value is stored as variable X7787.

5. Stock investments in IRA/Keogh account of each member of households. At questions Q1705A1B1-Q1705A1B4/Q1705A2B1-Q1705A2B4/Q1705A3B1-Q1705A3B4, they ask “How much is in your Roth IRA account(s)?”, “How much is in your roll-over IRA account(s)?”, “How much is in your regular or other IRA account(s)?”, and “How much is in your Keogh account(s)?”, respectively, to heads/spouses/other member of households, and their answers are stored as variables X6551-X6554/X6559-X6562/X6567-X6570 for heads/spouses/other member of households. At questions Q787A1 and Q790A1/Q787A2 and Q790A2/Q787A3 and Q790A3, they ask “How is the money in (this/these) account(s) invested?” and “About what percent is in stocks?”, and the answers are stored as variables X6551-X6556/X6563-X6564/X6571-X6572 for heads/spouses/other member of households. The percentage times the total value are accounted for stock investment.

6. Stock investments in annuities, more specifically, in variable annuities. At question Q911A1, it asks “How much would you receive if you cashed in these annuities?”, and the values are stored in variable X6577. At questions Q921A1 and Q1734A1, they ask “How is the money in (both types of/these) annuities invested?” and “About what percent is in stocks?”, respectively. And answers are stored as variables X6551 and X6552. The value of stock investments are calculated as value times corresponding percentage.

7. Stock investments in trust or managed investment accounts. At question Q911A2, it asks “How much would you receive if you cashed in these (trust or managed investment) accounts?”, and values are stored in variable X6587. At questions Q921A2 and Q1734A2, they ask “How is the money in these accounts invested?” and “About what percent is in stocks?”, respectively. And answers are stored in variables X6591 and X6592. The value of stock investments are calculated as value times corresponding percentage in stocks.

8. Stock investments in pensions for head and spouse from current main jobs. Only accounts similar to 401(k) or 403(b) will be counted, and the corresponding amount and percentage in stocks are used to calculated for amounts in stocks.

9. Stock investments in pensions other than IRAs or Keogh, from which households are receiving benefits. At questions Q1661A1-A6, they ask “Is this pension currently an account plan, such as a 401(k), where you could take the whole balance as one payment if you wanted to?” for different pension accounts, values are stored in variables X6461/66/71/76/81/86, correspondingly. At questions Q1663A1-A6, they ask “What is the current balance in this account?”, values are stored in variables X6462/67/72/77/82/87. At questions Q1665A1-A6
and Q1726A1-A6, they ask “How is the account invested? Is it all in stocks, all in interest earning assets, is it split between these, or something else?”, and “About what percent is in stocks?”, values are stored in variables X6933/37/41/45/49/53 and X6934/38/42/46/50/54, respectively. The value of such investment is counted if the account is qualified as account plan, and then using its values time percentage in stocks.

10. Stock investments in other pensions not mentioned above. At questions Q1336A1-A6, they ask “How much is in the account now?” and values are stored in variables X5604/12/20/28/36/44 for different such accounts. And at questions Q1643A1-A6 and Q1731A1-A6, they ask “How is the money in this account invested?” and “About what percent is in stocks?”, respectively. The amounts of investments are summation of all such accounts’ value in stocks.

A.3: Comovement between stock return and medical care inflation

We first construct real stock return and relative medical care inflation, and then look at the correlation between this two sequences.

To get both the real stock return and relative medical care inflation, we need price index to calculate the inflation. And we use annual CPI-U of U.S. city average and all items for this purpose. The CPI, or Consumer Price Index, is collected, calculated and published by Bureau of Labor Statistics. CPI measures the average change over time in prices paid for a market basket of consumer goods and services. The CPI-U sets this basket of goods and services to reflect spending pattern of all urban consumers, which accounts for 87% of total population.

The advantages of using CPI over other price index include, firstly, the social security benefit payments are adjusted according to change in CPI; and secondly, the CPI captures more out-of-pocket fashion expenses than other price index. We use CPI medical care to calculate firstly medical inflation and then relative medical inflation. The medical care accounts for 6.23% of the basket of goods. Medical care includes two major categories, one is medical care commodities and the other is medical care services. We calculate the medical care inflation rate for post war period (1947-2006) and then derive the relative medical care inflation rate by using medical care inflation subtracted by general inflation. Table 1 summarizes the statistics of post war relative medical care inflation and stock return. And Table 2 summarizes the strata and relative importance of components of medical care CPI.

A.4: Sensitivity Analysis of the comovement between stock return and medical care inflation

In a sensitivity analysis, for a sub-sample size ranged from 10 to 59 years, the original sample is broken into different blocks composed of consecutive years. For example, there are 2 possible blocks for a sub-sample as long as 59, while there are 31 possible blocks for a sub-sample having 20 consecutive years. The correlations are then calculated for these sub-samples. Table 3-5 list some statistics from this sensitivity analysis.
Table 1: The statistics of relative medical inflation and stock return (1947-2006)

<table>
<thead>
<tr>
<th></th>
<th>Sample Means</th>
<th>Sample Stdev</th>
<th>Correlation with Stock (p value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>0.0869</td>
<td>0.1542</td>
<td>1</td>
</tr>
<tr>
<td>Medical Inf</td>
<td>0.0179</td>
<td>0.0185</td>
<td>0.1257 (0.3385)</td>
</tr>
<tr>
<td>MCC Inf</td>
<td>-0.0072</td>
<td>0.0304</td>
<td>0.2551 (0.0491)</td>
</tr>
<tr>
<td>MCS Inf</td>
<td>0.0154</td>
<td>0.0262</td>
<td>0.0612 (0.6425)</td>
</tr>
</tbody>
</table>

Table 2: The strata of composition of medical care indexes, and relative importance as of December 2008

<table>
<thead>
<tr>
<th>Item (relative importance, in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medical Care (6.390)</td>
</tr>
<tr>
<td>Medical care commodities (1.625)</td>
</tr>
<tr>
<td>Prescription drugs (1.253)</td>
</tr>
<tr>
<td>Nonprescription drugs and medical supplies (0.372)</td>
</tr>
<tr>
<td>Internal and respiratory over-the-counter drugs (0.259)</td>
</tr>
<tr>
<td>Nonprescription medical equipment and supplies (0.113)</td>
</tr>
<tr>
<td>Medical care services (4.765)</td>
</tr>
<tr>
<td>Professional’s services (2.702)</td>
</tr>
<tr>
<td>Physicians’ services (1.364)</td>
</tr>
<tr>
<td>Dental services (0.752)</td>
</tr>
<tr>
<td>Eyeglasses and eye care (0.244)</td>
</tr>
<tr>
<td>Services by other medical professionals (0.342)</td>
</tr>
<tr>
<td>Hospital and related services (1.545)</td>
</tr>
<tr>
<td>Hospital services (1.337)</td>
</tr>
<tr>
<td>Nursing home and adult day care services (0.132)</td>
</tr>
<tr>
<td>Care of invalids, elderly and convalescents in the home (0.076)</td>
</tr>
<tr>
<td>Health Insurance (0.518)</td>
</tr>
</tbody>
</table>
### Table 3: Sensitivity Analysis for correlation between medical care and stock return

<table>
<thead>
<tr>
<th>Sub-sample size</th>
<th>Min correlation</th>
<th>Max correlation</th>
<th>Number of blocks with negative correlations (significant)</th>
<th>Number of blocks with positive correlations (significant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.6991</td>
<td>0.7616</td>
<td>20 (5)</td>
<td>31 (5)</td>
</tr>
<tr>
<td>20</td>
<td>-0.2084</td>
<td>0.3831</td>
<td>11 (0)</td>
<td>30 (1)</td>
</tr>
<tr>
<td>30</td>
<td>0.0559</td>
<td>0.2940</td>
<td>0 (0)</td>
<td>31 (0)</td>
</tr>
<tr>
<td>40</td>
<td>0.1017</td>
<td>0.2772</td>
<td>0 (0)</td>
<td>21 (4)</td>
</tr>
<tr>
<td>50</td>
<td>0.1396</td>
<td>0.2001</td>
<td>0 (0)</td>
<td>11 (0)</td>
</tr>
</tbody>
</table>

### Table 4: Sensitivity Analysis for correlation between MCC and stock return

<table>
<thead>
<tr>
<th>Sub-sample size</th>
<th>Min correlation</th>
<th>Max correlation</th>
<th>Number of blocks with negative correlations (significant)</th>
<th>Number of blocks with positive correlations (significant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.1463</td>
<td>0.7273</td>
<td>10 (0)</td>
<td>41 (2)</td>
</tr>
<tr>
<td>20</td>
<td>-0.0257</td>
<td>0.6278</td>
<td>2 (0)</td>
<td>39 (11)</td>
</tr>
<tr>
<td>30</td>
<td>0.1602</td>
<td>0.4428</td>
<td>0 (0)</td>
<td>31 (25)</td>
</tr>
<tr>
<td>40</td>
<td>0.2939</td>
<td>0.3633</td>
<td>0 (0)</td>
<td>21 (21)</td>
</tr>
<tr>
<td>50</td>
<td>0.2737</td>
<td>0.3383</td>
<td>0 (0)</td>
<td>11 (11)</td>
</tr>
</tbody>
</table>

### Table 5: Sensitivity Analysis for correlation between MCS and stock return

<table>
<thead>
<tr>
<th>Sub-sample size</th>
<th>Min correlation</th>
<th>Max correlation</th>
<th>Number of blocks with negative correlations (significant at 0.1)</th>
<th>Number of blocks with positive correlations (significant at 0.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.7468</td>
<td>0.6248</td>
<td>22 (6)</td>
<td>29 (2)</td>
</tr>
<tr>
<td>20</td>
<td>-0.2594</td>
<td>0.3401</td>
<td>14 (0)</td>
<td>27 (0)</td>
</tr>
<tr>
<td>30</td>
<td>0.0198</td>
<td>0.2236</td>
<td>0 (0)</td>
<td>31 (0)</td>
</tr>
<tr>
<td>40</td>
<td>0.0332</td>
<td>0.2201</td>
<td>0 (0)</td>
<td>21 (0)</td>
</tr>
<tr>
<td>50</td>
<td>0.0632</td>
<td>0.1332</td>
<td>0 (0)</td>
<td>11 (0)</td>
</tr>
</tbody>
</table>
A.5: Expenditures on health by older people: Evidence from CEX

To have a better understanding of household expenditures on health over the life cycle is important. It can help us to forge reasonable hypothesis on preference over consumption and health. It can offer us an approximation on household’s prior on risks such as distribution on future health expenditure.

The Consumer Expenditure Survey, or CEX is the ideal data set to look at the out of pocket expenditures on health. CEX is a quarterly survey of households consumption behavior and conducted by Bureau of Labor Statistics, or BLS. Each household surveyed is interviewed once every quarter for five consecutive quarters. During the first interview, demographic data is collected, while for the next four four interviews, information about expenditures and new savings are collected.

We assume there is no cohort effect and to control time effect, we focus on year 2002 data. We use data compiled by Harris and Sabelhaus, which is available at NBER (http://www.nber.org/data/ces_cbo.html). We restrict our samples based on Harris and Sabelhaus so that: First, we keep those who have full year data in 2002; Second, we keep those who has positive weight; Third, we keep only those who have key demographics, such as family size and clearly defined head of the household. And the resulting 2296 households is our main sample for analysis.
Health expenditures in CEX

Health expenditures include:
1) Drug preparations. Include prescription drugs and medicines;
2) Ophthalmic products and orthopedic appliances. Include purchase of eye glasses or contact lenses; kits and equipment, fittings, warranty expenses, and insurance; purchase of medical equipment for general use, such as blood pressure kits; purchase of supportive or convalescent medical equipment, such as wheelchairs; and hearing aids;
3) Physicians, dentists, other medical professionals. Include care for invalids, convalescents, handicapped or elderly; physicians’ services; dental care; eye exams, treatment or surgery; lab tests and X-Rays; services by medical professionals other than physicians, nursing services, and therapeutic treatments; other medical care service, such as blood donation, ambulance, emergency room, or outpatient hospital services; medical care in retirement community; medical care in retirement community; rental of supportive and convalescent equipment.
4) Hospitals. Include hospital room and meals; hospital service other than room, such as operating and recovery.
5) Nursing Homes. Include care in convalescent or nursing Home.
6) Health insurance. Include commercial health insurance; traditional fee for service health plan; preferred provider health plan; Blue Cross or Blue Shield; health maintenance plans; health maintenance organization; medicare payments; commercial medicare supplements, dental insurance and other health insurance; commercial medicare supplements; and other health insurance.

Aguiar and Hurst (2009) look at the decomposition or “deconstructing” of lifecycle expenditure. Their main findings include work related consumption categories (food, especially food away from home, nondurable transportation and clothing) account for decrease in mean expenditure after middle age; that the previous three categories can also account for the increase of variance of among households; and that “all other nondurable categories” show “no decline in mean expenditure over the life cycle nor do they show an increase in cross sectional dispersion over the life cycle”. They, however, didn’t look at health expenditure, while as we found, the average share of health expenditures increase from 2.7% in early life to 14.1% for households aged above 75.

The other major items, such as food, housing, and transportation vary relative much less than health does. Table 6 shows summarizes the information.
Table 6: Average expenditure shares over the lifecycle

<table>
<thead>
<tr>
<th>Age group</th>
<th>Food</th>
<th>Health Care</th>
<th>Housing</th>
<th>Transportation</th>
<th>Apparel</th>
<th>Entertainment</th>
<th>Personal insurance and pensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 25</td>
<td>14.1%</td>
<td>2.7%</td>
<td>32.6%</td>
<td>19.4%</td>
<td>5.0%</td>
<td>4.9%</td>
<td>8.3%</td>
</tr>
<tr>
<td>25-34</td>
<td>12.6%</td>
<td>3.7%</td>
<td>36.5%</td>
<td>19.1%</td>
<td>4.4%</td>
<td>5.2%</td>
<td>10.9%</td>
</tr>
<tr>
<td>35-44</td>
<td>12.5%</td>
<td>3.9%</td>
<td>35.6%</td>
<td>17.9%</td>
<td>4.0%</td>
<td>6.0%</td>
<td>11.8%</td>
</tr>
<tr>
<td>45-54</td>
<td>12.3%</td>
<td>4.8%</td>
<td>32.9%</td>
<td>17.4%</td>
<td>3.8%</td>
<td>5.4%</td>
<td>12.8%</td>
</tr>
<tr>
<td>55-64</td>
<td>11.6%</td>
<td>6.4%</td>
<td>32.0%</td>
<td>17.9%</td>
<td>3.5%</td>
<td>5.1%</td>
<td>11.5%</td>
</tr>
<tr>
<td>65-74</td>
<td>12.4%</td>
<td>11.8%</td>
<td>32.1%</td>
<td>18.1%</td>
<td>3.1%</td>
<td>6.2%</td>
<td>6.2%</td>
</tr>
<tr>
<td>Above 75</td>
<td>12.3%</td>
<td>14.1%</td>
<td>36.7%</td>
<td>12.4%</td>
<td>2.4%</td>
<td>4.1%</td>
<td>3.3%</td>
</tr>
</tbody>
</table>

References

Monetary Economics, 15, 145–161.