Three Essays on Skill Premium, College Choice, and Investment-Specific Technological Change

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This dissertation is dedicated to Ying, with love.
Abstract

The relative wage and quantity of skilled workers have evolved in the postwar U.S. economy. Both the skill premium, i.e., college wage premium, and the relative supply of workers holding college degrees have been increasing over the second half of the last century. Previous literature proposed several explanations including skill-biased technological change and demographic change. This thesis aims to quantitatively investigate the driving forces behind the dynamics of the skill premium and skill accumulation in a dynamic general equilibrium framework.

The first essay studies the driving forces behind the dynamics of the skill premium and college enrollment rate in the postwar US economy. I develop an overlapping generations general equilibrium model with an endogenous discrete schooling choice. The production technology features capital-skill complementarity as in Krusell et al. (2000). Within this framework, I quantitatively examine the effects on the skill premium and enrollment rate of two exogenous forces, investment-specific technological change (ISTC) and the demographic change known as “the baby boom and the baby bust”. I find that demographic change plays an important role in accounting for the dynamics of the skill premium before the late 1970s, while ISTC drives most of the changes in the skill premium since then. ISTC also explains about 30% of the increases in the enrollment rate for the period 1951-2000, while demographic change does not have a significant effect on the enrollment rate.

The second essay develops and calibrates a standard growth model, in which the dynamics of both skill accumulation and wage inequality arise as an equilibrium outcome driven by measured investment-specific technological change. Within this framework, we also examine quantitatively the effects of three hypothetical tax-policy reforms. We find that a revenue-neutral elimination of capital income taxes leads to a modest increase in wage inequality and a sizable welfare gain. An increase in the progressiveness of labor income taxes although mechanically reduces after-tax wage inequality, but leads to large declines in average productivity and welfare. In contrast, a policy that provides direct subsidies for human capital accumulation tends to encourage skill formation, alleviate the wage inequality, and improve welfare.

The third essay documents a dramatic reverse pattern of gender-specific college educational attainment over the past four decades and asks a quantitative question: to what extent can such changes be accounted for by changes in the gender-specific college wage premium? I develop and calibrate an overlapping generations model with a discrete schooling
choice. I also test two expectations hypotheses—perfect foresight (rational expectations) vs. naive expectations. The calibration results show that, for men the naive expectations prediction fits the data better, while for women, the rational expectations prediction captures adequately well the trend of enrollment rate up to 1979. For both genders, the naive expectations prediction captures (even overshoots) the dramatic increase of the college enrollment rates since 1980.
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Chapter 1

Skill Premium, College Choice, Skill-Biased Technological and Demographic Change

1.1 Introduction

The skill premium, which is defined as the ratio of skilled labor wage to unskilled labor wage, has gone through dramatic changes in the postwar U.S. economy. As shown in Figure 1.1, starting from 1949, the evolution of the skill premium exhibited an “N” shape: it increased in the 1950s and 1960s, then decreased throughout the 1970s, and has increased dramatically since then. Meanwhile, as is also shown in the figure, the relative supply of skilled labor (the ratio of weeks worked by workers holding college degrees to those holding high school diplomas) has been increasing over time.

In the last two decades the literature on the skill premium has been growing. Researchers have been asking why does the pattern of the skill premium look like this. The popular explanations include investment-specific technological change through capital-skill complementarity (see Krusell, Ohanian, Rios-Rull and Violante (2000), hereafter KORV), international trade induced skill-biased technological change (Acemoglu (2003)), and skill-biased technological change associated with the computer revolution (Autor, Katz and Krueger (1998)). Probably the most popular story is the one proposed in Katz and Murphy (1992). They claim that a simple supply and demand framework works well to explain the dynamics of the skill premium. As they say: “A smooth secular increase in the relative demand for college graduates combined with the observed fluctuations in the rate of growth of relative supply could potentially explain the movements in the college wage premium from 1963 to 1987.” (Katz and Murphy (1992), page 50.) Table 1.1 (taken from Autor, Katz and Krueger
Table 1.1: Growth of College/High School Relative Wage, Supply and Demand: 1950-1998 (Annualized Percent Changes)

<table>
<thead>
<tr>
<th></th>
<th>Relative Wage (%)</th>
<th>Relative Supply (%)</th>
<th>Relative Demand (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950-60</td>
<td>0.83</td>
<td>2.91</td>
<td>4.08</td>
</tr>
<tr>
<td>1960-70</td>
<td>0.69</td>
<td>2.55</td>
<td>3.52</td>
</tr>
<tr>
<td>1970-80</td>
<td>-0.74</td>
<td>4.99</td>
<td>3.95</td>
</tr>
<tr>
<td>1980-90</td>
<td>1.51</td>
<td>2.53</td>
<td>4.65</td>
</tr>
<tr>
<td>1990-98</td>
<td>0.36</td>
<td>2.25</td>
<td>2.76</td>
</tr>
</tbody>
</table>

(1998) demonstrates the basic idea. There has been a steady growth of relative demand for skilled labor starting from 1950, but the pattern of the growth rate of relative supply of skilled labor has fluctuated. It was quite stable from 1940 to 1970 (around 2.5% per year), then dramatically increased to 4.99% per year during the 1970s. After 1980 it dropped to the original average. Therefore, if we put the supply and demand changes together, we will see that the relative price of skilled labor, which is the skill premium, dropped during the 1970s, since supply exceeded demand, while it increased in other decades.

Why did the relative supply of skilled labor increase dramatically during the 1970s? Katz and Murphy attribute it to the baby boom. High fertility rates in the U.S. from 1946 to around 1960 implied a huge increase of college graduates in the labor force since the late 1960s. In turn, the passage of the baby boom cohorts into mid-career, together with the accelerating skill-biased technological change in the 1980s, contributed to the rising college wage premium since 1980. In other words, the demographic change, together with the trend in skill-biased technological change, explains the dynamics of the skill premium.

On the other hand, KORV (2000) try to understand the source of the latent skill-biased technological change. They claim that with the capital-skill complementarity in a neoclassical aggregate production function, the growth in the stock of capital equipment will complementarily increase the marginal product of skilled labor and hence raise its relative demand. They also quantitatively evaluate how much this capital-skill complementarity has affected the skill premium from 1963 to 1992 and find that changes in observed factor inputs can account for most of the variations in the skill premium over these 30 years.

One should realize, however, that the success of KORV’s (2000) calibration depends on the pattern of fluctuations in the relative supply of skilled labor. They can generate the “V” shape of the skill premium from 1970 in their model since the so-called capital-

---

1Katz and Murphy (1992) find this secular growth in the relative demand for college graduates can be proxied by a linear time trend 3.3% per year, which can be viewed as a proxy for the skill-biased technological change (SBTC). But what drives this secular trend remains a “black box” in their paper and subsequent work along this line. For example, Bound and Johnson (1992) also attribute much of the variation in the skill premium to a residual trend component that is interpreted as SBTC.
skill complementarity effect is dominated by the relative quantity effect\textsuperscript{2} in the 1970s due precisely to high growth in the relative supply of skilled labor, while after 1980 the pattern is reversed. Thus the pattern of an inverse “V” shape of the growth rate of the relative supply of skilled labor becomes key to their paper. They take this pattern as given and focus on the relative demand side of skilled labor. The implicit assumption they impose is the orthogonality between the relative supply and demand of skilled labor.

However, from the rational expectations perspective, the supply of skilled labor is a response to the expected skill premium. If the expected skill premium increases, high school graduates will be more willing to go to college since they expect higher wages after graduation. The resulting increase of college enrollment will in turn increase the future relative supply of skilled labor, which will also affect the skill premium in the future. In other words, the relative supply of skilled labor and the skill premium interact in a dynamic way. To understand the dynamics of the skill premium, we cannot ignore this interaction.

In this sense, the simple supply-demand framework as used in Katz and Murphy (1992) is also inadequate since it ignores this dynamic interaction between the relative supply of skilled labor and the skill premium. They both are equilibrium results emerging from a dynamic general equilibrium framework.

Therefore, looking at a single picture of the skill premium cannot tell us the whole story. To understand better the dynamics of the skill premium, we should also look at the dynamics of the relative supply of skilled labor. To understand the dynamics of both the skill premium and the college enrollment rate within a dynamic general equilibrium framework is the task of this chapter.

To achieve this goal, I develop a general equilibrium overlapping generations model with endogenous discrete schooling choice. The model includes three key features. First, with \textit{ex-ante} heterogeneity in disutility cost of schooling, individuals in each birth cohort (high school graduates) choose to go to college or not based on their expected future wage differentials, the forgone wages during the college years, the tuition payments, and the idiosyncratic disutility cost. This microfoundation gives us the standard features as found in the human capital investment literature. (See for instance Ben-Porath (1967).) Second, the production technology has the feature of capital-skill complementarity as in KORV (2000): Capital is more complementary to skilled than unskilled labor. Third, following Greenwood, Hercowitz, and Krusell (1997), I assume that there exists a technological change on investment goods and call it investment-specific technological change.

Under this theoretical framework, I calibrate the model and then quantitatively examine the effects of two widely discussed exogenous forces, \textit{investment-specific technological change}

\textsuperscript{2}Please refer to equation (1.13) in Section 3 for a detailed explanation of these two effects.
(ISTC) as described above and the demographic change represented by the growth rate of the cohort size of high school graduates, on the skill premium and enrollment rate over the U.S. postwar period 1951-2000. I find that in terms of the skill premium, demographic change dwarfs ISTC before the late 1960s and accounts for about one third of the decline of the skill premium in the 1970s. However, after the late 1970s, ISTC takes over to drive the dramatic increase in the skill premium. ISTC can also explain about 30% of the increase in the enrollment rate for the period 1951-2000, while demographic change does not have a significant effect on the enrollment rate over time.

The reason why this model can generate co-rising skill premium and the enrollment rate, particularly after the early 1980s, is due to the following simple economic mechanism. When ISTC speeds up, investment becomes increasingly efficient over time, so the relative price of the capital stock falls. This encourages higher investment; hence the capital stock increases. Due to capital-skill complementarity, increase in the capital stock raises the relative demand of skilled labor, which raises the skill premium. Forward-looking people anticipate the rising skill premium which will increase the benefits of college education, thus they are more willing to go to college.

However, demographic change affects the skill premium and the enrollment rate through a different channel. For example, growing birth cohorts (“baby boom”) change the age structure in the economy and make it skewed towards younger (college age population) cohorts. On the one hand, more people stay in college and meanwhile more unskilled workers join the labor force, therefore the relative supply of skilled labor decreases, which tends to raise the skill premium. On the other hand, young people hold fewer assets over the life cycle, therefore, change in age structure also slows down asset accumulation. The decrease in capital stock, through capital-skill complementarity, tends to lower the skill premium. Thus the total effect of demographic change on the skill premium and the enrollment rate is ambiguous and has to be investigated quantitatively.

In quantitative terms, before the late 1960s, the U.S. had undergone dramatic population growth (See Figure 1.2), while the change of ISTC was only moderate (See Figure 1.15). Demographic change outweighed technological change, therefore it dominated the impact on the skill premium. After the late 1970s, the magnitude of the baby bust was much smaller compared to the baby boom, meanwhile ISTC speeded up dramatically to become the major driving force.

\[\text{Fernandez-Villaverde (2001) proposes a similar mechanism, but targeting on a totally different question. He tries to answer why growth in per capita income has coexisted with fall in fertility during the past 150 years. The idea is capital-specific technological change brings more productive capital, which through the capital-skill complementarity, raises the skill premium, i.e., return to human capital. The increment in the skill premium induces parents to choose more education instead of more children, hence causing the declining fertility.}\]
This chapter extends the existing literature on the effects of the technological change on wage inequalities. In comparison to KORV(2000), I endogenize the supply of skilled labor and put their aggregate production function into a dynamic general equilibrium setup. Hence, I can test if capital-skill complementarity is the driving force behind the skill premium and the relative supply of skilled labor within a more comprehensive framework. I also view this paper as a dynamic general equilibrium extension of the work by Katz and Murphy (1992).

In spirit, this chapter is close to Heckman, Lochner and Taber (1998). They develop and estimate an overlapping generations general equilibrium model of labor earnings and skill formation with heterogenous human capital. They test their framework by building into the model a baby boom in entry cohorts and an estimated time trend of increase in the skill bias of aggregate technology. They find that the model can explain the pattern of wage inequality since the early 1960s. My model focuses on a different channel: capital-skill complementarity. In my model, ISTC is the source of skill-biased technological change. Thanks to the work by Greenwood, Hercowitz, and Krusell (1997) and Cummins and Violante (2002), I have data about this measured technological change. Therefore, instead of simulation, I am able to test my model by feeding in time series of ISTC and demographic change. This allows me to compare the model results with the data. A recent paper by Guvenen and Kuruscu (2006) presents a tractable general equilibrium overlapping generations model of human capital accumulation which is consistent with several features of the evolution of the U.S. wage inequality from 1970 to 2000. Their work shares the similar microfoundation of schooling choice as in this paper. But they do not have capital stock in the production technology, and hence no capital-skill complementarity, either. The only driving force in their paper is the skill-biased technological change which is calibrated to match the total rise in wage inequality in the U.S. data between 1969 and 1995. They do not have ISTC in their model, therefore the mechanism for rising wage inequality is different.

My model also extends another strand of literature about the effect of cohort size on schooling choices. Ahlburg et al. (1981) find that there does appear to be a significant statistical relationship between cohort size and the educational attainment of the cohort. Flinn (1993) develops a perfect foresight overlapping generations model to investigate the effects of cohort size on schooling decisions and cohort-specific welfare measures in a partial equilibrium environment. Under a set of sufficient conditions, he finds the existence of a unique mapping from any cohort size sequence to both the human capital rental rate and schooling choice sequences. His calibration exercise shows that the equilibrium response of schooling to perturbations in the cohort size sequence is small. Based on the structural estimation framework developed in Keane and Wolpin (1997), Lee (2005) extends Flinn’s partial equilibrium schooling choice model to a dynamic general equilibrium model of career
decisions (schooling, occupation and labor force participation choices). He then uses the model’s estimates to determine the impact of cohort size on human capital investment behavior and labor market outcomes. He has a Cobb-Douglas production and hence does not allow capital-skill complementarity in the technology. Capital stock, and capital and skilled labor share are all exogenously given. He also finds that the effect of demographic change (“the baby boom” and “the baby bust”) on the male college completion rate is insignificant (change as much as from -1% to 0.3%). This chapter extends and enriches the production side of Lee (2005) by including ISTC and capital-skill complementarity. Capital is endogenous in my model and the accumulation of capital stock is key to driving the skill premium. Within this framework, I still find that demographic change does not have a significant effect on schooling choice.

The remainder of this chapter is organized as follows. Section 1.2 documents some stylized facts about the dynamics of the cohort size of high school graduates, the college enrollment rate, and college tuition in the postwar U.S. economy. It also emphasizes the links among these facts. Section 1.3 presents my economic model of college going decisions, describes the market environment, and defines the general equilibrium in the model economy, thus laying out the theoretical foundation for the later data analysis and calibration exercise. Section 1.4 shows how to parameterize the model economy. Section 1.5 provides calibration results for the pre-1951 steady state. Section 1.6 computes the transition path of the model economy from 1951 to 2000 and compares the results with the data. It also conducts some counterfactual experiments to isolate the effects of investment-specific technological change and demographic change on the skill premium and the college enrollment rate, respectively. Finally, Section 1.7 concludes.

1.2 Stylized Facts

Figure 1.1 already shows the pattern of the skill premium and the relative supply of skilled labor. To understand the dynamics of the relative supply of skilled labor, we should ask the question of who provides skilled labor (workers with college degrees) and hence naturally turn our attention to the individual’s college education decisions. We should also be aware that the baby boom and baby bust only affect the cohort size of college age (in this chapter I restrict the college age to be 18-21 years old). From 1970 to 1980 the college age cohort increased substantially due to high fertility in the 1950s. If the college enrollment rate did not change through the 1970s, the growth of relative supply should not have changed drastically. If the college enrollment rate increased dramatically as well as the cohort size, then by combining these two effects we would not be surprised to see that the relative supply of college educated workers increased greatly, as we observe in the data.
Figure 1.2 shows the cohort size of high school graduates. It was very stable before the early 1950s, then began growing, reached a peak around 1976, and has decreased since then. Since the common age of high school graduation is around 18, we can view this graph as a 18-year lag version of U.S. fertility growth, i.e., it reflects the baby boom and baby bust.  

Figure 1.3 measures the college-age population. I report the 17-21 and 18-21 age population in the U.S. since 1955. They follow a similar pattern as in Figure 1.2. The baby boom pushed the college-age population up until the fertility rate reached its peak around 1960, corresponding to the peak of the college-age population around 1980. The baby bust then dragged the population size down.

The above two figures show changes in the population base of potential college students, but does the proportion of people going to college change over time? Figure 1.4 shows the college enrollment rate of recent high school completers. It began growing from the early 1950s until 1968 and then went down; the entire 1970s was a depressed decade for college enrollment, and it was not until 1985 that the enrollment rate exceeded the level in 1968. Starting from 1980 (notice the timing), the enrollment rate kept increasing for nearly 20 years. This pattern is also confirmed by other studies. (See Macunovich (1996) Figure 1.a, 1.b, 2.a, 2.b, Card and Lemieux (2000) Figure 3.)

By comparing the skill premium in Figure 1.1 and the college enrollment rate in Figure 1.4, I observe that interestingly they share a very similar pattern. This similarity implies a tight link between college-going decision and the expected skill premium. Future skill premium represents the expected gain from higher education. As the expected benefits increase, the enrollment rate increases.

To understand the behavior of the skill premium exhibited in Figure 1.1, we cannot ignore the behavior of schooling choice exhibited in Figure 1.4. The skill premium represents the benefit of a college education. To fully understand the determinants of schooling choice, we should also look at the cost side of college-going. In Figure 1.5 I report the real tuition, fee, room and board (TFRB) per student charged by an average four-year institution (average means the enrollment weighted average of four-year public and private higher education institutions, see Appendix A for details). Again, we see a pattern similar to that of the skill premium and the college enrollment rate. TFRB increased over time except in the 1970s. Starting from 1980 (notice this timing again), real TFRB has raised dramatically.

The similarity is not surprising since it reflects supply and demand in the higher education market. Higher demand for skilled labor in the 1980s and 1990s pushed up the skill premium as we see in Figure 1.1. More people wanted to go to college, hence the enrollment rate increased as shown in Figure 1.4. In turn, higher demand for college education raised

\textsuperscript{4}Cohort size has been increasing again since 1995 because the baby-boomer’s children reached college age since the mid 1990s.
the price of the college education, as shown in Figure 1.5.

The stylized facts relevant to this chapter can be briefly summarized as follows:

1. The skill premium rose during the 1950s and 1960s, then fell from 1971 to 1979, and has increased dramatically since 1980.

2. The relative supply of skilled labor has increased since the 1940s.

3. The college enrollment rate exhibits a similar pattern as the skill premium, as do the tuition payments.

The stylized facts we observe from Figure 1.1 (skill premium) and 1.4 (enrollment rate) are the targets of this chapter. To understand Figure 1.1, we also must understand Figure 1.4 because it tells us where the relative supply of skilled labor comes from. By modeling the dynamic interaction between them, I am able to examine the driving forces behind the dynamics of the skill premium and enrollment rate and answer an important quantitative question: By taking the demographic change as in Figure 1.2 and the measured investment-specific technological change as in Figure 1.15 exogenously given and feeding them into a dynamic general equilibrium model, what percentage of change in the skill premium and the enrollment rate can be explained by each of these two exogenous forces?

1.3 Model

In this section, I will present the economic model that will be used later for calibration. It is a discrete time overlapping generations (OLG) model. Individuals make the schooling choice in the first period. There is only one good in the economy that can be used either in consumption or investment.

1.3.1 Demographics

The economy is populated by overlapping generations. People enter the economy when they are 18 years old and finish high school, which I call the birth cohort and model as age $j = 1$. I assume people work up to age $J$, which is the maximum life span.\(^5\) To distinguish between the age of a cohort and the calendar time, I will use $j$ for the age, and $t$ for the calendar time. For example, $N_{j,t}$ is the population size of age-$j$ cohort at time $t$.

\(^5\)In an unreported experiment, I extend current model to including retirement, social security system and lifetime uncertainty. The quantitative results are very similar to the ones presented here, while life cycle profiles of consumption and asset holdings are more realistic (hump-shaped). This version is available upon request from the author.
In every period $t$ a new birth cohort enters the economy with cohort size $N_{1,t}$. It grows at rate $n_t$. Therefore, I have

$$N_{1,t} = (1 + n_t)N_{1,t-1}. \tag{1.1}$$

The fraction of age-$j$ cohort in the total population at time $t$ is

$$\mu_{j,t} = \frac{N_{j,t}}{N_t} = \frac{N_{j,t}}{\sum_{i=1}^{J} N_{i,t}}. \tag{1.2}$$

It will be used to calculate the aggregate quantities in the economy as cohort weights throughout the transition path.

The birth cohort in the model corresponds to the high school graduates (HSG) in Figure 1.2, and the growth rate of HSG cohort size is the data counterpart of $n_t$. Therefore, the “baby boom” corresponds to the 1951-1976 period when $n_t$ increased over time, while the “baby bust” period is from 1976 to 1990 when $n_t$ decreased over time.

### 1.3.2 Preferences

Individuals born at time $t$ want to maximize their discounted life time utility

$$\sum_{j=1}^{J} \beta^{j-1} u(c_{j,t+j-1}).$$

The period utility function is assumed to take the CRRA form

$$u(c_{j,t+j-1}) = \frac{c_{j,t+j-1}^{1-\sigma}}{1-\sigma}. \tag{1.3}$$

$\sigma$ is the coefficient of relative risk aversion, therefore $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution. Since leisure does not enter into the utility function, each individual will supply all her labor endowment, which is normalized to be one.

### 1.3.3 Budget Constraints

An individual born at time $t$ chooses whether or not to go to college at the beginning of the first period. I use $s \in \{c, h\}$ to indicate this choice. If an individual chooses $s = h$, she ends up with a high school diploma and goes on the job market to work as an unskilled worker up to age $J$, and earns high school graduate wage sequence $(w_{j,t+j-1}^h)_{j=1}^{J}$. Or, she can choose $s = c$, spend the first four periods in college as a full time student, and pay the tuition. I assume she can always successfully graduate from college (there is no some college or college dropout in the model). After that, she goes on the job market to
find a job as a skilled worker, and earns a college graduate wage sequence \( \{w_{c,j+t+1}\}_{j=1}^{f} \).

After this schooling choice, within each period, an individual makes consumption and asset accumulation decision according to her choice.

For \( s = c \), the budget constraints of the cohort born at time \( t \) are

\[
\begin{align*}
    c_{j,t+j-1} + \text{tuition}_{t+j-1} + a_{j,t+j-1} & \leq (1 + r_{t+j-1})a_{j-1,t+j-2} \quad \forall j = 1, 2, 3, 4 \quad (1.4) \\
    c_{j,t+j-1} + a_{j,t+j-1} & \leq (1 + r_{t+j-1})a_{j-1,t+j-2} + w_{c,j+t-1}c_{j} \quad \forall j = 5, \ldots, J \\
    c_{j,t+j-1} & \geq 0, a_{0,t-1} = 0, a_{j,t+j-1} \geq 0, \quad (1.5)
\end{align*}
\]

where \( \{c_j\}_{j=1}^{f} \) is the age-efficiency profile of college graduates. It represents the age profile of the average labor productivity for college graduates. Notice that individuals have zero initial wealth and cannot die in debt.\(^6\)

For \( s = h \), the budget constraints of the cohort born at time \( t \) are

\[
\begin{align*}
    c_{j,t+j-1} + a_{j,t+j-1} & \leq (1 + r_{t+j-1})a_{j-1,t+j-2} + w_{h,j+t-1}h_{j} \quad \forall j = 1, \ldots, J \quad (1.6) \\
    c_{j,t+j-1} & \geq 0, a_{0,t-1} = 0, a_{j,t+j-1} \geq 0.
\end{align*}
\]

Similarly, \( \{h_j\}_{j=1}^{f} \) is the age-efficiency profile of high school graduates.

1.3.4 Schooling Choice

Next, I would like to explicitly model an individual’s schooling choice. In order to be able to generate a positive enrollment rate in the model, I need to introduce some \emph{ex-ante} heterogeneity within each birth cohort. Without this within-cohort heterogeneity, the enrollment rate would be just only zero or one.

I assume that different individuals within each birth cohort are endowed with different levels of disutility cost of schooling. I index people by their disutility level \( i \in [0, 1] \), and the associated disutility cost that individual \( i \) bears is \( DIS(i) \). I assume \( DIS'(i) < 0 \). The Cumulative Distribution Function (CDF) of disutility cost is denoted by \( F \), \( F(i_0) = \Pr(i \leq i_0) \). Now an individual \( i \) born at time \( t \) has her own expected discounted life time utility

\[
\sum_{j=1}^{J} \beta^{j-1}u(c_{j,t+j-1}) - T_i \cdot DIS(i), \quad (1.7)
\]

\(^6\)Notice that the model does not have exogenous borrowing constraints. However, the standard properties of the utility function and the restriction that the agent cannot die in debt impose an endogenous borrowing constraint at every period.
where

\[ I_i = \begin{cases} 
1 & \text{if } s_i = c \\
0 & \text{if } s_i = h
\end{cases} \]

subject to the conditional budget constraints (1.4)-(1.5) or (1.6), depending on individual \( i \)'s schooling choice \( s_i \). Notice that idiosyncratic disutility cost \( DIS(i) \) does not enter into the budget constraints, so everyone within the same cohort and with the same education status will have the same life-time utility derived from physical consumption. I use \( UTIL_c^i \) to denote the discounted life-time utility derived from people who are born at time \( t \) and choose to go to college \((s = c)\) and \( UTIL_h^i \) to denote the discounted life-time utility derived from people who choose not to go \((s = h)\). Therefore, \( UTIL_c^i - UTIL_h^i \) represents the utility gain from attending college. Obviously, individual \( i \) will choose go to college if \( DIS(i) < [UTIL_c^i - UTIL_h^i] \), will not go if \( DIS(i) > [UTIL_c^i - UTIL_h^i] \), and is indifferent if \( DIS(i) = [UTIL_c^i - UTIL_h^i] \).

It is easy to show that this model implies

\[ UTIL_c^i - UTIL_h^i \gtrless 0 \text{ if } NPV_t \gtrless 0, \]

where

\[ NPV_t = \sum_{j=5}^{J} \frac{w^c_{t+j-1}v_j^h - w^h_{t+j-1}v_j^h}{\prod_{i=2}^{t}(1 + r_{t+i-1})} - \sum_{j=1}^{4} \frac{w^h_{t+j-1}v_j^h}{\prod_{i=2}^{t}(1 + r_{t+i-1})} - \sum_{j=1}^{4} \frac{tuition_{t+j-1}}{\prod_{i=2}^{t}(1 + r_{t+i-1})}. \]  

(1.8)

Here \( NPV \) stands for the net present value of higher education. It consists of three terms. The first term represents the benefit of schooling, college graduates can earn more through the skill premium. The second term represents the opportunity cost of schooling. It is the four year foregone wage income for the college students. The third term is the present value of tuition paid during college time, which represents the direct cost of schooling. From this representation it is very clear how the skill premium is going to affect an individual’s schooling decision. Keeping other things equal, an increase in the skill premium will raise the benefit of schooling, thus raising \( NPV \). A higher \( NPV \) will induce higher utility gain from schooling \( UTIL_c^i - UTIL_h^i \). If we assume that the distribution of disutility cost is stationary, higher utility gain from schooling means it is more likely that \( DIS(i) < [UTIL_c^i - UTIL_h^i] \), which implies that more people would like to go to college. This mechanism is going to generate the co-movement between skill premium and enrollment rate as observed in the data.
1.3.5 Production

I close the model by describing the production side of the economy. The representative firm in the economy uses capital stock \((K)\), skilled labor \((S)\), and unskilled labor \((U)\) to produce a single good. Here skilled labor consists of college graduates, and unskilled workers are high school graduates. Following KORV (2000), I adopt the form of aggregate production function with capital-skill complementarity as follows:

\[
Y_t = A_t F(K_t, S_t, U_t) = A_t [\mu U_t^\theta + (1 - \mu)(\lambda(B_t K_t)^\rho + (1 - \lambda)S_t^\rho)]^{1/\theta}
\]

where \(A_t\) is the level of total factor productivity (TFP), \(B_t\) is the level of capital productivity and represents capital-embodied technological change, and we have \(0 < \lambda, \mu < 1\) and \(\rho, \theta < 1\). This production technology is constant return to scale. The elasticity of substitution between capital-skilled labor combination and the unskilled labor is \(\frac{1}{\rho - 1}\) and the one between capital and skilled labor is \(\frac{1}{\theta - 1}\). For the capital-skill complementarity, we require \(\frac{1}{1 - \rho} < \frac{1}{1 - \theta}\), which means \(\rho < \theta\).

The difference between my production function and the one in KORV (2000) is that I do not distinguish between structure and equipment, so the capital \(K\) in my model is just the total capital stock.

The representative firm rents capital, skilled labor and unskilled labor from households at the rates \(r_t\), \(w_t^c\) and \(w_t^h\). Its profit maximization implies the first order conditions below

\[
\begin{align*}
 r_t &= \lambda(1 - \mu)A_t B_t^{\rho} H_t (\lambda(B_t K_t)^\rho + (1 - \lambda)S_t^\rho)^{\frac{\theta - 1}{\theta}} - \delta \\
 w_t^c &= (1 - \mu)(1 - \lambda)A_t H_t (\lambda(B_t K_t)^\rho + (1 - \lambda)S_t^\rho)^{\frac{\theta - 1}{\theta}} - S_t^\rho \\
 w_t^h &= \mu A_t H_t U_t^{\theta - 1}
\end{align*}
\]

where \(H_t = [\mu U_t^\theta + (1 - \mu)(\lambda(B_t K_t)^\rho + (1 - \lambda)S_t^\rho)]^{\frac{1}{\theta - 1}}\). \(\delta\) is the capital depreciation rate. Dividing (1.11) by (1.10), I derive the expression for the skill premium

\[
\frac{w_t^c}{w_t^h} = \frac{(1 - \mu)(1 - \lambda)}{\mu} [\lambda(B_t K_t)^\rho + (1 - \lambda)]^{\theta - 1} S_t^{\rho - 1} [U_t^{-1}].
\]

Log-linearizing (1.13), differentiating it with respect to time, and using “hat” to denote the rate of change \((\hat{X} = \frac{\dot{X}}{X})\), I obtain (ignoring the time subscript for the convenience)

\[
\left(\frac{\dot{w}_t^c}{\dot{w}_t^h}\right) \approx \lambda(\theta - \rho)(\frac{B_t K_t}{S})^\rho [\hat{B} + \hat{K} - \hat{S}] + (\theta - 1)[\hat{S} - \hat{U}].
\]

This equation is exactly as same as in KORV (2000) except for the \(B\) term. It says that the
growth rate of the skill premium is determined by two components. One is the growth rate of the relative supply of skilled labor \( \dot{S} - \dot{U} \). Since \( \theta < 1 \), relatively faster growth of skilled labor will reduce the skill premium. This term is called “relative quantity effect” in KORV (2000). Another term \( \lambda(\theta - \rho)(\frac{B}{S})^\rho[\dot{B} + \dot{K} - \dot{S}] \) is called the “capital-skill complementarity effect”. If capital grows faster than skilled labor, this term will raise the skill premium due to \( \rho < \theta \). The dynamics of the skill premium depend on the trade-off between these two effects.

The law of motion for the capital stock in this economy is expressed as

\[
K_{t+1} = (1 - \delta)K_t + X_t q_t,
\]

where \( X_t \) denotes capital investment. Following Greenwood, Hercowitz and Krusell (1997, GHK hereafter), I interpret \( q_t \) as the current state of the technology for producing capital, hence changes in \( q \) represent the notion of investment-specific technological change. When \( q \) increases, investment becomes increasingly efficient over time.

To simplify the computation, following Fernandez-Villaverde (2001), I map investment-specific technological change into the changes in the capital productivity level \( B_t \).\(^7\) Therefore, increases in \( q_t \) will transform into increases in \( B_t \). As shown in equation (1.14), when \( B_t \) increases, through capital-skill complementarity effect, it will raise the skill premium. Investment-specific technological change thus is also skill-biased.\(^8\)

Finally, the resource constraint in the economy is given by

\[
C_t + TUITIOn_t + X_t = Y_t,
\]

where \( C_t \) is the total consumption and \( TUITIOn_t \) is the total tuition payment.

### 1.3.6 The Recursive Competitive Equilibrium

The model above is a standard OLG setting with discrete schooling choices. I assume people have perfect foresight so that they can forecast all the future wages they are going to earn. Suppose an individual \( i \) born at time \( t \) has already made the schooling decision \( s_{i,t} \). Conditional on this choice, I can present her utility maximization problem in terms of dynamic programming representation.

\(^7\)The transformation is \( B_t = 1 + (1 - (1 - \delta)^{t-1})(\frac{B}{q_1} - 1) \). I normalize \( q_t = 1 \) in initial steady state. Please refer to Fernandez-Villaverde (2001) for the details.

\(^8\)More accurately, it can be easily shown that an economy with investment-specific technological change \( q \), but without capital-embodied technological change \( B \) (which is my benchmark economy), can be equivalent to another economy with capital-embodied technological change \( B \), but without ISTC in terms of allocation. (See Greenwood, Hercowitz and Krusell (1997) for details.) Since changes in \( B_t \) will increase the skill premium, due to the equivalence, the ISTC in my benchmark economy has the same effect.
For \( s_{i,t} = c \), let \( V^c_{t+j-1}(a_{j-1,t+j-2}, j) \) denote the value function of an age-\( j \) individual with asset holding \( a_{j-1,t+j-2} \) at beginning of time \( t+j-1 \), and it is given as the solution to the dynamic problem

\[
V^c_{t+j-1}(a_{j-1,t+j-2}, j) = \max_{\{c_{j,t+j-1}, a_{j,t+j-1}\}} \{u(c_{j,t+j-1}) + \beta V^c_{t+j}(a_{j,t+j-1}, j+1)\} \tag{1.15}
\]

subject to (1.4)-(1.5).

For \( s_{i,t} = h \), the corresponding value function is

\[
V^h_{t+j-1}(a_{j-1,t+j-2}, j) = \max_{\{c_{j,t+j-1}, a_{j,t+j-1}\}} \{u(c_{j,t+j-1}) + \beta V^h_{t+j}(a_{j,t+j-1}, j+1)\} \tag{1.16}
\]

subject to (1.6).

Individuals solve their perfect foresight dynamic problem by using backward induction. Back to age 1, an individual with disutility index \( i \) will choose \( s_{i,t} \) based on the criteria below

\[
s_{i,t} = \begin{cases} c & \text{if } V^c_t(a_{0,t-1} = 0, 1) - \text{DIS}(i) > V^h_t(a_{0,t-1} = 0, 1) \\ h & \text{if } V^c_t(a_{0,t-1} = 0, 1) - \text{DIS}(i) < V^h_t(a_{0,t-1} = 0, 1) \\ \text{indifferent} & \text{if } V^c_t(a_{0,t-1} = 0, 1) - \text{DIS}(i) = V^h_t(a_{0,t-1} = 0, 1). \end{cases} \tag{1.17}
\]

Based on the individuals’ dynamic program and schooling choice criteria above, the definition of the competitive equilibrium in this model economy is standard.

**Definition 1** Let \( A = \{a : -b \leq a \leq a_{\text{max}}\} \), \( S = \{h, c\} \), \( J = \{1, 2, \ldots, J\} \), \( D = [0, 1] \) and \( T = \{1, 2, \ldots, T\} \). Given the age structure \( \{\{\mu_{j,t}\}_{j=1}^{T}\}_{t=1}^{T} \), a Recursive Competitive Equilibrium is a sequence of individual value functions \( V^s_t : A \times J \to \mathbb{R} \); individual consumption decision rules \( C^s_t : A \times J \to R_+ \); individual saving decision rules \( A^s_t : A \times J \to A \) for \( s \in S \) and \( t \in T \); a period one individual \( i \)'s schooling choice \( s^s_{i,t} \) for \( s \in S \), \( i \in D \) and \( t \in T \); an allocation of capital and labor (skilled and unskilled) inputs \( \{K_t, S_t, U_t\}_{t=1}^{T} \) for the firm; a price system \( \{w^c_t, w^h_t, r_t\}_{t=1}^{T} \); and a sequence of measures of individual distribution over age and assets \( \lambda^s_t : A \times J \to R_+ \) for \( s \in S \) and \( t \in T \) such that:

1. Given prices \( \{w^c_t, w^h_t, r_t\} \), the individual decision rules \( C^s_t \) and \( A^s_t \) solve the individual dynamic problems (1.15) and (1.16).

2. Optimal schooling choice \( s^s_{i,t} \) is the solution to the schooling choice criteria (1.17) for each individual \( i \).
3. Prices \( \{w^f_t, w^h_t, r_t\} \) are the solutions to the firm’s profit-maximization problem (1.10)-(1.12).

4. The time-variant age-dependent distribution of individuals choosing \( s \) follows the law of motion
   \[
   \lambda^s_{t+1}(a', j + 1) = \sum_{a:a' \in A_t^s(a, j)} \lambda^s_t(a, j).
   \] (1.18)

5. Individual and aggregate behaviors are consistent.

   \[
   K_t = \sum_j \sum_a \sum_s \mu_{j,t} \lambda^s_t(a, j) A^s_t(a, j - 1)
   \] (1.19)

   \[
   S_t = \sum_j \sum_a \mu_{j,t} \lambda^c_t(a, j) \varepsilon^c_j
   \] (1.20)

   \[
   U_t = \sum_j \sum_a \mu_{j,t} \lambda^h_t(a, j) \varepsilon^h_j.
   \] (1.21)


   \[
   \sum_{j=1}^J \sum_a \sum_s \mu_{j,t} \lambda^s_t(a, j) C^s_t(a, j) + \sum_{j=1}^4 \sum_a \mu_{j,t} \lambda^c_t(a, j) tuition_{j,t} + X_t = Y_t
   \] (1.22)

   *i.e.*

   \[
   C_t + TUITION_t + X_t = Y_t.
   \]

When ISTC and demographic change both stabilize at some constant levels, i.e., \( q_t = q \) and \( n_t = n \), \( \forall t \), the economy reaches a steady state. In such a steady state, the age structure, the distribution of individuals over assets and age, and the individual decision rules are all age-dependant but time-invariant. Therefore, I can define the stationary competitive equilibrium accordingly.

### 1.4 Parameterization

In this section, I calibrate the model economy to replicate certain properties of the U.S. economy in the pre-1951 initial steady state. More specifically, my strategy is to choose parameter values to match *on average* features of the U.S. economy from 1947 to 1951.\(^9\)

\(^9\)I choose the U.S. economy from 1947 to 1951 as the initial steady state based on the observations that both the ISTC and demographic changes were quite stable for this time period.
1.4.1 Data Work of Cohort-specific Skill Premium

The skill premium data I report in Figure 1.1 is the average skill premium across all age groups in a specific year. However, since the model presented here is a cohort-based OLG model, each cohort’s college-going decision is based on this cohort’s specific life time skill premium profile. For example, for the cohort born at time $t$, the life time cohort-specific skill premium is $\left\{ \frac{w_{t+j+1-1}^c}{w_{t+j+1-1}^h} \right\}_{j=1}^J$. In order to understand the mechanism of schooling decision for each cohort, I need to find the data counterpart of this cohort-specific skill premium.

I use March CPS data from 1962 to 2003, plus 1950 and 1960 Census data to construct the cohort-specific skill premium profiles for the 1948-1991 cohorts. (I choose to end the sample in 1991 due to the data quality. 1991 cohort only has twelve year HSG wage and eight year CG wage data.) In order to make my results comparable to the literature, I follow Eckstein & Nagypál (2004) in restricting the data. (please refer to their paper for the details.) The sample includes all full-time full-year (FTFY) workers between age 18 and 65. To be consistent with the model, I only look at high school graduates (HSG) and college graduates (CG). The wage is the annualized real wage (in terms of year 2002 U.S. dollars). In Figure 1.6, I show the mean CG and HSG wages (top panel) and the skill premium (bottom panel) which is the ratio between these two means for the sample period 1949-2002. It is similar to the pattern of the skill premium shown in Figure 1.1 which includes post college graduates in the skilled labor group. However, the decline during the 1970s is flatter and the magnitude of the increase since 1980 is smaller. This is because the wage of post college graduates increases even faster than CG.\footnote{Eckstein & Nagypál (2004) find this fact. Please refer to their paper for more details.} Including them (as Autor and Katz (1999) do) further widens the wage gap between skilled and unskilled labor.

Since CPS is not a panel data set, theoretically speaking I cannot keep tracking specific cohorts from it. However, since it is a repeated cross-sectional data set, I can use a so-called “synthetic cohort construction method” to construct a proxy of a cohort’s specific skill premium. For example, the 1962 cohort (18 year old HSG in 1962)’s life time (18-65 year old) HSG wage profile $\left\{ w_{1961+j+1}^h \right\}_{j=1}^{48}$ is constructed as following: I take the 18 year old HSGs in 1962 and calculate their mean wage. Then I take the 19 year old HSGs in 1963 and calculate their mean wage. Next, the 20 year old HSGs in 1964, the 21 year old HSGs in 1965, and so on, until the 58 year old HSGs in 2002. Later I will show how to predict the mean HSG wage after age 58 to complete the life cycle wage profile for this cohort. See Figure 1.7 for the construction.

I perform a similar procedure to construct the 1962 cohort’s CG wage profile $\left\{ w_{1961+j+1}^c \right\}_{j=1}^{48}$. But I start from 1966 because if someone from the 1962 cohort chose to go to college, she would spend four years in college, graduate in 1966, and start to earn CG wage thereafter.
Therefore, I take the 22 year old CGs in 1966, calculate their mean wage, and then follow the above procedure again. See Figure 1.8 for the construction.

Using this method repeatedly for each birth cohort, I have the original data sequences of cohort-specific HSG and CG life-time wage profiles for the 1948-1991 cohorts. However, due to the time range of the CPS data, some data points are missing for a complete life-time profile for every cohort. For example, some cohorts are missing at the late age data points (cohorts after 1962) and some are missing at the early age data points (e.g., cohort 1948-1961). I use the econometric method to predict the mean wage at those specific age points to interpolate the missing data. I predict them by either second or third order polynomial, or conditional Mincer equation as follows

\[
\begin{align*}
\log[HSG\text{wage}(age)] &= \beta_0^h + \beta_1^h \text{experience}_h + \beta_2^h \text{experience}_h^2 + \epsilon^h, \text{ experience}_h=age-18 \\
\log[CG\text{wage}(age)] &= \beta_0^c + \beta_1^c \text{experience}_c + \beta_2^c \text{experience}_c^2 + \epsilon^c, \text{ experience}_c=age-22.
\end{align*}
\]

The criteria is basically the goodness of fit. I also check with the neighborhood cohorts to make sure the predicted value is reasonable. The “rule of thumb” of hump-shaped profile also applies here to help make choices. As an example, in Figure 1.9 and 1.10 I show the prediction for the 1955 cohort by using the trendlines of second and third order polynomial. Obviously the third order one fits the data better, therefore, I use it to predict the missing data points for this cohort.

Through this procedure, I obtain the complete life-time wage profiles for the 1948-1991 cohorts. As the first cut, I want to feed this data into the building block of the schooling choice problem (see Section 1.3.4) to see if it can shed some light on how people will react to these cohort-specific wage profiles. In Figure 1.11, I show the calculation of \(\text{NPV}\) (assume \(r_t+j = 6\% \ \forall t, \forall j\)) for the 1948-1991 cohorts as in (1.8), together with the internal rate of return (IRR) to higher education for the same period.\(^{11}\) From this figure, we see both \(\text{NPV}\) and IRR have increased since 1970, but the biggest jump occurred in 1980. The tremendous fluctuation after 1986 is partially due to the bad data quality. (We have less data points for cohorts after 1986. For example, cohort 1987 only has 16 data points for HSG wage profile from age 18 to age 35 and 12 data points for CG wage profile.) If people react to this \(\text{NPV}\) pattern, we should expect that in the model the enrollment rate increases in 1950s, then beginning in 1960, declines slightly until 1970. After 1970, it should start to increase again and jump to a very high level around 1980.

Two findings need to be mentioned for the time path of the cohort-specific return to schooling. First, they show the possible Vietnam War Draft effect. If the schooling choice

\(^{11}\) My estimation of IRR (range is between 9% and 14%) is closely in line with the mainstream research about the human capital investment, which claims IRR of higher education is around 9-16%.
is purely based on a cost-benefit analysis as human capital investment theory suggests, we would see that the enrollment rate decreased in the 1960s, then increased since 1970. In this sense the spike centered around 1968 that we observe from data (Figure 1.4) could be related to the Vietnam War. To avoid the draft, males raised their enrollment into college.\textsuperscript{12} (Of course to clearly pin down this effect, I have to distinguish male enrollment rate from the female one. See the third chapter in this thesis for the further work on this topic.) Second, the cohort-specific NPV captures the dramatic increase in the skill premium since 1980, so it also implies that the enrollment rate should follow, which is consistent with the data.

### 1.4.2 Distribution of Disutility Cost

Now the distribution of disutility cost becomes very crucial in my computation because it is this distribution that determines the enrollment rate in the model. The problem is how to obtain it.

The schooling choice criteria embodied in (1.17) actually sheds some light on how to compute the distribution of disutility cost. Note that the person $i^*$ who is indifferent between going to college or not will have

$$V^c_t(a_{0,t-1} = 0, 1) - DIS(i^*) = V^h_t(a_{0,t-1} = 0, 1),$$

i.e., her disutility cost is exactly the difference between two conditional value functions. Since disutility cost is a decreasing function of index $i$, people with disutility index $i > i^*$ will go to college. Therefore, for a specific cohort $t$, if we calculate the difference between two conditional value functions $V^c_t(a_{0,t-1} = 0, 1) - V^h_t(a_{0,t-1} = 0, 1)$, it gives us the cut-off disutility cost for this cohort. If we also know the enrollment rate of this cohort, it tells us the proportion of people in this cohort who have less disutility than $i^*$ at that specific cut-off point of disutility cost. By doing so, I can pin down one point on the CDF of disutility cost. Applying this procedure to different cohorts will give me a picture of how disutility cost is distributed.

Fortunately I have cohort-specific life time wage profile data from 1948 to 1991. I set interest rate equal 3\%, discount parameter $\beta = 1.03$, and the preference parameter $\sigma = 1.5$.\textsuperscript{13} For each cohort born at time $t$, I normalize 18 year old HSG wage (which is $w^h_{18}$ in the model) to one and feed in the cohort-specific life time wage profiles. I go

\textsuperscript{12}My transition path results also confirm this finding. See section 1.6.1 for more details.

\textsuperscript{13}Since I obtain the disutility cost from the partial equilibrium computation given the preference parameter $\sigma$, the discount rate $\beta$ and the interest rate $r$, later when I calibrate the model to match pre-1951 steady state, I need to make sure these three values are consistent with those used in the general equilibrium. The values I give here are consistent with those I obtain later in calibrating the initial steady state.
through the backward induction of Bellman equation as described in Section 1.3.6 to obtain the value function difference $V^c_t(a_{0,t-1} = 0, 1) - V^h_t(a_{0,t-1} = 0, 1)$ for every cohort $t$. In Figure 1.12, by plotting them against enrollment rate data in the same time range, I have 44 points on the possible CDF of the disutility cost. I then use OLS to estimate the CDF function locally. Later in the computation of the stationary equilibrium and transition path, during each iteration when I obtain factor prices $\{w_t^c, w_t^h, r_t\}$, I can go do backward induction of Bellman equations loop to get the conditional value functions. Feeding the difference between these two functions into the estimated CDF, I have the corresponding enrollment rate.

1.4.3 Demographic

The model period is one year. Agents enter the model at age 18 ($j = 1$), work up to age 65 and die after that age ($J = 48$).

The growth rate of cohort size $n$ is calculated as the average growth rate of the HSG cohort size from 1948 to 1951, which is 0%.

1.4.4 Preferences and Endowments

I pick CRRA coefficient $\sigma = 1.5$, which is in the reasonable range between 1 and 5 and is widely used in the literature (e.g., Gourinchas and Parker (2002), Attanasio et al. (1999), and Chen et al. (2004) among others).

The age-efficiency profile of high school graduates $\{\varepsilon^h_j\}_{j=1}^J$ and college graduates $\{\varepsilon^c_j\}_{j=1}^J$ are calculated as follows: from the 1962-2003 CPS and the 1950 and 1960 Census data I calculate the mean HSG and CG wages across all ages for the time period 1949-2002, then I obtain the mean HSG and CG wage in the same time period for each age group. Thus the age-efficiency profiles are expressed as

$$\varepsilon^h_j = \frac{\text{HSG wage}_j}{\text{HSG wage}}, \quad \varepsilon^c_j = \frac{\text{CG wage}_j}{\text{CG wage}}, \quad \forall j = 1, ..., 48.$$ 

The result is shown in Figure 1.13. Both profiles exhibit a clear hump shape and reach a peak around age 55. Also notice that $\varepsilon^c_j = 0, \forall j = 1, ..., 4$ since CGs never work during study.

1.4.5 Technology

Two key elasticity parameters in the production function, the coefficient for elasticity of substitution between capital and skilled labor $\rho = -0.495$ and the coefficient for elasticity of substitution between unskilled labor and capital-skilled labor combination $\theta = 0.401$,
Table 1.2: Parameter Values from Outside Sources

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Match the moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td>$48$, corresponding to age 65 in the real life</td>
<td></td>
</tr>
<tr>
<td>${s_j}_{j=1}^J$, $s = c, h$</td>
<td>age-efficiency profiles</td>
<td>1962-2003 CPS, 1950, 1960 Census Data</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>CRRA coefficient</td>
<td>1.5, Gourinchas and Parker (2002)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>elasticity b/w $U$ and $K$</td>
<td>0.401, taken from KORV</td>
</tr>
<tr>
<td>$\rho$</td>
<td>elasticity b/w $S$ and $K$</td>
<td>-0.495, taken from KORV</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.069, İmrohoroglu, İmrohoroglu and Joines (1999)</td>
</tr>
</tbody>
</table>

Table 1.3: Parameter Values from Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Match the moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1.03</td>
<td>$\frac{K}{Y} = 2.67$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.645</td>
<td>Capital income share= 27.57%</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.418</td>
<td>Skill premium= 1.4556</td>
</tr>
<tr>
<td>$s\delta$</td>
<td>3.10</td>
<td>enrollment rate= 41.54%</td>
</tr>
</tbody>
</table>

are taken directly from KORV (2000). This implies the elasticity of substitution between capital and skilled labor is 0.67 and the one between unskilled and skilled labor is 1.67. Capital-skill complementarity is satisfied.

In the initial steady state, both TFP level $A$ and capital productivity $B$ are normalized to unity. I set the depreciation rate of capital $\delta = 0.069$ by following İmrohoroglu, İmrohoroglu and Joines (1999). They calculated this parameter from the annual U.S. data since 1954.

Table 1.2 summarizes the choices of parameter values from the outside sources.

This leaves four parameter values to be calibrated. The subjective discount rate $\beta$ is set equal 1.03 to replicate the target capital-output ratio 2.67 which is the average value from 1947 to 1951.\textsuperscript{14} The income share of capital in capital-skilled labor combination $\lambda = 0.645$ is chosen to match income share of capital in NIPA for the 1947-1951 period. The income share of unskilled labor $\mu = 0.418$ is chosen to match the average skill premium 1.4556 in the 1949 Census data. The scale factor of the disutility cost $s\delta$ (See the appendix B for the detail) is chosen to match the average enrollment rate between 1947 and 1951. Table 1.3 summarizes the discussion above.

The computation method of the steady state is described in detail in the Appendix B.

1.5 Steady State Results

In this section, I report the numerical simulations for the stationary equilibrium of the benchmark economy and compare the results with the pre-1951 U.S. data. The macro

\textsuperscript{14}It is plausible for a subjective discount factor greater than one in an overlapping generations setting. Please see Imrohoroglu et al. (1995) for detailed discussion.
The simulations show that the model does well in matching the data. It matches our targets—skill premium, enrollment rate, capital-output ratio \( K/Y \), and labor income-output ratio \( (w^sS + w^hU)/Y \) by construction. Additionally, several key macro aggregate ratios such as consumption-output ratio \( C/Y \) and investment-output ratio \( X/Y \) are also in line with the U.S. average data. The model also matches the relative supply of skilled labor very well. The risk-free real interest rate is 3%. Average CG enjoys higher life-time utility than average HSG because a CG has a higher consumption over the life cycle. We can see this clearly in the life cycle profiles below.

### 1.5.1 Life Cycle Profiles

This model also generates the life-cycle profiles for CG and HSG, respectively. Figure 1.14 shows the life-cycle profiles of wealth accumulation, consumption, and income for CG and HSG. Panel A shows that since CGs have no income in the first four periods, they have to borrow to pay for tuition and consumption. Therefore they accumulate negative wealth over the first four periods. After graduating from college, they start to earn the CG wage and are able to pay the loans. By age 34, CGs pay back all the loans borrowed from previous years and begin to accumulate positive wealth, reaching the peak around their mid-50s. At that time they begin to dissave. There is no bequest motive in this model, therefore people die with zero assets remaining. The same hump shape is also observed for HSG, except that they accumulate positive assets from the beginning.

The life-cycle profile of consumption in Panel B is worthy of explanation. It keeps increasing until the deaths of both CG and HSG. The reason is very clear when we look at
the intertemporal Euler equation derived from the model

\[ \left( \frac{c_{j+1}}{c_j} \right)^\sigma = \beta(1 + r). \]  

(1.23)

Given \( \beta = 1.03, r = 0.03 \) as in the results, \( \beta(1 + r) = 1.061 \). Therefore, the right hand side of the equation (1.23) is larger than 1, inducing a positive growth rate of consumption over the life cycle.

Although CGs do not have any income during first four periods, they have higher consumption than HSGs at any age. Because in this deterministic model, consumption path is determined by the permanent income, and CGs have higher discounted life time income.

Finally, panel C shows the hump-shaped life-time labor income profiles for CG and HSG, which are affected by the hump-shaped CG and HSG age-efficiency profiles \( \{\varepsilon_j^c\}_{j=1}^J \) and \( \{\varepsilon_j^h\}_{j=1}^J \).

### 1.5.2 Comparative Static Experiments

Based on the steady state results, I carry out some comparative static exercises to study the effects of the growth rate of the cohort size by changing \( n \) and the effects of investment-specific technological change by changing \( q \). I summarize the corresponding results in Table 1.5 and 1.6 respectively. In Table 1.5, 0% is the average growth rate of the HSG cohort size from 1947 to 1951, which is our benchmark case. 4.06% is the average growth rate of the HSG cohort size from 1952 to 1976, the “baby boom” period. -1.57% is the average growth rate from 1977 to 1991, the period when \( n_t \) continuously decreased. The results show that as the growth rate of the HSG cohort size increases, the skill premium and enrollment rate both increase, and vise versa. However, the effect is not quantitatively significant. In particular, the effect on the enrollment rate is very small.

Why does the increase in the HSG cohort size cause an increase in the skill premium and the college enrollment rate? The intuition is as follows: an increase in \( n \) will change the age structure \( \{\mu_j\}_{j=1}^J \) in the economy, making it skewed towards younger cohorts. Keeping the enrollment rate unchanged, more people from the college age cohort stay in college. Meanwhile more people from the college age cohort also join the labor force as unskilled labor. It results in relatively less out-of-school skilled labor in the current labor market. This
is shown in Table 1.5. When $n$ increases up to around 4%, the relative supply of skilled labor $S/U$ decreases by 9.1%. This change tends to raise the relative price of skilled labor which is the skill premium through the relative quantity effect. In turn, it will encourage people to go to college. However, change in age structure also has an impact on asset accumulation. Recall that the life cycle profile of asset holding for CG and HSG in Figure 1.14. People accumulate fewer assets during early working years. A shift towards younger cohorts in demographic structure thus decreases incentive of asset accumulation in the economy. As a result, the capital-output ratio $(K/Y)$ decreases from 2.77 in the benchmark case to 2.57 in $n = 4.06\%$ case. It also leads to a decrease in effective capital-skilled labor ratio $(BK/S)$. It then, through capital-skill complementarity effect, tends to decrease the skill premium and thus the enrollment rate. Quantitatively, the impact of demographic change on the relative supply of skilled labor dominates that on the relative demand of skilled labor through capital-skill complementarity. Thus, both the skill premium and the enrollment rate increase.

On the other hand, decrease in $n$ will make age structure favor the older cohort, and hence will increase the relative supply of skilled labor and raise the incentive for asset accumulation. These two impacts again tend to offset each other. Quantitatively, a change in $n$ from 0% to -1.57% only slightly decreases the skill premium and enrollment rate.

Next, I show the effect of a permanent change in $q$ on the steady state. In this model, as shown in GHK(1997) and KORV(2000), due to the existence of the ISTC, the relative price of capital goods is equal to the inverse of the investment-specific technological change $q$. Therefore I can use the relative price of capital to identify ISTC $q$. I take the price index of the personal consumption expenditure from NIPA, and the quality-adjusted price index of the total investment (equipment and structure) from Cummins and Violante (2002) for the time period 1951-2000. I divide these two sequences to obtain the data counterpart of the $q$. Figure 1.15 shows the log of time series of $q_t$. It was fairly stable before 1957, then started to grow. The average growth rate of $q$ in the 1960s and 1970s was 1.8% and 1.7%, respectively. It has speeded up since the early 1980s. The average growth rate in the 1980s was 3.2% and it was even higher in the 1990s (4.4%).

I then use the mapping mentioned before to transform the sequence of $q_t$ to the changes in the capital productivity level $B_t$. Normalizing the initial steady state value $B_{1951} = 1$, I

\footnote{Alsalam (1985) also points out this mechanism and concludes the dynamic behavior of enrollment rate is derived from various time series of cohort size of HSGs. But his model is totally silent on the other features of my model, namely the capital-skill complementarity, and the investment-specific technological change. Therefore, his model does not have the interaction effect of demographic change on the demand side of the skilled labor as I mention in the text. He actually calls for a complete model of enrollment rates suitable for estimation and testing, and suggests that this model “would be flexible enough to allow for technological change or an increase in the demand for college educated labor relative to less educated workers.” My model does answer this call.}
have $B_{2000} = 3.28$. During these fifty years, the decline of the relative price of the capital goods is equivalent to increases in the capital productivity by approximately 3.3 times.

Suppose that the U.S. economy reaches the steady state again after 2000. Keeping other things equal, Table 1.6 shows the effects of this permanent change from $B_{1951}$ to $B_{2000}$.

Investment-specific technological change, through the capital-skill complementarity, increases both the skill premium and the enrollment rate significantly. The mechanism is as follows: investment-specific technological change raises the capital productivity $B_t$, hence raising the effective capital stock $B_tK_t$. Since capital is complementary with skilled labor, increases in effective capital also raise the demand for skilled labor. Therefore it tends to increase the skill premium. Forward-looking individuals predict that the skill premium will increase. The response by college age adults is to enroll in college. Thus we also observe the increases in the enrollment rate.

However, as more people go to college and earn degrees, the relative supply of skilled labor increases. The relative quantity effect thus dampens the increase of the skill premium. The quantitative results in Table 1.6 confirm that the first order impact of ISTC is on the demand side of skilled labor through capital-skill complementarity. This impact dominates the repercussion effect from the relative supply side. Hence the skill premium increases from 1.4541 to 1.8516 in the model, which is quite close the data in 2000. The results also show that both CG and HSG significantly benefit from this technological change.

However, the model underpredicts the increase in the enrollment rate significantly. From 1951 to 2000, the enrollment rate increases from 41.54% (1947-1951 average) to 63.33%, while the model indicates only an increase from 41.51% to 47.46%. Increases in $q_t$ can only explain around 27% of the increases in the enrollment rate from 1951 to 2000. The reason probably lies in the estimation of the CDF of the disutility cost. The OLS method I use allows me to obtain a fairly flat line for the CDF. The advantage of this method is that I obtain the convergence in steady state easily, but the disadvantage is that I cannot generate enough variations in the enrollment rate.
1.6 Transition Path

The comparative static exercises I have done above (especially the one with ISTC) show that we are on the right track to explain the increases in the skill premium and enrollment rate over time. However, to see how far the model can go to match the time series data of the skill premium and enrollment rate in the postwar U.S. economy, comparative static analysis is not enough. I have to solve the model along a time path.

Following the spirit of the computation method as in Chen, İmrohoroğlu and İmrohoroğlu (2004) and Conesa and Kreuger (1999), I compute the model along a transition path from initial pre-1951 steady state towards a final steady state in a far future. The computation algorithm is described in detail in Appendix C.

1.6.1 Benchmark Case

In the benchmark case, I feed into the model the exogenous path of capital-specific technological change $f_{B_{t}}^{2000}_{t=1951}$ and demographic change embodied in the change in the growth rate of the HSG cohort size $f_{n_{t}}^{2000}_{t=1951}$. I also feed in the normalized tuition payments $f_{tuition_{t}}^{2000}_{t=1951}$. I then assume capital-specific technological change gradually decelerates until it becomes stable in 2030, continues at this constant level until 2050. For simplicity, I also assume that after 2000 there is no demographic change after 2000 and the tuition payment is constant at 2000 level. Since I want to focus on the effect of ISTC, the neutral TFP change has been normalized to unity all the time periods through the transition. I compute the transition path of the benchmark economy between 1951 and 2050 and truncate it to the 1951-2000 period. The results are shown in Figure 1.16 and 1.17.

In Figure 1.16, the simulated skill premium from the benchmark economy overshoots the actual data since 1965, but it captures the increase after 1980 very well. From 1951 to 2000, the data show that the skill premium increases from 1.4546 to 1.8357 and the average growth rate (per year) during these fifty years is 0.49%. In the model the skill premium increases from 1.4543 to 1.8793 and the average growth rate is 0.54%. If we compare the data to the model in detailed periods, from 1951 to 1959, the average growth rate of the skill premium in the data is 0.73%, while in the model is -0.32%. The model does not capture the increase through that decade. From 1963, I have annual data of the skill premium so the comparison between the data and the model performance is more accurate. From 1963 to 1969, the average growth rate is 1.30% in the data and 1.52% in the model. From 1969 to 1981, the skill premium decreases at the average rate of 0.45%, but the model misses this decline by predicting an almost flat skill premium over this period, The average growth rate is 0.02%. The skill premium starts to increase dramatically beginning in 1981. From 1981 to 1990, the average growth rate in the data is 1.45%, while the model predicts 0.82%.
That is, the model captures 56% of the changes in the skill premium during this decade. From 1990 to 2000, the average growth rate in the data slows down to 0.96%, and the model predicts 1.01% which overshoots the growth. Overall, for the three episodes in the “N” shape of the skill premium from 1963 to 2000, the model captures 117% of the changes in the skill premium for the period 1963-1969 and 77% for the period 1981-2000. But the model fails to replicate the declining part of the skill premium from 1969 to 1981.

The model also raises the enrollment rate from 41.52% to 48.07% for the period 1951-2000; while the growth rate in the data is from 41.54% to 63.3%. In other words, the model can explain about 30% of the increases in the enrollment rate during this period. Breaking down into episodes, from 1951 to 1968, the enrollment rate grows at average rate 1.8% in the data. The average growth rate in the model is 0.48%. Therefore, the model accounts for about 27% of the growth in this period. From 1968 to 1980, the enrollment rate decreases. Its average growth rate is -0.88% in the data, while in the model the enrollment rate still increases slightly at a rate 0.34%. Since 1980, the enrollment rate starts to increase again at a average growth rate 1.34% and the model predicts 0.12%, which means that the model can only account for around 9% of changes in the enrollment rate for the period 1980-2000.

Compared to the skill premium, the model does less successfully in accounting for the fluctuations in the enrollment rate. There are two possible reasons: the first is as I mentioned before, the OLS estimation of the disutility cost of schooling gives me a easy convergence, but at the expense of generating enough variations; the second is that the model here is a highly abstract one which excludes many of important determinants of an individual’s schooling choice. For example, from Figure 1.16, the effect of the Vietnam War Draft is obvious. It increased the college enrollment rate from 1963 to 1968, then the draft effect gradually phased out, dragging the enrollment rate down. This policy change is not included in our model. Therefore, it is not surprising that the model only predicts a moderate increase in the enrollment rate over this period. In addition, there were dramatic social norm changes since the 1970s that have affected especially the female college-going behavior and increased female enrollment rate. Increases in the female enrollment rate contribute to the large increase in college enrollment rate in the data. The model is also silent about this driving force behind rising enrollment rate.16

As shown in equation (1.13), the skill premium is determined by the two competing effects: the relative quantitative effect where the key determinant is the relative supply of skilled labor \((S/U)\), and the capital-skill complementarity effect where the effective capital-

16Goldin (2006) calls the period since the late 1970s a “quiet revolution” for women’s increased involvement into the economy. She documents that during this period, women had expanded horizons, cared more about their individual identities, and faced shifts in relative earnings and occupations. It would be interesting to see the effects of these factors on the female enrollment rate in my dynamic general equilibrium framework.
skilled labor ratio ($BK/S$) plays a key role. In Figure 1.17, I show the model performance along these two dimensions. Panel A shows the effective capital-skilled labor ratio in the model and data. The data are taken from KORV (2000) and the time range is from 1963 to 1992. It is the ratio of the combination of real capital structure and quality-adjusted equipment stocks divided by total working hours of skilled labor. I normalize the data point in 1963 to be consistent with the one predicted in the model. The model does a good job in replicating the dramatic increase in this ratio. But since the model does less successfully in replicating changes in the enrollment rate, the relative supply of skilled labor is significantly underpredicted compared to the data. This could be the major reason why the model overshoots the skill premium data.

1.6.2 Counterfactual Decomposition

To answer the quantitative question raised in the introduction, I conduct the following counterfactual experiments to isolate each exogenous change and investigate its impact on the skill premium and enrollment rate.

I first shut down the investment-specific technological change, so the only exogenous force remaining is the demographic change. The results are shown in Figure 1.18 and 1.19. Panel A in Figure 1.18 shows that the model fits the skill premium data fairly well until 1980, and it does generate the declining part of the skill premium during the 1970s. However, it cannot capture the dramatic increase since 1980 as shown in the data. More specifically, from 1963 to 1969, the model generates around 1% average growth rate in the skill premium, which can explain 77% of the changes in this period. The “demographic change only” model also captures 31% of the decline of the skill premium from 1969 to 1981. But from 1981, it predicts a slight decrease in the skill premium (0.007% per year), in contrast to the dramatic increase shown in the data.

Additionally, Panel B in Figure 1.18 depicts that the model generates little variation in the enrollment rate. From 1951 to 2000, model predicts that the enrollment rate slightly decreases from 41.52% to 40.88%, while the data are from 41.54% to 63.33%. The average growth rate of the enrollment rate is 0.95% in the data, while in the model is -0.03%. Demographic change does not have a significant effect on the enrollment rate over this period.

Figure 1.19 shows the driving forces in shaping the pattern of the skill premium. In Panel A, we see that except for first few years (1963-1966) the “demographic change only” model is a total failure in replicating the effective capital-skilled labor ratio in the data. From 1966 to 1976, $BK/S$ ratio in the model actually decreases from 5.83 to 5.29. Then from 1977 to 2000, it increases from 5.27 to 5.87. This pattern basically is consistent with
the mechanism mentioned in the comparative static experiments before (see section 1.5.2), the “baby boom” decreases $BK/S$ ratio while the “baby bust” increases it. But without ISTC, we cannot generate such a dramatic increase in $BK/S$ ratio as shown in the data.

Panel B demonstrates the dynamics of the relative supply of skilled labor. Since the model fails to generate increases in the enrollment rate over time, hence it also fails to replicate the increasing trend of $S/U$ over the period 1951-2000.

Figure 1.19 help us to understand the performance of the model in the skill premium. From 1966 to 1976, decreasing relative supply of skilled labor tends to raise the skill premium through the relative quantity effect, but the change in age structure also brings capital accumulation down. Decreases in $BK/S$ ratio, through capital-skill complementarity, tends to decrease the skill premium. Things reverse after 1976. The shape of the skill premium thus is a result of the trade-off of these two forces.

Second, I shut down the demographic change. What remains is only the investment-specific technological change. Figure 1.20 and 1.21 show that the results are similar to those in the benchmark case. The skill premium overshoots the data after the early 1970s and keeps increasing afterwards. Therefore it captures the dramatic increase since the late 1970s, but misses the declining part of the skill premium during the 1970s. From 1963 to 1969, the average growth rate of the skill premium in the model is 0.55%. The model thus can explain about 42% of the changes in the skill premium over this period. From 1969 to 1981, the data show that the skill premium decreases at an average rate 0.45%, while the model goes the wrong direction to predict a growth at a rate 0.26% on average. However, after 1981, “ISTC“ only model generates a 0.89% average growth rate of the skill premium over the period 1981-2000, which can explain about 74% of the increases in the skill premium in the data.

In Panel B, the enrollment rate rises from 41.52% in 1951 to 48.07% in 2000, as same as in the benchmark case. The model also captures the dramatic rise in capital-skilled labor ratio. Since there is no demographic change in this model, changes in the relative supply of skilled labor are only driven by the increase in the enrollment rate. The model predicts continuously increasing $S/U$ but much less drastic than in the data due to underprediction in the enrollment rate. This underprediction could contribute to the overshooting of the skill premium in the model both in this one and benchmark case. If the model could do better job to match the changes in the enrollment rate, $S/U$ would increase and drag the skill premium down.

The decomposition results will be more clear if we combine three cases in one graph. In Figure 1.22, I show the simulations of the skill premium in the benchmark, shutting down the technological change (“Demographic only“), and shutting down the demographic change (“ISTC only“) cases. We can see that the “Demographic only” outcome is very close to the
Table 1.7: Average Annual Growth Rate of the Skill Premium: Model vs. Data

<table>
<thead>
<tr>
<th>Period</th>
<th>Data (%)</th>
<th>Benchmark (%)</th>
<th>Demographic (%)</th>
<th>ISTC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963-2000</td>
<td>0.68</td>
<td>0.73</td>
<td>0.11</td>
<td>0.63</td>
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<td>1963-1969</td>
<td>1.30</td>
<td>1.52</td>
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<td>0.02</td>
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<td>1981-1990</td>
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<td>0.06</td>
<td>0.78</td>
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<tr>
<td>1990-2000</td>
<td>0.96</td>
<td>1.01</td>
<td>-0.07</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table 1.7: Average Annual Growth Rate of the Skill Premium: Model vs. Data

Table 1.8: Decomposition of the Contribution to the Dynamics of the Skill Premium

benchmark outcome until 1965. Then they diverge. On the other hand, the “ISTC only” outcome closely follows the benchmark outcome since the late 1970s. From this observation, we can draw the conclusion that demographic change dwarfs the ISTC before 1966, and it does contribute to the decline of the skill premium in the 1970s. However, things reverse after the late 1970s. ISTC takes over to drive the increase in the skill premium.

Table 1.7 compares the average annual growth rate of the skill premium in the data and in the three model cases for different periods.

The contribution of each force to the dynamics of the skill premium is summarized in Table 1.8. Here the contribution is measured by the ratio of the average annual growth rate of the skill premium in the model and in the data. Overall, ISTC is much important than demographic change in explaining the pattern of the skill premium from 1963 to 2000 (93% vs. 17%). But demographic change dwarfs ISTC in the 1960s (77% vs. 42%). It can also explain about one third of the declining part for the period 1969-1981 while ISTC goes into a wrong direction. The relative importance of demographic change decreases dramatically since 1981, while ISTC becomes the major driving force behind the skill premium.

We should also be aware of that these two exogenous forces are not mutually exclusive to each other. That’s the reason why the summation of each contribution in separate case is not equal to the one in the benchmark case. For example, in 1981-1990 period, the contribution by demographic change is 4% and the one by ISTC is 54%. We should expect the total effect by combining two forces together, as in the benchmark case, to be 58%, if the effects on the skill premium from these two forces are orthogonal, but the actual number is 56%. The reason lies in the interaction between these two forces (refer to section 5.2 for
<table>
<thead>
<tr>
<th>Period</th>
<th>Data (%)</th>
<th>Benchmark (%)</th>
<th>Demographic (%)</th>
<th>ISTC (%)</th>
</tr>
</thead>
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<tr>
<td>1951-2000</td>
<td>0.95</td>
<td>0.30</td>
<td>-0.03</td>
<td>0.30</td>
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<tr>
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<tr>
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<td>0.34</td>
<td>0.00</td>
<td>0.29</td>
</tr>
<tr>
<td>1980-2000</td>
<td>1.34</td>
<td>0.12</td>
<td>-0.10</td>
<td>0.12</td>
</tr>
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</table>

Table 1.9: Average Annual Growth Rate of the College Enrollment Rate: Model vs. Data

<table>
<thead>
<tr>
<th>Period</th>
<th>Data (%)</th>
<th>Benchmark (%)</th>
<th>Demographic (%)</th>
<th>ISTC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1951-2000</td>
<td>100</td>
<td>31.47</td>
<td>-3.26</td>
<td>31.50</td>
</tr>
<tr>
<td>1951-1968</td>
<td>100</td>
<td>26.80</td>
<td>1.46</td>
<td>28.93</td>
</tr>
<tr>
<td>1968-1980</td>
<td>100</td>
<td>-38.85</td>
<td>-0.04</td>
<td>-32.67</td>
</tr>
<tr>
<td>1980-2000</td>
<td>100</td>
<td>8.98</td>
<td>-7.40</td>
<td>9.03</td>
</tr>
</tbody>
</table>

Table 1.10: Decomposition of Contribution to the Dynamics of the College Enrollment Rate

the discussion). Both forces contribute to the dynamics of the capital-skilled labor ratio \(BK/S\) and relative supply of skilled labor \(S/U\). And the skill premium is nonlinearly determined by these two ratios.

Figure 1.23 shows the simulations of enrollment rate in these three cases. Clearly the “Demographic only” case generates little variation in enrollment rate. On the other hand, the “ISTC only” case is very similar to the benchmark one, which implies that investment-specific technological change, not the demographic change, is a driving force of the increases in enrollment rate. Table 1.9 summarizes the average annual growth rate of the enrollment rate in the data and in the three model cases for different periods. Table 1.10 shows the contribution of each force on the enrollment rate for each period.

Since the effective capital-skilled labor ratio \(BK/S\) and relative supply of skilled labor \(S/U\) are the two major determinants of the skill premium in the model, I also show simulations of both ratios in Figure 1.24 and 1.25, respectively. Figure 1.24 displays that \(BK/S\) in the benchmark case tracks closely with the one in “demographic change only” case from 1951 to 1965, which implies that demographic change dominates ISTC over this period. The ups and downs from 1959 to 1965 are due to the dramatic demographic change during that time. From 1959 to 1960, \(n_t\) drops from 14.2% to 5.7%. This decrease in the growth rate of HSG cohort size brings increase in \(BK/S\) (recall the mechanism for the “baby bust” in section 1.5.2). On the other hand, \(n_t\) increases from 1.3% in 1962 to 17.5% in 1963. This huge increase changes the age structure significantly and decreases the asset accumulation in the economy. Therefore, \(BK/S\) decreases drastically. Since 1965, \(BK/S\) in the benchmark case turns to follow closely with the one in “ISTC only” case. It shows that ISTC becomes the major driving force to affect this ratio. And it is the drastic rising capital-skilled labor ratio since the late 1970s, through capital-skill complementarity, that
drives the rising skill premium.

Figure 1.25 shows the relative supply of skilled labor in benchmark and two decomposition cases. “Demographic change only” case predicts decreasing $S/U$ from 1951 to 1976 and increasing $S/U$ since then. This is consistent with the mechanism mentioned in section 1.5.2. “Baby boom” decreases $S/U$, while “baby bust” does the opposite. In contrast to this case, “ISTC only” model shows a continuously rising $S/U$ ratio by generating an increase in the enrollment rate over time. $S/U$ in the benchmark case is in between these two decomposition cases. It decreases until 1966, then begins to increase and converges to the ratio in “ISTC only” model after 1990. This “J” pattern confirms that demographic change is a dominating force in driving the evolution of the relative supply of skilled labor before 1966, but the effect phases out after that. ISTC catches up to become the major driving force.

We should be aware that due to the weakness of the model in replicating enrollment rate data, the model underpredicts the growth of the relative supply of skilled labor in the postwar U.S. economy significantly. This weakness contributes to the overshooting of the skill premium compared to the data. Improvement in the enrollment rate will increase $S/U$ in the model and hence alleviate the overshooting problem.

1.6.3 Sensitivity Analysis

In this subsection, I show that my results are robust in the choice of the model setting, alternative values of key parameters, data sequences of the investment-specific technological change, and the timing of the final steady state.

My current model does not include retirement. Another model setting including retirement and social security system (people retire at age 66 and live up to age 100) gives very similar results.\textsuperscript{17}

Since the two elasticity parameters in the production function: $\theta$ and $\rho$ are the key parameters in the model, I also do the sensitivity analysis according to different values of these two parameters. More specifically, I use the values $\theta = 0.33$ and $\rho = -0.67$ which are taken from Fernandez-Villaverde (2001) since we share the same specification of the production function. This implies the elasticity of substitution between capital and skilled labor is 0.60, the one between unskilled and skilled labor is 1.49. I then recalibrate the model according to these new values. All the comparative static and transition path results are similar to the benchmark one shown here.

The data sequence of the investment-specific technological change is taken from Cummins and Violante (2002). I also check the alternative sequence taken from GHK(1997).

\textsuperscript{17}This version of the model and the quantitative results are available on request from the author.
Their model divides the capital into two categories: structure and equipment. Investment-specific technological change is assumed to affect equipment only. Therefore, their \( q \) is identified with a quality-adjusted relative price index of newly equipment, which is slightly different from our specification. Feeding in GHK's \( q \) data into the model, I have similar results to the benchmark one. But now the magnitude of the technological change is bigger, therefore, the effects on the skill premium and the enrollment rate is higher than those in the benchmark case.

I set the timing of the final steady state in some arbitrary way. However, I notice that the choice of this timing does not affect my results significantly. For example, if the model reaches the final steady state right after 2000, I still obtain very similar results.

1.7 Conclusion

The skill premium (college wage premium) in the U.S. increased in the 1950s and 1960s, decreased in the 1970s, and has increased again dramatically since 1980. What are the driving forces behind this “N” shape? The previous literature has proposed several explanations including skill-biased technological change (SBTC) and demographic change. In this chapter, I find that the college enrollment rate shares a similar pattern as that of the skill premium, which motivates me to try to understand the dynamic interaction between the enrollment rate and the skill premium. To serve this goal, I establish and compute an overlapping generation general equilibrium model with endogenous schooling choice to answer an important quantitative question: what percentage of change in the skill premium and the enrollment rate can be explained by these two widely discussed exogenous driving forces: demographic change and investment-specific technological change (ISTC)?

In this model, ISTC and demographic change drive the equilibrium outcomes of the skill premium and the enrollment rate by dynamically affecting the relative demand and supply of skilled labor. ISTC, through the key feature of capital-skill complementarity in the production technology, increases the relative demand of skilled labor, thus raises the skill premium. The rising skill premium encourages skill formation and hence increases the relative supply of skilled labor. In contrast to ISTC, demographic change affects the age structure in the economy. Change in the age structure has a direct impact on the relative supply of skilled labor. In addition, since people have different saving tendency along the life cycle, change in the age structure also has an influence on the relative demand of skilled labor through changing asset accumulation in the economy. The ultimate effects of these two forces on the skill premium and the enrollment rate depend on the quantitative magnitude of both demand and supply effects.

I calibrate the model to match U.S. data for the period 1947-1951 as the initial steady
state. Then, by feeding in the investment-specific technological change data from Cummins
and Violante (2002) and the growth rate of the HSG cohort size from 1951 to 2000, I conduct
perfect foresight deterministic simulations to compare with the data of the period 1951-2000
and counterfactual decomposition experiments to identify the effects of each force.

My results show that in terms of the skill premium, demographic change dwarfs ISTC
before the late 1960s and accounts for about one third of the decline of the skill premium
in the 1970s. However, ISTC takes over to drive the dramatic increase in the skill premium
since the early 1980s. It explains about three fourth of the increases in the skill premium
since 1981. ISTC is also an important driving force behind the increasing trend in the
enrollment rate. It explains about 30% of the increases in the enrollment rate during
the post war period, while demographic change does not have a significant effect on the
enrollment rate.

As shown in the text, the benchmark model overshoots the skill premium data and can
only capture limited variations of the enrollment rate. (See Figure 1.16.) The possible
reason could lie in the model-generated disutility cost of schooling. Future work should
include other important determinants of the schooling choice into the model to improve
its performance. For example, I can add a time trend in the disutility cost associated
with the change in social norm of schooling. By doing so, I would have a closer match
with the enrollment rate data. This would increase the relative supply of skilled labor and
hence alleviate the overshooting of the skill premium. Another extension is to allow for
unemployment. The United States experienced very high unemployment rate in the 1970s.
Adding in different unemployment shock associated with educational choice could help to
explain the remaining part of the decline of the skill premium which cannot be captured by
the demographic change.

The model I present here also provides a platform for further research topics in ed-
ucational policy, health economics and population economics. First, one could study the
general equilibrium effect of tuition policies on the skill premium and enrollment rate in this
framework. Second, the empirical evidence suggests that survival probability is different
across educational groups. In this model, one could endogenize the survival probability and
link it to human capital investment and hence could open up another avenue through which
schooling choice may affect the economy. Finally, many countries are undergoing dramatic
demographic transition to an aging society (for example, Asian countries such as China and
Japan). This model can also help to predict what would happen to wage inequality and
educational attainment in these countries through the transition.
1.8 Appendix A. Data Sources and Construction

The skill premium and relative supply of skilled labor data in Figure 1.1 are taken from Katz and Autor (1999). Data are from the 1940, 1950, 1960 Censuses and the 1964-1997 March CPS. The skill premium in their paper is the coefficient on workers with a college degree or above relative to high school graduates in a log weekly wage regression. The sample includes full-time full-year workers aged between 18 and 65. The relative supply of skilled labor is the ratio between college equivalents and non-college equivalents, using weeks worked as weights. Here college equivalents = CG + 0.5 × workers with some college, non-college equivalents = High School Dropout + HSG + 0.5 × workers with some college. Figure 1.1 is also as same as Figure 1 in Acemoglu (2003). Please refer to Katz and Autor (1999) or Data Appendix in Acemoglu (2003) for detailed data construction.

HSG cohort size data in Figure 1.2 are from National Center for Education Statistics (NCES): *Digest of Education Statistics (DES)* 2002, Table 103, data of 1941, 1943 and 1945 are from *DES* 1970 Table 66. In this figure year means school year, for example, 1939 means the school year 1939-1940.

The 18-21 year college age population data in Figure 1.3 are from NCES: *DES* 2002 Table 15 for 1970-2000, NCES: *DES* 1995 for 1960-1969, Standard Education Almanac 1968 Table 1 for 1955-1959. The 17 year old population data are from NCES: *DES* 2002 Table 103.

College enrollment rates of HSG in Figure 1.4 are from NCES: *DES* 2002 Table 183 for 1960-2001 data. 1948-1959 data are calculated by the author. To construct them, first I take the 1948-1965 first-time freshmen enrolled in institutions of higher education data from NCES: *DES* 1967 Table 86, divided by the HSG cohort size as in Figure 1.2. Since first-time freshmen are not necessarily from recent HSG, I use the overlapped year 1960-1965 to calculate the average difference between our own calculation and the true data, then adjust our own calculation for the 1948-1959 period according to this difference.

Average TFRB charges data in Figure 1.5 are constructed as following. First, I obtain data about estimated Average Charges to Full-Time Resident Degree-Credit Undergraduate Students between 1956-57 and 1966-67 school year from *Standard Education Almanac* 1969, Table 120; 1967-68 to 1973-74 data from *Standard Education Almanac* 1981-82, P. 231-232; 1974-75 to 1983-84 data from *Standard Education Almanac* 1984-85, P. 328-329; 1984-85 to 2003-04 four year institution data from “The Trends in College Pricing 2003”, the College Board, Table 5a, 5b. 1948-1955 data are from Table 102: “Estimated Costs of Attending College, Per Student: 1931-1981” on *Standard Education Almanac* 1968. To make it consistent with the data after 1955, I use the overlapped 1956 data to adjust. Second, I focus only on public or private four-year institutions. I obtain the TFRB charges
for those institutions. Third, I calculate the enrollment share of public and private four-year institutions respectively. For the 1948-1964 data, I obtain the total fall enrollment in degree-granting institutions by control of institution (private vs. public) from NCES: DES 2002 Table 172, noticing that it is for all higher education institutions. Then from Table 173, I have total fall enrollment in degree-granting institutions by control and type of institution from 1965 to 2000. Fourth, I weight the average TFRB charges of public and private four-year institutions by the enrollment share, then use the Personal Consumption Expenditure deflator from NIPA to express them in constant 2002 dollars. Finally, the third order moving average method is used to smooth the data.

The construction of the cohort-specific skill premium data is in the text. (See section 1.4.1.)

The skill premium data used in this paper is the ratio of the real mean annualized wage of CG and HSG as in Figure 1.6. The data counterpart of the relative supply of skilled labor \( \frac{S}{U} \) is the ratio of weeks worked of CG plus some college and HSG. I obtain these data from Katz and Autor (1999)'s dataset.

1.9 Appendix B. Computation Algorithm to Stationary Equilibrium

Given the parameter values as shown in the text, I compute the stationary equilibrium as following:

1. Guess the initial values for (per capita, also aggregate since the measure of agents is one) capital stock \( K_0 \) and initial enrollment rate \( e_0 \).

2. Given initial guesses, I can calculate the skilled labor \( S_0 \) and the unskilled labor \( U_0 \). Notice that for every \( j \), \( \sum_a \lambda^c(a,j) = \) enrollment rate, \( \sum_a \lambda^h(a,j) = 1 - \) enrollment rate, so by (1.20) and (1.21) I have

\[
S_0 = (\text{enrollment rate}) \cdot \sum_j \mu_j \varepsilon^c_j
\]
\[
U_0 = (1 - \text{enrollment rate}) \cdot \sum_j \mu_j \varepsilon^h_j.
\]

Given all the inputs, from the firm’s FOCs (1.10)-(1.12), I can compute interest rate \( r \), wage rates \( w^c \) and \( w^h \).

3. Discretize the asset level \(-b \leq a \leq a_{\text{max}}\) (make sure that the borrowing limit \( b \) and the maximum asset \( a_{\text{max}} \) will never be reached). Given prices \( \{w^c, w^h, r\} \), feed into
the normalized tuition data. By using backward induction (remember $a_J = 0$), I can solve the conditional value function $V^s(a, 1)$, for $s = h, c$, therefore obtain the cut-off disutility cost $\text{DIS}(i^*) = (V^c(a = 0, 1) - V^h(a = 0, 1))/sd$, where $sd$ is the scale factor of disutility cost which is calibrated to replicate the enrollment rate data.

4. From the CDF function of disutility cost, corresponding to $\text{DIS}(i^*)$, I will have the new enrollment rate $e_1$. Check the convergence criteria ($\frac{|e_0 - e_1|}{e_0} \leq tol_e$). If it is not satisfied, update it by relaxation method

$$e_2 = \kappa_e e_0 + (1 - \kappa_e)e_1,$$

where $0 < \kappa_e < 1$ is the relaxation coefficient for the enrollment rate.

5. Using the decision rules I obtain from step 3, $A^s(a, j) \forall j, \forall a$ and following (1.18), I compute the age-dependent distributions by forward recursion. Then I use these distributions $\lambda^s(a, j)$ and the age shares $\mu_j$ to compute the per capita (next period) capital stock $K_1$, and new transfers $\xi_1$ as follows

$$K_1 = (\sum_j \sum_a \sum_s \mu_j \lambda^s(a, j)A^s(a, j))/(1 + n)$$

where $n$ is the growth rate of cohort size. Check the convergence criteria ($\frac{|K - K_1|}{K} \leq tol_K$) to see if it needs to stop. If not, use the relaxation method to update $K$

$$K_2 = \kappa_K K_0 + (1 - \kappa_K)K_1$$

where $0 < \kappa_K < 1$ are the relaxation coefficients for the capital. Then set $K_0 = K_2$, $e_0 = e_2$, and go to step 1. The iteration will stop once all errors fall into the tolerance ranges.

6. Compute the aggregate consumption, investment, tuition expense, and output by using the decision rules, age-dependent distributions and age shares

$$C = \sum_{j=1}^{J} \sum_a \sum_s \mu_j \lambda^s(a, j)C^s(a, j)$$

$$\text{TUITION} = \sum_{j=1}^{4} \sum_a \mu_j \lambda^c(a, j)\text{tuition}_j$$

$$X = (n + \delta)K.$$
7. Finally, check if market clearing condition given by (1.22) holds. If it does, stop.

1.10 Appendix C. Computation Algorithm to Transition Path

In this paper I follow Chen, Imrohoroglu and Imrohoroglu (2004) (also see Conesa and Kreuger (1999)) in computing a transition path from initial pre-1951 steady state towards a final steady state. In this way, I view 1952-2000 as a part of the transitional path. For notation of the time period, I have $t = 1$ for 1951, $t = T$ for the final steady state, $t = 2, ..., T - 1$ for the transitional period. I take the following steps in computation.

1. Compute the pre-1951 initial steady state by following the method described in Appendix B. Save the distribution $\lambda^s(a, j), \forall j, \forall s$ for the later calculation.

2. Feed in the exogenous changes in growth rate of HSG cohort size $\{n_t\}_{t=1}^T$, transformed investment-specific technological change $\{B_t\}_{t=1}^T$ and the tuition payments $\{tuition_t\}_{t=1}^T$.

3. Compute the final steady state at $t = T$. Save the value function $V^s(a, j), \forall j, \forall s$ for the later calculation.

4. Take the initial and final steady state values of capital stock $K$ and enrollment rate $e$, use linear interpolation to guess the sequences of $\{K_t\}_{t=1}^T$ and $\{e_t\}_{t=1}^T$. This is the initial guess for the transition path computation.

5. Start from $T - 1$, take the value function of final steady state as the terminal values $V^s(a, j, T)$, solve the individual optimization problem by backward induction, obtain the decision rules for all cohorts through the transition path.

6. Using the distribution of pre-1951 steady state as the initial asset distribution, together with the decision rules collected from step 5, calculate $\{\lambda^s(a, j, t)\}_{t=2}^T$ by forward recursion. Then use them to calculate new $\{K_t\}_{t=1}^T$ and $\{e_t\}_{t=1}^T$.

7. Compare the new sequences of endogenous variables $\{K_t\}_{t=1}^T$ and $\{e_t\}_{t=1}^T$ with initial guess and iterate on them until convergence.
Figure 1.1: The Skill Premium and Relative Supply of Skilled labor: 1949-1996

Figure 1.2: High School Graduates Cohort Size
Figure 1.3: College Age Population: 1955-2000

Figure 1.4: College Enrollment Rate of High School Graduates: 1948-2001
Figure 1.5: Average Real TFRB charges: 1948-2001

Figure 1.6: Real Annualized Mean CG and HSG Wage: 1949-2002
### Figure 1.7: Synthetic 1962 HSG Cohort

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### Figure 1.8: Synthetic 1962 CG Cohort

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41
Figure 1.9: Life-Cycle HSG Wage Profile: 1955 Cohort

Figure 1.10: Life-Cycle CG Wage Profile: 1955 Cohort
Figure 1.11: Cohort-Specific Return to Schooling: 1948-1991

Figure 1.12: CDF of the Disutility Cost
Figure 1.13: Age Efficiency Unit Profile: 1949-2002 Average
Figure 1.14: Life Cycle Profiles of HSG and CG
Figure 1.15: Investment-Specific Technological Change (Log Units)

Source: Cummins and Violante (2002)
Figure 1.16: Model vs. Data: Benchmark Case
Figure 1.17: Model vs. Data: Benchmark Case Con’d
Figure 1.18: Model vs. Data: Demographic Change Only
Figure 1.19: Model vs. Data: Demographic Change Only Con’d
Figure 1.20: Model vs. Data: ISTC Only
Figure 1.21: Model vs. Data: ISTC Only Con’d
Figure 1.22: Skill Premium: Model vs. Data
Figure 1.23: College Enrollment Rate: Model vs. Data
Figure 1.24: Capital-Skill Labor Ratio: Decomposition

Figure 1.25: Relative Supply of Skilled Labor: Decomposition
Chapter 2

Investment-Specific Technological Change, Skill Accumulation, and Wage Inequality

2.1 Introduction

In the postwar period, the U.S. economy has experienced steady growth in per capita income, accompanied by substantial changes in income inequality. As shown in Figure 2.1, income inequality measured by the relative wage of college-educated workers (i.e., college wage premium) increased in much of the 1960s, then declined modestly in the 1970s, and has since increased substantially starting in the early 1980s. In the meantime, the number of skilled workers (e.g., those with college degrees) has steadily grown relative to the number of unskilled workers (e.g., those with high school diplomas), as is evident in Figure 2.2. Understanding potential causes of the observed dynamics in wage inequality and skill accumulation is of great interest to both academic economists and policy makers.

The literature on wage inequality is large and growing. Most studies attribute the dynamics of wage inequality to skill-biased technological change (SBTC). One possible mechanism through which SBTC may affect wage inequality is proposed by Katz and Murphy (1992). Based on a simple supply and demand framework, they argue that, if there is a constant secular trend in SBTC, then the increase in the relative supply of skilled workers in the 1970s associated with the baby boom generation leads to a temporary fall in inequality, which, before moving back to its secular trend, is bound to increase at an accelerated
rate (see also Bound and Johnson, 1992). It is unclear, however, what drives the trend in SBTC. Acemoglu (1998) proposes that SBTC can be endogenous and can respond to the market size for skilled workers. As the relative supply of skilled workers increases, there will be a larger market size and more monopoly rents for skill-complementary technologies. This provides a greater incentive for innovating firms to upgrade the productivity of skilled workers. As a result, the skill premium initially falls and then rises.1

Most studies in the SBTC literature do not examine the quantitative contributions of the underlying mechanisms that may drive wage inequality. Yet, to understand the driving mechanisms of the changes in inequality and other labor market phenomena, “it is necessary to formulate dynamic models that can quantitatively include the main alternative explanations so that one can measure the impact of each one of them” (Eckstein and Nagypál, 2004, p. 26).

In an important contribution, Krusell et al. (2000, henceforth KORV) build a quantitative framework to study the evolution of wage inequality. They show that, if capital equipments are more complementary to skilled workers than to unskilled workers (e.g., Griliches, 1969), then variations in the quantities of input factors help account for much of the observed changes in college wage premium in the postwar U.S. economy. They interpret equipment-skill complementarity as a form of SBTC. They further suggest that the observed changes in capital equipments can be attributable to investment-specific technological change in the spirit of Greenwood, Hercowitz, and Krusell (1997, henceforth GHK). The study by KORV (2000) is particularly important from a macroeconomic perspective because it relates the driving forces of the relative demand for skilled workers and skill premium to input factors that can be explicitly measured.

A common feature of these SBTC-based theories — including KORV (2000) — is that technological change drives wage inequality through affecting the relative demand for skilled workers, taking as given the relative supply of skilled workers.2 In the current chapter,

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1 Other theories on wage inequality include, for example, openness to international trade, changes in the unionization rate, and changes in real minimum wages. A general consensus is that SBTC theories provide a more compelling story than these other theories. For a survey of this literature, see, for example, Acemoglu (2002) and Aghion (2002).
2 There are a few notable exceptions. For instance, Heckman, Lochner, and Taber (1998) develop and estimate an overlapping generations (OLG) model with heterogenous skills, endogenous (once-for-all) schooling choice, and post-school on-the-job investment to study college wage premium and skill formation. For their purpose, they approximate SBTC by a trend estimated from an aggregate technology, rather than using direct measures, such as that based on observed changes in the relative price of equipments. Greenwood and Yorukoglu (1997), on the other hand, emphasize the role of declining prices of producer durables and equipments in explaining the rise in wage inequality and the slowdown in productivity growth. Unlike KORV (2000), both of these studies abstract from capital-skill complementarity. For a more recent quantitative study of the changes in college wage premium and college enrollment rate, see Chapter 1 in this thesis, which constructs an OLG model that incorporates demographic change, investment-specific technological change, and capital-skill complementarity.
we would like to turn the question around and ask: *What quantitative framework helps account for the dynamics of both skill accumulation and wage inequality, taking as given some measures of SBTC?* In other words, we would like to build a quantitative model with endogenous skill accumulation (instead of taking the supply of skills as given), and to examine whether the observed changes in wage inequality and the relative quantity of skilled workers can arise as an *equilibrium* outcome driven by measured technological change.

For this purpose, we build a general equilibrium model with vintage capital, in which production of capital equipments becomes increasingly efficient over time (as in GHK, 1997). To examine the quantitative effects of such capital-embodied (or investment-specific) technological change on the equilibrium dynamics of wage inequality and skill accumulation, we assume that capital equipments are more complementary to skilled workers than to unskilled workers (as in KORV, 2000), and that skill accumulation requires scarce resources and time (as in Ben-Porath, 1967; Trostel, 1993). With reasonable parameter values, we find that the model driven solely by measured investment-specific technological change is able to account for much of the steady growth in the relative quantity of skilled labor in the postwar U.S. economy, and the model does well in replicating the substantial rise in wage inequality after the early 1980s. Further, we find that investment-specific technological change in our model accounts for about 52% of the average annual growth rate of output per hour during the postwar period, which is close to the finding in GHK (1997), who abstract from equipment-skill complementarity and skill accumulation.

Our model contains a simple mechanism that propagates the investment-specific technological change (denoted by $q$) to generate the observed patterns in skill accumulation and wage inequality. As $q$ grows over time, the relative price of capital equipments falls, which encourages investment in new equipments. Given equipment-skill complementarity, the expectation that the stocks of equipments will rise in the future provides incentive for increased investment in skill accumulation, since increases in equipments would raise the marginal productivity of skilled workers and lower the marginal productivity of unskilled workers. Of course, the increase in the relative quantity of skilled labor dampens the rise in the skill premium. With plausible equipment-skill complementarity and a calibrated skill accumulation process, the model is able to deliver both the steady growth in the relative quantity of skilled labor during the postwar period and the substantial rise in wage inequality after the early 1980s.

An implication of the model’s mechanism is that, not only changes in $q$, but other factors that can raise the stocks of capital equipments can also raise wage inequality. To investigate this possibility, we present a counterfactual experiment based on the calibrated model. In the experiment, we lower the capital income tax rate from 39.7% to 0 in the spirit of the optimal Ramsey taxation literature (e.g., Chamley, 1986), and we examine
the effects of this capital-income tax reduction on skill accumulation and wage inequality. When we eliminate capital income taxes, we adjust the labor income tax rate to keep the present value of the tax revenue unchanged. We assume that the same time series for $q$ drives equilibrium dynamics before and after the tax reform. We also examine the effects of the tax reform on welfare, which is measured by a consumption equivalence in the spirit of Lucas (1987). We find that lowering the capital tax rate to zero leads to a substantial increase in the stock of capital equipments and in the relative quantity of skilled labor. The tax reform also creates a sizable increase in welfare. Yet, perhaps surprisingly, its effect on wage inequality is small.

The reduction in capital income taxes works through three channels to affect wage inequality. First, the reduction in capital taxes raises the stocks of capital equipments and, with equipment-skill complementarity, raises the relative marginal productivity of skilled workers as well. Second, related to the first, the reduction in capital taxes encourages skill accumulation and thereby lowers the skill premium, since the relative supply of skilled workers increases. Third, to keep the present value of tax revenue unchanged, the reduction in capital taxes requires an increase in the labor income tax rate. Raising the labor income tax lowers the benefit of skill accumulation since skilled labor income is taxed at a higher rate; it also lowers the opportunity cost of time investment for skill accumulation. However, since goods investment for skill accumulation is not tax deductible, the higher labor tax reduces only part of the cost of skill accumulation. The total cost is thus reduced by less than the reduction in the benefit, and skill accumulation is discouraged (e.g., Trostel, 1993). As such, raising the labor income tax tends to increase the skill premium. Under calibrated parameters, the net effect of the capital-tax reduction on wage inequality is small.

In a second counterfactual experiment, we examine the effectiveness of two (revenue-neutral) policy changes that aim at reducing income inequality. One such policy is to raise the progressiveness of labor income taxes, and the other is to provide subsidies for human-capital investment. Increasing the progressiveness of labor taxes, although mechanically reduces the after-tax skill premium, is not effective in reducing wage inequality. Such a policy change discourages skill accumulation and thus increases the scarcity of skilled labor. Further, by lowering the average skill level, a more progressive labor taxation system can lead to a decline in average productivity and inflict a substantial welfare loss. In contrast, subsidizing skill accumulation can effectively reduce the skill premium through raising the relative quantity of skilled workers, and the policy change is welfare-improving.

In what follows, we present the model in Section 2.2, describe the calibration and solution methods in Section 2.3, discuss the main results in Section 2.4, present the counterfactual

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More accurately, we are comparing two economies with the same $q$ series (and the same tax revenue), but with different factor-income tax rates.
policy experiments in Section 2.5, and conclude in Section 2.6. In an appendix, we describe the data sources and computation methods.

2.2 The Model

We now present a general equilibrium model with vintage capital. The model features (i) investment-specific technological change, under which production of new capital equipments becomes increasingly efficient over time; (ii) equipment-skill complementarity, under which capital equipments are more complementary to skilled workers than to unskilled workers; and (iii) endogenous skill accumulations.

2.2.1 The Economic Environment

Time is discrete. The economy is populated by a large number of identical, infinitely lived households. The representative household is formed by a measure-one of workers, who supply inelastically to the market one unit of time. In each period, a fraction of workers is skilled and the rest are unskilled. There exists a technology to transform unskilled labor into skilled labor, and such transformation requires time and goods as inputs. The household derives utility from consumption of a final good, which is produced by a large number of firms using skilled labor, unskilled labor, capital equipments, and capital structures. The final good is also used for accumulations of physical capitals (equipments and structures) and human capital (skilled labor). A government collects revenues through proportional taxes on labor incomes and capital incomes, and rebates the proceeds to the representative household through lump-sum transfers. All agents have perfect foresight.

The representative household has a life-time discounted utility function

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma},$$

where $\beta \in (0, 1)$ is a subjective discount factor, $c_t$ is consumption, and $\sigma > 0$ is a relative risk aversion parameter.

The representative firm has access to a constant-returns production technology to produce the final good. The production function is given by

$$y_t = \kappa_{st}^{\phi} \left[ \mu(z_t \tilde{u}_t)^{\nu} + (1 - \mu)[\lambda \kappa_{et}^{\phi} + (1 - \lambda)(z_t \tilde{s}_t)^{\phi}]^{\nu/\phi} \right]^{1-\phi},$$

where $y_t$ denotes output, $\kappa_{st}$ denotes input of capital structures, $\kappa_{et}$ denotes input of capital equipments, $\tilde{u}_t$ denotes input of unskilled workers, $\tilde{s}_t$ denotes input of skilled workers, and
$z_t$ denotes a (neutral) labor-augmenting technological change. The parameter $\theta \in (0, 1)$ measures the elasticity of output with respect to capital structures, and the parameters $\phi$ and $\nu$ determine the elasticities of substitution between equipments and skilled labor and between the skill-equipment composite and unskilled labor, respectively. If $\phi < \nu < 1$, then equipments are more complementary to skilled workers than to unskilled workers and the production technology features equipment-skill complementarity in the spirit of KORV (2000).

Physical capitals depreciate over time. Denote by $\delta_s$ and $\delta_e$ the depreciation rates for capital structures and equipments, respectively. Then, the laws of motion for these capital stocks are given by

$$k_{s,t+1} = (1 - \delta_s)k_{st} + i_{st},$$  \hspace{1cm} (2.3) \\
and

$$k_{e,t+1} = (1 - \delta_e)k_{et} + i_{et}q_t,$$  \hspace{1cm} (2.4)

where we assume that new investments in capital structures $i_{st}$ and in capital equipments $i_{et}$ are both non-negative and $k_{st}$ and $k_{et}$ are the current stocks of such capitals. We interpret the term $q_t$ in (2.4), in the spirit of GHK (1997), as investment-specific technological change (ISTC) that enhances the productivity of newly formed capital equipments. One can also interpret $1/q_t$ as the relative price of new capital equipments, which, according to the evidence provided by GHK (1997) and Cummins and Violante (2002), is declining for much of the postwar period, and the decline has accelerated since the early 1980s. We will discuss more about $q_t$ in the calibration section.\footnote{Our model can be reinterpreted as a two-sector model, in which one sector produces consumption good and capital structures, and the other produces equipments. Each sector is subject to a sector-specific productivity shock. Then, under some conditions (e.g., perfect factor mobility and identical capital-labor ratio across sectors), such a two-sector model is isomorphic to the model used in our quantitative analysis. See also GHK (1997) for a similar result in an environment with Cobb-Douglas technologies.}

We now describe the skill accumulation technology. The representative household consists of a measure one of workers, who supply inelastically one unit of time to the market. In each period $t$, a fraction $s_t \in (0, 1)$ of workers is skilled and a fraction $u_t = 1 - s_t$ is unskilled. Denote $i_{ht} \geq 0$ the goods invested in skill accumulation and $e_t \in (0, 1)$ the fraction of time of the unskilled workers used for skill transformation. The technology that transforms unskilled labor into skilled labor is given by

$$s_{t+1} = (1 - \eta)s_t + B \left( \frac{i_{ht}}{z_t} \right)^\alpha \left[ e_t(1 - s_t) \right]^{1-\alpha} \xi, \hspace{0.5cm} \eta, B > 0, \hspace{0.5cm} \alpha, \xi \in (0, 1),$$  \hspace{1cm} (2.5)

where $\eta$ measures the depreciation rate of existing skills; and $B$ measures the efficiency, $\alpha$ measures the relative importance of goods input vs. time input, and $\xi$ measures the
transformation model consistent with balanced growth, under which the investment-specific technological change $q_t$ settles down at a constant level while the neutral technological change $z_t$ grows at a constant rate, we divide goods investment $i_{ht}$ by the level of the neutral technological change $z_t$.

Some studies assume that effective time is the only input in human capital production (e.g., Heckman, 1976; Haley, 1976); some other studies assume that goods are the only input (e.g., Stokey, 1996). We follow Ben-Porath (1967) and Trostel (1993) and assume that skill accumulation requires both time and goods as inputs. This specification has important implications for studying the effects of taxation (as we do in the policy experiments below). As in Ben-Porath (1967) and Trostel (1993), we impose a unitary elasticity of substitution between goods and time invested in skill accumulation and we assume that the production of new skills exhibits decreasing returns to scale (i.e., $\xi < 1$) to ensure an interior solution. Unlike Ben-Porath (1967) and Trostel (1993), our specification here implies that adding skilled workers also subtracts from the unskilled.

The government collects tax revenues through proportional taxes on the household’s capital income and labor income. In calculating the tax base for capital income taxes, there is a depreciation allowance. The government rebates tax revenues to the representative household through lump-sum transfers, so that

$$\tau_k[(r_{st} - \delta_s)k_{st} + (r_{et} - \delta_e/q_t)k_{et}] + \tau_l(w_{st}s_t + w_{ut}u_t(1 - e_t)) = T_t,$$  \hspace{1cm} (2.6)

where $\tau_k$ and $\tau_l$ are the tax rates on capital income and labor income, $r_{st}$ and $r_{et}$ are the rental rates on structures and equipments, $w_{st}$ and $w_{ut}$ are the wage rates for skilled and unskilled workers, and $T_t$ is the lump-sum transfer.

### 2.2.2 Competitive Equilibrium

The representative household owns the physical capital (equipments and structures), which she rents to the representative firm at the competitive rental rates $r_{et}$ for equipments and $r_{st}$ for structures. A fraction $s_t$ of the members of the household supplies skilled labor to the firm at a competitive wage $w_{st}$, a fraction $u_t(1 - e_t)$ supplies unskilled labor to the firm at a competitive wage $w_{ut}$, and the remaining members of the household (of measure $u_te_t$) invest their time for skill accumulation. The household takes the wage rates and the rental rates as given and chooses consumption $c_t$, investment in capital equipments $i_{et}$, investment in capital structures $i_{st}$, investment in human capital in terms of both goods $i_{ht}$ and foregone
time $e_t$ to maximize the discounted utility (2.1) subject to a sequence of budget constraints

$$c_t + i_{et} + i_{st} + i_{ht} \leq (1 - \tau_t)(w_{st}s_t + w_{ut}(1 - e_t)(1 - s_t))$$

$$+(1 - \tau_k)(r_{et}k_{et} + r_{st}k_{st}) + \tau_k(\delta_c k_{et}/q_t + \delta_s k_{st}) + T_t,$$  \hspace{1cm} (2.7)

and the laws of motion (2.3), (2.4), and (2.5) for the physical capitals and the human capital, along with non-negativity constraints on $c$, $i_e$, $i_s$, $i_h$, and $e_t$.

The firm takes the wage rates and the rental rates as given, and chooses the quantities of inputs $\{\tilde{k}_{et}, \tilde{k}_{st}, \tilde{u}_t, \tilde{s}_t\}$ to solve a profit-maximizing problem

$$\max \pi = y_t - w_{st}\tilde{s}_t - w_{ut}\tilde{u}_t - r_{et}\tilde{k}_{et} - r_{st}\tilde{k}_{st},$$  \hspace{1cm} (2.8)

where the output $y_t$ is related to the inputs through the production function (2.2). As the production technology exhibits constant returns and the firm faces perfectly competitive markets, profit is zero in equilibrium.

A competitive equilibrium consists of a set of allocations $c_t, k_{et,t+1}, k_{st,t+1}, s_{t+1}, i_{et}, i_{st}, i_{ht},$ and $e_t$ for the representative household; a set of allocations $y_t, \tilde{k}_{et}, \tilde{k}_{st}, \tilde{u}_t,$ and $\tilde{s}_t$ for the representative firm, a set of prices $r_{et}, r_{st}, w_{st},$ and $w_{ut},$ and a profile of government policy $\{\tau_k, \tau_l, T\}$, such that

1. Taking the prices and the policy as given, the household’s allocations solve its utility maximizing problem.

2. Taking the prices and the policy as given, the firm’s allocations solve its profit maximizing problem.

3. Markets for the input factors and for the final good clear so that

$$\tilde{k}_{et} = k_{et}, \quad \tilde{k}_{st} = k_{st}, \quad \tilde{s}_t = s_t, \quad \tilde{u}_t = u_t(1 - e_t), \quad s_t + u_t = 1,$$

and

$$y_t = c_t + i_{st} + i_{et} + i_{ht}.$$  \hspace{1cm} (2.9)

### 2.2.3 Equilibrium Dynamics

We now characterize the equilibrium dynamics. The household’s optimizing conditions can be reduced to three intertemporal Euler equations with respect to the three forms of capital: structures, equipments, and skills and one intratemporal decision with respect to the time allocated to skill accumulation.
The Euler equation for capital structures is given by
\[ c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left[ (1 - \tau_k)(r_{s,t+1} - \delta_s) + 1 \right]. \tag{2.10} \]

The left-hand side of the equation gives the marginal utility loss of foregoing one unit of consumption good to invest in capital structures in period \( t \). The right-hand side of the equation gives the present value of marginal-utility gain in period \( t+1 \) from such investment, which equals the after-tax return on capital structures. In equilibrium, the household breaks even, so that the utility gain and the utility loss are equal.

The Euler equation for capital equipments is given by
\[ \frac{c_t^{-\sigma}}{q_t} = \beta \frac{c_{t+1}^{-\sigma}}{q_{t+1}} \left[ (1 - \tau_k)(r_{e,t+1}1_{q_{t+1} - \delta_e}) + 1 \right]. \tag{2.11} \]

This equation is similar to (2.10), except that the units need to be appropriately converted using the relative price \( 1/q_t \) for new equipments.

To help derive and interpret the Euler equation for skill accumulation, we define a function
\[ f(i_{ht}, e_t, s_t) = B \left[ \left( \frac{i_{ht}}{z_t} \right)^\alpha e_t(1 - s_t)^{1-\alpha} \right]^\xi, \]
where \( f_{i_{ht}} = \frac{\partial f}{\partial i_{ht}} > 0 \), \( f_{e_t} = \frac{\partial f}{\partial e_t} > 0 \), and \( f_{s_t} = \frac{\partial f}{\partial s_t} < 0 \). The Euler equation for skill accumulation can then be written as
\[ \frac{c_t^{-\sigma}}{f_{i_{ht}}} = \beta e_t^{-\sigma} \left\{ \frac{1 - \eta + f_{s_{t+1}}}{f_{i_{ht},t+1}} + (1 - \tau_l)[w_{s,t+1} - w_{u,t+1}(1 - e_{t+1})] \right\}. \tag{2.12} \]

To understand this equation, note that each unit of consumption good invested in skill accumulation results in \( f_{i_{ht}} \) units of new skills. In other words, \( 1/f_{i_{ht}} \) measures the shadow price of newly formed skills. The left-hand side of (2.12) then represents the period-\( t \) marginal utility loss from investing goods for producing an additional unit of skilled labor. The right-hand side represents the present value of the marginal utility gain from having this additional unit of skilled labor. In particular, the utility gain consists of two components: (i) the remaining value of the skills after taking into account of skill depreciation (\( \eta > 0 \)) and the reduction in the number of unskilled workers available to be transformed into skilled workers (\( f_{s} < 0 \)), and (ii) the marginal increase in the after-tax wage income for adding a unit of skilled labor (and thereby subtracting a unit from the unskilled). In equilibrium, the utility gain and loss are equal, so that the household remains indifferent at the margin of the skill accumulation decisions.
The optimizing decision on the time invested for skill accumulation is given by
\[ f_{et} = f_{ht}(1 - \tau_t)w_{ut}(1 - s_t). \] (2.13)

This equilibrium relation reflects that, at the margin, the household should remain indifferent between investing time versus investing goods in skill accumulation. Investing a marginal unit of time produces \( f_{et} \) units of skilled labor; alternatively, investing a marginal unit of consumption goods produces \( f_{ht} \) units of skilled labor. The time investment bears an opportunity cost equal to the after-tax labor income of the unskilled workers. In equilibrium, the household is indifferent between these two alternative means of producing an additional unit of skilled labor.

The firm’s optimizing decisions equate the prices of input factors to their marginal products. To simplify expressions, we define
\[ \tilde{y}_t = [\mu(z_tu_t(1 - e_t))^{\nu} + (1 - \mu)(\lambda k_{et}^{\phi} + (1 - \lambda)(z_t s_t)^{\phi}/\phi)]^{1/\nu}, \] (2.14)

so that the production function can be written as \( y_t = k_{st}^{\theta} \tilde{y}_t^{1 - \theta} \). The factor prices are given by
\[ r_{st} = \theta \left( \frac{\tilde{y}_t}{k_{st}} \right)^{1 - \theta}, \] (2.15)
\[ r_{et} = \lambda(1 - \theta)(1 - \mu)k_{et}^{\theta} \tilde{y}_t^{1 - \theta - \nu}[\lambda k_{et}^{\phi} + (1 - \lambda)(z_t s_t)^{\phi}/\phi]^{\frac{\nu - \phi}{\phi}} k_{et}^{\phi - 1}, \] (2.16)
\[ w_{et} = (1 - \lambda)(1 - \theta)(1 - \mu)k_{et}^{\theta} \tilde{y}_t^{1 - \theta - \nu}[\lambda k_{et}^{\phi} + (1 - \lambda)(z_t s_t)^{\phi}/\phi]^{\frac{\nu - \phi}{\phi}} z_t^{\phi - 1} s_t^{\phi - 1}, \] (2.17)
\[ w_{ut} = (1 - \theta)\mu k_{et}^{\theta} \tilde{y}_t^{1 - \theta - \nu}(u_t(1 - e_t))^{\nu - 1} z_t^{\phi - \nu}. \] (2.18)

Denote by \( \pi_{st} = \frac{w_{ut}}{w_{et}} \) the skill premium. From equations (2.17) and (2.18), the skill premium is given by
\[ \pi_{st} = \frac{(1 - \mu)(1 - \lambda)}{\mu} \left[ \lambda \left( \frac{k_{et}}{z_t s_t} \right)^{\phi} + (1 - \lambda) \right]^{\nu - \phi} \left[ \frac{u_t(1 - e_t)}{s_t} \right]^{1 - \nu} z_t^{\phi - \nu}. \] (2.19)

If \( 1 > \nu > \phi \), then capital equipments are more complementary to skilled labor than to unskilled labor. With such equipment-skill complementarity, we have
\[ \frac{\partial \pi_s}{\partial (k_{e}/s)} > 0, \quad \frac{\partial \pi_s}{\partial [s/(u(1 - e))] < 0. \]

In other words, the skill premium increases with the equipment-skill ratio (the equipment-skill complementarity effect), and decreases with the skilled-unskilled ratio (the relative
quantity effect).

To summarize, we have 12 equilibrium conditions, including the three Euler equations (2.10)–(2.12), the intratemporal decision on the time input for skill accumulation (2.13), the three capital accumulation equations (2.3)–(2.5), the four factor-price equations (2.15)–(2.18), and the aggregate resource constraint (2.9). These equilibrium conditions jointly determine the equilibrium values of 12 variables \{c_t, i_{st}, i_{et}, e_t, k_{e,t+1}, k_{s,t+1}, s_{t+1}, r_{et}, r_{st}, w_{st}, w_{ut}\}_{t=0}^{\infty}.

2.2.4 Balanced Growth

Since investment-specific technological change \(q_t\) is capital augmenting rather than labor augmenting, the model economy with a CES production function as the one in (2.2) would attain balanced growth only if there is no secular growth in \(q_t\) (e.g., Hornstein and Krusell, 2003). Of course, if the production function is Cobb-Douglas, as in GHK (1997), then balanced growth can be attained even if \(q_t\) grows at a constant rate. More formally, we have the following proposition.

**Proposition 1**: The balanced growth path (BGP) in this economy cannot be achieved unless there is no secular growth in the investment-specific technological change or the elasticities of substitution between input factors are unitary.

**Proof**: By contradiction. Without loss of generality, suppose that the neutral technological change \(z_t\) stays constant. Suppose there were a BGP with a positive growth in the investment-specific technological change \(q\). Along the BGP, the growth rates of \(y, k_e\), and \(k_s\), and the levels of \(s\) and \(u\) are all constant. Let \(\gamma_x\) denote the growth rate of a variable \(x\). Then, from the resource constraint and the laws of motion of physical capital stocks, we have \(\gamma_y = \gamma_{k_s}\) and \(\gamma_{k_e} = \gamma_y + \gamma_q\). The production function \(y_t = k_{st}^{\theta} e_t^{1-\theta}\) implies that \(\gamma_y = \gamma_{\bar{y}}\), which leads to a contradiction because \(\gamma_{\bar{y}}\) is in general not a constant, as \(k_e\) grows at a constant rate, while the levels of \(s, e,\) and \(u\) remain constant on a BGP.

In the case with unitary elasticities of substitution between input factors (i.e., Cobb-Douglas production function), there exists a BGP, as shown by GHK (1997). Q.E.D.

We do not consider the case with a Cobb-Douglas production function because it is inconsistent with the evidence of equipment-skill complementarity; we do not restrict \(q_t\) to be constant because we would like to examine the role of investment-specific technological change (i.e., a time-varying \(q_t\)) in accounting for the observed dynamics in wage inequality and skill accumulation. To isolate the role \(q_t\), we shut off the neutral technological change by assuming that \(z_t = 1\) for all \(t\). In computing the equilibrium dynamics, we interpret the growth in the investment-specific technological change as a transition from some initial steady state to a new steady state, and we focus on computing the transition dynamics of wage inequality and the relative quantity of skilled labor in the economy where agents have
perfect foresight about the future time path of $q_t$.

2.3 The Calibration and Solution Methods

We now describe our approach to calibrating the parameters to be used in our computation of the transition dynamics.

The parameters to be calibrated include $\beta$, the subjective discount factor; $\sigma$, the relative risk-aversion parameter; $\theta$, $\mu$, and $\lambda$, which determine the income shares of capital structures, capital equipments, and labor skills; $\phi$ and $\nu$, which determine the elasticities of substitution between equipments and skilled labor and between the equipment-skill composite and unskilled labor; $\delta_s$, $\delta_e$, and $\eta$, the depreciation rates of structures, equipments, and skills; $B$, $\alpha$, and $\xi$, parameters in the skill transformation technology; and $\tau_k$ and $\tau_l$, the tax rates on capital and labor incomes. The calibrated parameter values are summarized in Table 2.1.

We set $\sigma = 1.5$, a standard value used in the literature. We follow GHK (1997) and set $\theta = 0.13$, $\delta_s = 0.056$, and $\delta_e = 0.124$. We set $\phi = -0.495$ so that the elasticity of substitution between capital equipments and skilled labor is about 0.67, which is the value estimated by KORV (2000). Based on the estimation by Duffy et al. (2004), we set $\nu = 0.79$ as a benchmark. Since the empirical literature provides a wide range of estimates for $\nu$ (see Hamermesh, 1993), we also consider some other values of $\nu$ in our policy experiments. Heckman’s (1976) estimates suggest that the returns to scale parameter $\xi$ varies in the range between 0.51 and 0.81. We set $\xi = 0.7$ as a benchmark value, which lies within the range of Heckman’s estimates and is also the value used by Stokey (1996). In our policy experiments, we also consider some other values of $\xi$ in the range between 0.5 and 0.8. According to Heckman’s (1976) estimates, the rate of human capital depreciation ranges from 0.04 to 0.09. We set $\eta = 0.08$, which is also the value used by Stokey (1996). We set the average capital income tax rate to $\tau_k = 0.397$, a value used by Domeji and Heathcote (2004), and the average labor income tax rate to $\tau_l = 0.277$, following the work

<table>
<thead>
<tr>
<th>Preferences</th>
<th>$\sigma = 1.5$, $\beta = 0.9880$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor income shares</td>
<td>$\theta = 0.13$, $\mu = 0.4228$, $\lambda = 0.4912$</td>
</tr>
<tr>
<td>Elasticities of substitution</td>
<td>$\phi = -0.495$, $\nu = 0.79$</td>
</tr>
<tr>
<td>Depreciation rates</td>
<td>$\delta_s = 0.056$, $\delta_e = 0.124$, $\eta = 0.08$</td>
</tr>
<tr>
<td>Skill accumulation</td>
<td>$B = 0.3492$, $\xi = 0.70$, $\alpha = 0.60$</td>
</tr>
<tr>
<td>Income tax rates</td>
<td>$\tau_k = 0.397$, $\tau_l = 0.277$</td>
</tr>
</tbody>
</table>

Table 2.1: Calibrated Parameter Values
by McGrattan (1994) and Mendoza et al. (1994). This leaves five parameters to be calibrated, including $B$, $\alpha$, $\beta$, $\lambda$, and $\mu$. We assign values to these parameters so that the initial steady state in the model matches five moment conditions observed in the data in 1949. These five moment conditions are as follows:

1. The college wage premium (i.e., the average annual wage of college graduates relative to that of high-school graduates) is 1.456 in 1949 (Census data).
2. The expenditure in skill accumulation as a fraction of GDP is about $\frac{i_h}{y} = 0.018$ in 1949 (Data source: National Center for Educational Statistics, DES 2003).
3. The average capital-output ratio is 2.659 for the period between 1947 and 1949 (NIPA data).
4. The average income share of capital stock is 0.267 for the period between 1947 and 1949 (NIPA data).
5. The ratio of skilled labor (i.e., college graduates) to unskilled labor (i.e., high-school graduates) is 0.288 in 1949 (Census data).

Table 2.1 reports the values of these five parameters required to match the five initial moment conditions. There we have $B = 0.349$, $\alpha = 0.60$, $\beta = 0.988$, $\lambda = 0.491$, and $\mu = 0.423$.

To measure the investment-specific technological change series $q_t$, we rely on the study by Cummins and Violante (2002), who construct a quality-adjusted time series of the price index for 24 types of equipments and softwares during the period from 1947 to 2000, in the spirit of an earlier study by Gordon (1990). Upon obtaining the price index for equipments and softwares, we divide it by the price index of consumer non-durables and services reported in the National Income and Product Accounts (NIPA) to obtain a (quality-adjusted) relative price of new equipments and softwares. The investment-specific technological change (i.e., the $q_t$ series) is then the inverse of this relative price. The resulting $q_t$ series is plotted in Figure 2.3. The figure shows that $q_t$ has been increasing for most of the postwar period, and its growth has accelerated since the early 1980s. The average growth rate of $q_t$ was 3.45% for the period 1950-1980 and increased to 5.83% for the period 1981-2000.

As we have discussed earlier, we interpret the growth in the ISTC during the period from 1949 to 2000 as part of a transition from some initial steady state to a new steady state, where the ISTC becomes stable. To compute the transition dynamics, we assume that

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5We choose 1949 as the initial steady state, since our $q_t$ series constructed based on Cummins and Violante (2002) appears fairly stable between 1947 and 1949.
the growth rate of the ISTC slows down linearly starting in year 2001, and reaches zero in 2050. To ensure convergence to the final steady state, we further extend the (hypothetical) sample period for the ISTC series to year 2108. This way, we obtain a time series for $q_t$ with a length of 160 years, consisting of 52 years of actual observations between 1949 and 2000 taken from Cummins and Violante (2002) and 108 additional years for $q_t$ to settle down at a new steady state.\(^6\)

To compute the transition dynamics in the model, we first solve for the initial steady state and the final steady state given the values of the $q_t$ series in the initial and the final steady states. Table 2.2 summarizes the solutions for some key variables in the initial steady-state (with $q_1 = 1$) and compares these solutions with the corresponding moments in U.S. data (i.e., the values in 1949). The first five moment conditions in Table 2.2 match the data by construction. The model does fairly well on the other two dimensions. The average consumption-output ratio in the model is about 77\%, which is close to the observed value of 81\%; the ratio of capital equipments to capital structures in the model is 0.56, which is not far from that in the data (0.64).\(^7\)

Upon obtaining the solutions for the initial and the final steady states in the model, we compute the transition dynamics using a non-linear solution methods in the spirit of Conesa and Krueger (1999), Chen, İlrohoroglu and İlrohoroglu (2004), and Chapter 1 in this thesis. The details of the solution algorithm are described in Appendix B.

\(^6\)Our quantitative results are not sensitive to alternative assumptions about what happens to the ISTC after 2001. For instance, when we assume that the ISTC stops growing in 2010 (instead of 2050), we obtain almost identical results.

\(^7\)The equipment-structure ratio in the data is the average value for the period 1963-1992 taken from KORV (2000). Ideally, we should compare the model’s initial steady state value with the value in 1949 in the data. Unfortunately, quality-adjusted data of capital equipments in 1949 are not available. As the relative productivity of equipments has been rising since 1949, it is reasonable to believe that the equipment-structure ratio in 1949 should be lower than the average value for 1963-1992, and therefore be closer to our model’s prediction.
2.4 Dynamic Implications of the Model

We now describe the equilibrium dynamics of wage inequality and the relative quantity of skilled labor driven solely by the measured investment-specific technological change. We compare the model’s predictions with the observations in the U.S. data.

Figure 2.4 plots the skill premium (in log units) generated from the model and that in the data for the period between 1963 and 2000. The model does well in accounting for the substantial rise in wage inequality since the early 1980s. In particular, wage inequality measured by the relative wages of skilled workers in the U.S. economy has increased by about 19% between 1984 and 2000. The model predicts an increase of 14%. The model fails to capture the earlier episodes in the evolution of wage inequality, especially that in the 1970s. This is perhaps not surprising, since other factors such as demographic changes associated with the baby boom generation, which we do not model here, might also be driving the observed changes in wage inequality in the 1970s (e.g., Katz and Murphy, 1992; Chapter 1 in this thesis).

Figure 2.5 plots the dynamics of the relative quantity of skilled labor predicted by the model and that observed in the data. The model’s prediction tracks the data surprisingly well for the entire sample period from 1963 to 1996. The result here suggests that the observed secular increase in the relative quantity of skilled labor can be mostly accounted for by investment-specific technological change.

Our model contains a simple mechanism that propagates the investment-specific technological change to generate the observed patterns in skill accumulation and wage inequality. As $q_t$ grows over time, the relative price of capital equipments falls, which encourages the household to invest in new equipments so that the stock of capital equipments grows over time. Because of equipment-skill complementarity, the increase in the stock of equipments raises the marginal productivity of skilled workers and lowers that of unskilled workers and thereby driving up the skill premium. As such, the household finds it optimal to invest more in human capital and the skilled-unskilled ratio rises over time. Although the increase in the skilled-unskilled ratio dampens the rise in the skill premium, the equipment-skill complementarity effect dominates. Under calibrated parameters, our model predicts that

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8 A main discrepancy between the model’s predictions and the data seems to be the two spikes in the skill premium predicted (incorrectly) by the model. The timing of these spikes appears to coincide with those in the $q_t$ series (see Figure 2.3). It is not clear what drives the spikes in the $q_t$ series. The timing suggests that oil price shocks in the 1970s may have been a contributing factor. High oil prices render some capital equipments obsolete, leading to higher equipment investment and a higher equipment price. As the effects of oil shocks are expected to dissipate, the price of equipments is expected to fall, and thus productivity of the equipment sector (i.e., $q_t$) is expected to rise after the oil shocks. Cummins and Violante (2002) also notice that 1975 data point of $q$ acts as an outlier and ask readers should be aware that the sharp drop in $q$ in that year does not reflect (negative) technical change.
both the skill premium and the skilled-unskilled ratio rise over time, as in the data.\footnote{In an unreported experiment, we examine the quantitative importance of endogenous skill accumulation for capturing the trend in the skill premium. For this purpose, we consider an extreme case with $\xi = 0$, so that skill accumulation is prohibitively costly and the equilibrium relative quantity of skilled labor is constant. In this case, the model substantially overstates the trend in the skill premium: the skill premium rises by about 35\% in the model for the period 1984-2000, but only 19\% in the data for the same period. Thus, we need both equipment-skill complementarity and endogenous skill accumulation to match the time path of the skill premium.}

The propagation mechanism in the model implies that, as production of new equipments becomes more efficient over time, the average labor productivity measured by output per hour grows as well. GHK (1997) find that investment-specific technological change accounts for about 58\% of the average annual growth rate in output per hour in the United States for the period 1954–1990. Our model generalizes the model in GHK (1997) by incorporating equipment-skill complementarity and endogenous human-capital accumulation. The presence of these new elements, however, does not alter the main quantitative results obtained by GHK (1997). Our model driven solely by investment-specific technological change predicts that the average growth rate of output per hour is 0.64\% during the period 1954–1990, compared to 1.24\% in the data. In other words, investment-specific technological change accounts for about 52\% of the average growth in output per hour observed in the U.S. economy.

Our model has also interesting implications for measured total factor productivity (TFP). Since we assume a constant neutral technology in our model and the ISTC is the only source of equilibrium dynamics, changes in output reflects changes in measured input factors only. We can thus measure TFP by the difference between actual GDP in the data and output in the model. The TFP series so measured displays a productivity slowdown since the early 1980s: the average annual growth rate of TFP was 2.38\% for 1950-1979 and became lower at 1.09\% for 1980-2000. The slowdown in TFP growth coincides with the acceleration in the ISTC growth since the early 1980s, and it is a simple consequence of growth accounting: in the latter sample period, faster growth in the ISTC leads to faster growth in input factors and thus a larger fraction of output growth being accounted for by input growth.

To summarize, our results suggest that the ISTC can be an important source of growth in average labor productivity and the acceleration in the ISTC growth since the early 1980s may have contributed to the observed productivity slowdown. More importantly, our model accounts for much of the steady growth in the relative quantity of skilled labor in the postwar U.S. economy, and it does well in replicating the substantial rise in wage inequality since the early 1980s.
2.5 Tax Reforms and Efficiency-Inequality Trade-offs

The dynamic behavior of wage inequality in our model is driven by two competing forces between a “relative quantity effect” and an “equipment-skill complementarity effect.” Thus, not only investment-specific technological change, but other factors that affect capital accumulation might also affect wage inequality. In this section, we first illustrate this possibility by considering a counterfactual capital-income tax reform that encourages capital accumulation. We then examine the effectiveness of some hypothetical tax policies that aim at reducing income inequality.\(^\text{10}\)

2.5.1 Counterfactual Experiment I: Eliminating Capital Income Taxes

We now examine the quantitative effects of eliminating capital income taxes on wage inequality and skill accumulation. When we eliminate capital income taxes, we adjust labor income taxes so that the present value of the tax revenues during the entire transition period remains the same as in the benchmark economy. Since a zero capital income tax is consistent with the Ramsey optimal tax policy (e.g., Chamley, 1986), we also calculate the welfare gains from the tax reform.

In our quantitative experiment, we compare wage inequality and welfare in two economies, both driven by the same investment-specific technological change (i.e., our \(q_t\) series). The two economies are identical except for their tax policies. One economy is our benchmark model, which has positive tax rates on both capital and labor, with \(\tau_k = 39.7\%\) and \(\tau_l = 27.7\%\). The other economy has a zero tax on capital, but a higher tax on labor so that the present value of the total tax revenue during the transition period remains the same as in our benchmark economy. The required labor income tax rate in this latter economy is \(\tilde{\tau}_l = 32.84\%\).\(^\text{11}\)

Figure 2.6 plots the wage inequality for the two economies with different tax policies. Apparently, eliminating capital income taxes raises wage inequality modestly. For the period of our interest, 1949–2000, wage inequality in the economy with a zero capital income tax is on average 3.3\% higher than that in the benchmark economy. The effects of the tax reform on wage inequality also vary with time. Beginning in the early 1980s, as the growth in the investment-specific technological change accelerates, the tax reform has a larger impact on wage inequality than in the earlier periods.

\(^\text{10}\)For some recent quantitative studies about the effects of changes in tax policies on wage inequality and welfare, see, for example, Blankenau (1999) and Blankenau and Ingram (2002). A key difference between these studies and ours is that we emphasize the role of investment-specific technological change in driving wage inequality.

\(^\text{11}\)In calculating the present value, the discount factor that we use is the “state price” \(D_{t,t+j} = \beta^j (c_{t+j}/c_t)^{-\sigma}\). Since we assume a complete asset market, the state price is unique.
As wage inequality measured by the skill premium depends on both the equipment-skill ratio and the skilled-unskilled ratio (see equation (2.19)), it is instructive to examine the effects of the capital tax reform on these two determinants. Figure 2.7 and 2.8 plot the effects of eliminating capital income taxes on the relative quantity of skilled labor and on the equipment-skill ratio. They reveal that the reduction in capital taxes raises both the relative quantity of skilled labor and the equipment-skill ratio. The gap between the skilled-unskilled ratio before and after the capital tax reduction appears to become larger over time. The gap between the equipment-skill ratio displays substantial time variations, and becomes larger in the post-1980 period.

These results suggest that there are interesting interactions between the capital income tax reform and investment-specific technological change in shaping the dynamics of wage inequality and skill accumulation. As production of new equipments becomes more efficient over time, eliminating capital income taxes would create further incentive for capital accumulation, which, through equipment-skill complementarity, leads to greater wage inequality and more skill accumulation. The overall effects of the tax reform on skill accumulation and the equipment-skill ratio seem to be large, with an average increase of about 24\% in the skilled-unskilled ratio and about 20\% in the equipment-skill ratio relative to the pre-reform levels during the period between 1949 and 2000. Since the skilled-unskilled ratio and the equipment-skill ratio drive the skill premium to opposite directions, the overall effect of the tax reform on wage inequality is modest at about 3.3\% relative to the benchmark economy.\footnote{We have also computed the average differences between wage inequality and skill accumulation for the entire transition period (with 160 years). The average increase in wage inequality associated with the elimination of capital taxes in this extended sample becomes 1.49\% of its pre-reform level, and the average effect on the relative quantity of skilled labor becomes 14.29\%.}

Why does eliminating capital income taxes lead to a modest increase in wage inequality and a substantial rise in the relative quantity of skilled labor? As we have alluded to in the introduction, the elimination of capital income taxes can affect wage inequality and skill formation through three channels. First, eliminating capital taxes encourages the household to invest in physical capitals which, through the equipment-skill complementarity, raises the relative marginal productivity of skilled workers and hence the skill premium. Second, related to the first, the expectation of a higher future skill premium provides an incentive for the household to invest in skill accumulation, which raises the skilled-unskilled ratio and reduces the skill premium. Third, to keep the present value of tax revenue unchanged, the reduction in capital taxes requires an increase in the labor income tax rate. Raising the labor income tax lowers the benefit of skill accumulation since skilled labor income is taxed at a higher rate; it also lowers the opportunity cost of time invested in human capital.
However, since goods invested in human capital are not tax deductible, the higher labor tax reduces only part of the cost of human capital investment. The reduction in total cost is thus less than the reduction in the benefit, and skill accumulation is discouraged (e.g., Trostel, 1993). As such, raising the labor income tax tends to increase the skill premium. Under calibrated parameters, the capital tax reduction leads to a modest increase in wage inequality and a substantial rise in the relative quantity of skilled labor.

Since a zero capital income tax rate is consistent with Ramsey optimal fiscal policy (e.g., Chamley, 1986), one should expect the tax reform to increase social welfare in our model. Indeed, it does. We measure welfare gains by a consumption equivalence in the spirit of Lucas (1987). In particular, we define welfare gains from the reduction of capital income taxes as the permanent percentage increase in consumption that is required for the representative household to remain indifferent between living in two economies: the benchmark economy with positive capital and labor taxes, and an alternative economy with no capital income taxes but a higher labor income tax rate. For the period between 1949 and 2000, we find that the welfare gain from eliminating capital income taxes is equivalent to a 1.33% permanent increase in consumption. The size of the welfare gains here is quite close to that obtained by Domeji and Heathcote (2004) in a representative-agent version of their model (1.5%), and is sizable relative to the welfare cost of business cycle fluctuations calculated, for example, by Lucas (1987).

Eliminating capital income taxes creates a sizable welfare gain for two reasons. First, it removes intertemporal distortions in capital accumulation. Second, and more important, it raises average productivity through encouraging skill accumulation. The first channel is familiar in the Ramsey tax literature, but the second is new and is unique to our model with equipment-skill complementarity and with endogenous skill accumulation.

To examine the robustness of these results, we consider variations in two key parameters: \( \nu \) and \( \xi \). The parameter \( \nu \) determines the importance of equipment-skill complementarity (for a given value of \( \phi \)), through which investment-specific technological change and tax policies can affect wage inequality. The parameter \( \xi \) measures the returns to scale in human capital production: a lower value of \( \xi \) means that transforming unskilled labor into skilled labor is more costly and thus the effects of \( q_t \) or tax policies on skill accumulation is more muted. As we have discussed in the calibration section, the empirical literature provides a wide range of estimates for \( \nu \) and \( \xi \). Thus, it is important to know to what extent our results depend on the values of these parameters.

We first examine the quantitative effects of eliminating capital income taxes on wage inequality and skill accumulation for alternative values of \( \nu \), while holding \( \xi \) at its benchmark value (0.7). In addition to our benchmark calibration with \( \nu = 0.79 \), we consider two alternative values of \( \nu \) used in the literature. One is estimated by KORV (2000), which
A. Sensitivity to $\nu$

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$w_s/w_u$</th>
<th>$s/u$</th>
<th>$K_e/s$</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.79 (benchmark)</td>
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<td>23.86%</td>
<td>20.03%</td>
<td>1.33%</td>
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<tr>
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<td>14.02%</td>
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<td>0.401</td>
<td>1.04%</td>
<td>6.82%</td>
<td>18.23%</td>
<td>0.96%</td>
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B. Sensitivity to $\xi$

<table>
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<th>$K_e/s$</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
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<td>2.36%</td>
<td>28.98%</td>
<td>19.77%</td>
<td>1.48%</td>
</tr>
<tr>
<td>0.70 (benchmark)</td>
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<td>23.86%</td>
<td>20.03%</td>
<td>1.33%</td>
</tr>
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<td>0.60</td>
<td>4.22%</td>
<td>19.17%</td>
<td>20.36%</td>
<td>1.23%</td>
</tr>
<tr>
<td>0.50</td>
<td>5.11%</td>
<td>14.79%</td>
<td>20.65%</td>
<td>1.16%</td>
</tr>
</tbody>
</table>

Table 2.3: Effects of Eliminating Capital Income Taxes

gives $\nu = 0.401$; the other is estimated by Denny and Fuss (1977), which gives $\nu = 0.65$. We do the same counterfactual experiments of tax reforms using these alternative values of $\nu$. The results are reported in Table 2.3 (Panel A). The table shows that, as $\nu$ becomes smaller, the effects of eliminating capital income taxes on wage inequality, the skill-unskilled ratio, the equipment-skilled ratio, and welfare become more muted. A smaller value of $\nu$ implies weaker equipment-skill complementarity, so that the rise in equipment investment induced by the tax reform is associated with a smaller increase in the skill premium and a weaker incentive for skill accumulation. But for all the values of $\nu$ that we have considered here, the tax reform leads to a sizable welfare gain and a modest rise in wage inequality.

We next examine the effects of eliminating capital income taxes for alternative values of $\xi$, while holding $\nu$ at its benchmark value (0.79). Table 2.3 (Panel B) displays the results. As the value of $\xi$ becomes smaller (from 0.8 to 0.5), transforming unskilled labor into skilled labor becomes more costly. As such, the increase in equipment investment induced by the tax reform leads to a smaller increase in the skilled-unskilled ratio (from about 29% to about 15%) and a slightly larger increase in the equipment-skill ratio (from about 20% to about 21%). As the rise in the skilled-unskilled ratio is dampened (while the rise in the equipment-skill ratio remains roughly unchanged), the rise in the skill premium is magnified (from 2.36% to 5.11%). As skill accumulation becomes more costly, the welfare gain from eliminating capital income taxes becomes smaller (from 1.48% to 1.16%). But even for the lowest value of $\xi$ that we consider here, the rise in wage inequality remains modest (at about 5% compared to the benchmark economy) and the welfare gain remains sizable (at above 1% of consumption equivalence).

To summarize, eliminating capital income taxes can have large effects on skill accumulation and can lead to sizable welfare gains, but it has modest effects on wage inequality.
This result is robust to alternative values of key parameters. Our experiment thus suggests that a capital tax reform such as the one in 1986 is unlikely to be a good candidate for explaining the substantial rise in wage inequality since the 1980s.

2.5.2 Counterfactual Experiment II: Policies Designed to Reduce Inequality

We now examine the effectiveness of two tax policies, both designed to reduce income inequality. One such policy is to increase the progressiveness of labor income taxes by imposing a higher tax rate on skilled labor income (denoted by \( \tau_s \)) than on unskilled labor income (denoted by \( \tau_u \)). When we change the labor income taxes, we adjust the capital income taxes so that the present value of the tax revenues during the entire transition period remains the same as in the benchmark economy. The other policy is to provide subsidies for human capital accumulation, while adjusting the labor income tax rate (common to both types of labors) to keep the policy change revenue neutral.

Table 2.4 reports the effects of increasing the progressiveness of labor income taxes on wage inequality, skill accumulation, and welfare. The table shows that, when the uniform labor tax in the benchmark model with \( \tau_s = \tau_u = 27.7\% \) is replaced by a modestly progressive tax system with \( \tau_s = 30.22\% \) and \( \tau_u = 25.18\% \) (so that \( \tau_s/\tau_u = 1.2 \) and \( (\tau_s + \tau_u)/2 = 27.7\% \)), the after-tax skill premium falls by 2.25%; but the before-tax skill premium, which captures the general equilibrium effect, rises by 4.81%. Following such a policy change, the skilled-unskilled ratio falls by 18.32% and the welfare is reduced by 1.24% of consumption equivalence. When the progressiveness further increases, the after-tax skill premium falls by more, but no more than 10.4% even when the tax rate for skilled labor goes up to twice that for the unskilled. Meanwhile, when \( \tau_s/\tau_u \) rises from 1 to 1.6 and then to 2.0, the before-tax skill premium rises from 0% to 12% and then to 16%; the skilled-unskilled ratio falls from 0% to −44% and then to −63%; and the welfare loss rises from 0% to 4.3% and then to 8.9%. These results suggest that raising the progressiveness of labor income taxes, although mechanically redistributes income, is not effective in reducing wage inequality. Such a policy discourages skill accumulation and can lead to large welfare losses.

Progressive labor income taxes affect the skill premium and skill accumulation through three channels. First, raising the progressiveness reduces the benefit of skill accumulation since the labor income of skilled workers is taxed at a higher rate. It also raises the opportunity cost of time invested for skill accumulation since the after-tax wage income for unskilled workers goes up. Thus, raising the progressiveness discourages skill accumulation, which drives up the skill premium. Second, holding the capital income tax constant, as the
quantity of skilled labor falls through the first channel, the equipment-skill ratio should rise, which, through the equipment-skill complementarity effect, tends to drive up the skill premium as well. Third, to keep the policy change revenue neutral requires raising the capital income tax rate, which discourages physical capital accumulation, so that the equipment-skill ratio and therefore the skill premium may fall. As Table 2.4 shows, the equipment-skill ratio rises slightly for modestly progressive labor taxes (as the reduction in skilled labor dominates) but falls slightly for large progressiveness (when the reduction in the stock of capital equipments dominates). Our results reveal that the first channel dominates, so that progressive labor taxes lead to large declines in the relative quantity of skilled labor, large losses in welfare, but not much reduction in (after-tax) wage inequality.

We now consider an alternative policy that, instead of making labor income taxes more progressive, provides subsidies to human capital investment. Denote the subsidy rate by $\tau_h$. Under the subsidy policy, the household’s budget constraint (2.7) and the government budget constraint (2.6) should be modified accordingly. In particular, in the household’s budget constraint, the term $i_h$ should be replaced by $(1 - \tau_h)i_h$; and in the government budget constraint, the expenditure associated with the subsidy in the amount of $\tau_hi_h$ should be subtracted from the tax revenues. To maintain the present value of tax revenues the same as in the benchmark economy without subsidies, we adjust the labor income tax rate $\tau_l$ when we increase the value of $\tau_h$.

Table 2.5 reports the effects of subsidizing human capital accumulation. The table shows that, as the subsidy rate increases, the skill premium and the equipment-skill ratio both decline, and the relative quantity of skilled labor and welfare both increase. Even a modest increase in the subsidy rate, say from 0 (the benchmark economy) to 8%, can result in a sizable reduction in the skill premium (1.17%), a significant increase in the relative quantity of skilled labor (4.48%), and a non-trivial welfare gain (0.58%). Subsidizing human capital investment provides incentive for skill accumulation, and thereby raises the skilled-unskilled ratio and lowers the equipment-skill ratio, both of which tend to reduce the skill premium.

<table>
<thead>
<tr>
<th>$\tau_s/\tau_u$</th>
<th>$\frac{w_s}{w_u}$ (after tax)</th>
<th>$\frac{w_s}{w_u}$ (pre-tax)</th>
<th>$s/u$</th>
<th>$K_e/s$</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
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<td>1 (benchmark)</td>
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<td>0%</td>
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</tr>
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</tr>
<tr>
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<td>-63.03%</td>
<td>-13.04%</td>
<td>-8.88%</td>
</tr>
</tbody>
</table>

Table 2.4: Effects of Increasing Progressiveness of Labor Income Taxes
By raising the relative quantity of skilled labor, a subsidy leads to more labor-tax revenue for any given labor income tax rate; as such, to keep revenue neutral, the required labor tax goes down. Thus, a subsidy to human capital counteracts some of the distortions associated with labor taxes and improves welfare. This result suggests that subsidizing human capital accumulation does not seem to involve a trade-off between equity and efficiency.

### Table 2.5: Effects of Subsidizing Skill Accumulation Investment

<table>
<thead>
<tr>
<th>(\tau_h)</th>
<th>(\frac{w_s}{w_u})</th>
<th>(\frac{s}{u})</th>
<th>(\frac{K_c}{s})</th>
<th>Welfare</th>
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</thead>
<tbody>
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<td>7.49%</td>
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</table>

2.6 Conclusion

Understanding the driving forces of wage inequality is of great interest to both academic researchers and policy makers. In the literature, many potential mechanisms are proposed for explaining the qualitative changes in wage inequality. Yet, quantitative studies of the relative importance of these mechanisms are scarce. In the current paper, we have examined the quantitative importance of a skill-biased technological change, a popular mechanism proposed in the literature, in explaining the dynamics of wage inequality and skill accumulation. We measure the skill-biased technological change by the relative efficiency in the production of new capital equipments, and we examine the quantitative effects of such measured technological change on wage inequality and skill accumulation in a general equilibrium model. We find that, working through equipment-skill complementarity and endogenous skill accumulation, investment-specific technological change is able to account for much of the observed dynamics in the relative quantity of skilled labor in the postwar U.S. economy, and the model does fairly well in replicating the substantial rise in wage inequality since the early 1980s. In our counterfactual experiments, we find that a revenue-neutral elimination of capital income taxes leads to a modest increase in wage inequality and a sizable welfare gain. We also find that a revenue-neutral increase in the progressiveness of labor income taxes is not effective in reducing income inequality and, since it discourages skill accumulation, can potentially lead to large declines in average productivity and welfare. In contrast, a policy that provides direct subsidies for human capital accumulation tends to raise the
We focus on the role of investment-specific technological change in explaining the dynamic evolution of wage inequality and skill accumulation mainly because such technological change can be explicitly measured, thanks to the empirical work by Gordon (1990), GHK (1997), KORV (2000), and Cummins and Violante (2002). Such technological change turns out to be a quantitatively important mechanism in explaining wage inequality, but we do not claim it is the only mechanism. Future work should incorporate other mechanisms such as demographic changes (that affect human capital accumulation) or institutional reforms (that affect the relative returns to education), and evaluate the quantitative importance of these alternative mechanisms in explaining the dynamics of wage inequality, especially for the period before 1980.

Another direction to extend our study is to introduce consumer heterogeneity. Since our focus is on income inequality, we have taken a representative-agent approach, which implicitly assumes perfect risk-sharing between households. As such, there is no consumption inequality in our model. Incorporating consumer heterogeneity can be potentially important for evaluating the quantitative trade-offs between equity and efficiency when designing a public policy reform, such as the counterfactual policy experiments that we have considered in the current paper. Future work along these lines should help further improve our understanding of the causes and consequences of income inequality, and is thus both important and promising.

2.7 Appendix

In this appendix, we describe our data sources and computation methods.

Appendix A: Data

Our measure of wage inequality (i.e., skill premium) is the ratio of the mean wage for college graduates to that for high-school graduates, where the wages are annualized real wages (in 2002 dollars). To construct the wage data for different education cohorts, we follow Eckstein and Nagypál (2004) in selecting our sample. The sample includes data for all full-time, full-year workers between ages 18 and 65. The main source of the data is the March Current Population Survey (CPS) from 1962 to 2003. Earlier observations are taken from the 1950 and 1960 Census data.

Our measure of the relative quantity of skilled workers is the ratio of the number of college graduates to that of high-school graduates. These time series are taken from Katz and Autor (1999), who also use the Census and the CPS as their data source. Their sample
includes all workers between ages 18 and 65, and we focus on college graduates and high-
school graduates.

Our measure of the expenditure for skill accumulation is the current-fund expenditures and educational and general expenditures of degree-granting higher education institutions. The source of the data is National Center for Educational Statistics, DES 2003.

**Appendix B: Computation**

We solve the model by using the following algorithm:

1. Given $q_1 = 1$, we solve the initial steady state. We save the values of initial consumption $c_1$, equipment $k_{e,1}$, structure $k_{s,1}$, skilled labor $s_1$, fraction of time invested in skill accumulation $e_1$, goods invested in human capital $i_{h,1}$, in equipment $i_{e,1}$, and in structure $i_{s,1}$.

2. Given the terminal value of the ISTC series $q_T$, we solve the final steady state. We save the values of final-period consumption $c_T$, equipment $k_{e,T}$, structure $k_{s,T}$, skilled labor $s_T$, fraction of time invested in skill accumulation $e_T$, goods invested in human capital $i_{h,T}$, in equipment $i_{e,T}$, and in structure $i_{s,T}$.

3. Through linear interpolations between the initial steady state and the final steady state, we obtain a sequences of each of the eight variables $\{c_t, k_{e,t}, k_{s,t}, s_t, e_t, i_{h,t}, i_{e,t}, i_{s,t}\}_{t=1}^T$. We use these sequences as an initial guess for solving the system of non-linear equations, which consists of equations (2.3)–(2.5) and (2.9)–(2.13), together with non-negativity constraints on these variables. We have $8 \times T$ equations in this system. We solve this system of equations using standard non-linear numerical methods.

4. We make $T$ sufficiently large so that the transition dynamics between 1949–2000 are not affected by small variations in $T$. 

80
Figure 2.1: College Wage Premium (Log Units): 1963–2000

Figure 2.2: Relative Quantity of College Skills: 1963–1996
Figure 2.3: Investment-Specific Technological Change: 1949–2000
Figure 2.4: Dynamics of Wage Inequality (Log Units): Model vs. Data
Figure 2.5: Dynamics of the Relative Quantity of Skills: Model vs. Data
Figure 2.6: Effects of Capital-Tax Reform on the Skill Premium (Log Units)
Figure 2.7: Effects of Capital-Tax Reform on the Relative Quantity of Skills

Figure 2.8: Effects of Capital-Tax Reform on the Capital-Skill Ratio (Log Units)
Chapter 3

College Gender Gap and Gender Differences in Skill Premium

3.1 Introduction

In 2001, 56.3% of college students are women, men account for only 43.7% of the higher education enrollments (National Center for Educational Statistics, 2003, Table 174). Back to three or four decades ago, the pattern is exactly opposite. As shown in Figure 3.1, in 1947, this figure is around 71% for the men, 29% for the women. Enrollment numbers of the both sexes increased thereafter, male enrollment reached the peak in 1975, then became stable. Female’s enrollment kept increasing, eventually caught up with that of male in 1978 and surpassed it after then. Women today also more likely tend to go to college. In 2002, the female college enrollment rate of recent high school completers (individuals age 16 to 24 who graduated from high school or completed a GED during the preceding 12 months) is 68.4%, 6.3 percentage point higher than that of males (National Center for Educational Statistics, 2003, Table 186). Again, back three decades this is just opposite. Figure 3.2 shows that in 1955, male college enrollment rate is 59.1%, 24.5% higher than females! The trend of the enrollment rates by sex is also worthy of mentioning. Since 1955, male enrollment rate decreased a little until 1963, then increased by about 11% to the peak in 1968. After that it turned to decrease again, reached the bottom around 1980, bounced up since then. But it did not recover to the 1968 level until 1997, nearly three decades late. Roughly speaking, the male enrollment rate exhibits a “V” shape, and 1980 is the turning point. On the female side, there is a different story. It kept increasing steadily until 1976 when the female enrollment rate exceeded that of males at the first time. From 1976 to 1987, it moved together with the male one. After 1987, it surpassed the male enrollment rate. This pattern can be seen more clearly in Figure 3.3 with the three year moving average
smoothed data.

The message conveyed by these figures is very clear. Starting from 1950s, the college gender gap was shrunk and eventually reversed in the late 1970s. The question is why. What are the determinants of the gender differences in college enrollment behavior over time?

There are several studies documented the stylized facts mentioned above and attempted to give the answer. Among them, Averett and Burton (1996) try to link the gender differences in college attendance to the gender differences in college wage premium. They use a human capital model to examine gender differences in the college-going decision, positing that this decision is a function of family background characteristics and the expected future earnings differential between college and high school graduates. Using the NLSY79 dataset, they study how one cohort (those age 14 to 21 in 1979) has responded to the recent jump in the college wage premium after 1980. They find for men, the effect of college wage premium is positive and statistically significant, while for women it is much smaller and statistically insignificant. Therefore they conclude that for women the college wage premium is not nearly as important to the decision to attend college as it is for men. But they only focus on one cohort, and the future earnings are in a short time horizon. More specifically, they only include people who were interviewed subsequently in 1981 and 1991, which means the oldest age in the dataset is 33, at best people can only predict their future wage up to age 33. Therefore, exactly as they claim in the paper, “We cannot interpret our results as reflecting the impact of life-cycle earnings on education choice.” Hence the application of their results on the time series trend is questionable.

Card and Lemieux (2000) study the trends in enrollment separately by sex, and notice the different trend in enrollment rate by gender as we mentioned above. Their main focus is trying to explain “what went wrong in the 1970s?” Basically they link the answer to the big supply shock caused by baby boom generations attended and finished school in the 1970s and hence the depressed wage premium during that decade. They claim for women, the slowdown in the education in the 1970s (more accurately, the first half of the 1970s) was a temporary response to large cohort sizes and low returns to education. But for men, it is a permanent downward shift in the inter-cohort trend. But they do not explain why the male and female education attainment trend is different.

Jacob (2002) uses the National Educational Longitudinal Study (NELS) 1988 dataset to study why women have higher attendance rates than men. He focuses on two explanations: college wage premium which is proxied by earnings differential of 25-34 year old full-time workers and non-cognitive skills which is measured by middle school grades and the number of hours spent on homework per week in eighth grade, a composite measure of disciplinary incidents and an indicator of whether the child had ever been retained in grade during the elementary school. Using the Oaxaca decomposition method, he shows higher returns to
college and the greater non-cognitive skills among women account for nearly 90 percent of the gap. But due to the limitation of data availability, his research still focuses on cross-sectional pattern, hence cannot fully address the question I raised above.

Anderson (2002) tries to answer the same question by looking at different cohorts over time. Using CPS dataset, she constructs five cohorts who were born during 1953-1957, 1958-1962, 1963-1967, 1968-1972, and 1973-1977. She looks at male-female differences both within and across cohorts for 20-year olds. She also employs the standard Oaxaca method to do within and across cohort enrollment decomposition. She finds an important component of the increase in female enrollment is the behavior of older women, who enrolled less frequently than males when young, but later make up for this lack of higher education. Other important factors are the males have higher dropout rate in high school, are more likely to be in the prison and military and the social changes in the form of women delaying marriage for careers.

Charles and Luoh (2003) argue the reason why the standard approach with its focus on the college wage premium cannot explain very well the pattern of male and female schooling outcomes is because it misses an important aspect of educational decision, namely the uncertainty of two investment options. Inheriting the idea from Altonji (1993), they claim that risk averse students also care about the riskiness of these different options when choosing between them. In other words, now in an extended human capital investment model, not only the expected earnings differential, but also the anticipated dispersion of the future earnings determine the people’s educational investment decisions. Using the CPS data, they show these anticipated future dispersions have evolved over time very differently for men and women. For women, the dispersion of future college earnings decreased over the past three decades, it is opposite for men. Therefore, this trend would encourage women more likely go to college.

In their paper, Charles and Luoh impose an assumption to use the information about current wages for proxy of the expected future wages, which is quite common in the empirical literature. But is there any better way to obtain the data about the expected wages? If people have perfect foresight, or there is change in the trend of life cycle wage profiles, this assumption then is not that reliable.

In this chapter, I am going to ask a question to what extent can the trend in the college gender gap be explained by the trend in the gender differences in college wage premium. In this sense, my work is quite close to the original idea proposed in Averett and Burton (1996), but deeper in term of three aspects. First, my model is a full-blown life-cycle model, people enter into the model at age 18 and face the choice go to college or not. After education, they work until age 65. Within this framework, we can analyze the effect of life-cycle earnings on educational choice. Second, by assuming people have perfect foresight, I actually construct
the cohort-specific wage premium from the data, and use it for the expected future earnings. For a comparison purpose, I also adopt the naive expectations hypothesis similar to Charles and Luoh (2003) in the calibration. It turns out these two expectations hypotheses make a big difference in terms of the model’s prediction power. Third, different from Charles and Luoh (2003), I stick to the standard human capital investment approach. What I want to see is in a standard life cycle framework, how many percentage of change in the college gender gap can be accounted for by the change in the gender differences in college wage premium. Accordingly, instead of using econometric technique as used in the most of literature, I calibrate the model to compare the model-generated enrollment rate with the data. As far as I know, this is probably the first attempt to do so.

The reminder of the paper is organized as follows. Section 3.2 presents a simple model of college going decision, lays out the theoretical foundation for the later data analysis and calibration exercise. Section 3.3 describes the data and analyze some findings from the data. In the section 3.4, I calibrate the model and present the results. Section 3.5 provides the extension to include the trend of gender difference in labor force participation and employment rate to analyze how they are going to change our results in the previous section. Section 3.6 concludes.

3.2 Model

In this section, I will present the economic model that will be used later for calibration. The model is very similar to the one used in Chapter 1 in this thesis.

3.2.1 Demographics

The economy is populated by overlapping generations. People enter into (or “born”) the economy with zero initial assets when they are 18, which is the common age of high school graduates. I call them the birth cohort and model as age $j = 1$. I assume people live and work up to age $J$, which is the maximum life span. To distinguish between the age of a cohort and the calendar time, I will use $j$ for the age, and $t$ for the calendar time.

3.2.2 Preferences

Each individual born at time $t$ wants to maximize her discounted life time utility

$$
\sum_{j=1}^{J} \beta^{j-1} u(c_{j,t+j-1}).
$$
The period utility function is assumed to take the CRRA form
\[
    u(c_{j,t+j-1}) = \frac{(c_{j,t+j-1})^{1-\sigma}}{1-\sigma},
\]
where \( c_{j,t+j-1} \) is the consumption for the age \( j \) individual at time \( t+j-1 \). \( \sigma \) is the coefficient of relative risk aversion, \( \frac{1}{\sigma} \) is the intertemporal elasticity of substitution. Since leisure does not enter into utility function, each individual will supply all of her labor endowment, which is normalized to be one.

### 3.2.3 Budget Constraints

An individual chooses go to college or not at the beginning of the first period. I use \( s \in \{c, h\} \) to indicate this choice. If an individual chooses \( s = h \), she ends up with a high school diploma and goes on the job market to work as an unskilled labor, and earns high school graduate wage sequence \( \{w_j^h\}_{j=1}^J \). Or, she can also choose \( s = c \), spend the first four years in college as a full time student, and pay the tuition. I assume that she can always successfully graduate from college (there is no some college or college dropout in the model). After that, she goes on the labor market to find a job as a skilled worker, and earns college graduate wage sequence \( \{w_j^c\}_{j=1}^J \). We assume there is no unemployment.

For \( s = c \), the budget constraints of the cohort born at time \( t \) are
\[
    c_{j,t+j-1} + tuition_{t+j-1} + a_{j,t+j-1} \leq (1 + r_{t+j-1})a_{j-1,t+j-2} \quad \forall j = 1, 2, 3, 4
\]
\[
    c_{j,t+j-1} + a_{j,t+j-1} \leq (1 + r_{t+j-1})a_{j-1,t+j-2} + w_{j,t+j-1}^c \quad \forall j = 5, ..., J
\]
\[
    c_{j,t+j-1} \geq 0, a_{0,t-1} = 0, a_{J,t+J-1} \geq 0.
\]

In the first four periods, she pays the tuition \( tuition_{t+j-1} \), consumes \( c_{j,t+j-1} \), and saves \( a_{j,t+j-1} \). After the graduation, she earns wage \( w_{j,t+j-1}^c \) at the age \( j \), and consumes and saves subject to what she earns and accumulates previously. Notice there is no borrowing constraint in this economy. Since they do not have any initial assets, college graduates need to borrow money for consumption and paying tuition during first four periods, and they pay back the loans later.

For \( s = h \), the budget constraints of cohort born at time \( t \) are
\[
    c_{j,t+j-1} + a_{j,t+j-1} \leq (1 + r_{t+j-1})a_{j-1,t+j-2} + w_{j,t+j-1}^h \quad \forall j = 1, ..., J
\]
\[
    c_{j,t+j-1} \geq 0, a_{0,t-1} = 0, a_{J,t+J-1} \geq 0.
\]
3.2.4 Schooling Choice

To let the model be able to generate enrollment rate, I need to introduce some ex-ante heterogeneity within each birth cohort. Without this within-cohort heterogeneity, the enrollment rate would be just either zero or one.

I assume that different individuals within the birth cohort are endowed with different levels of ability. And ability will only affect individual’s disutility cost of schooling. If we rank people’s ability level in an ascending order, the disutility cost of schooling is a strictly decreasing function of ability.

We index people by their ability $i \in [0, 1]$, the CDF of ability distribution is denoted by $F$, $F(i_0) = \Pr(i \leq i_0)$. Now an individual $i$ born at time $t$ has the discounted life time utility

$$
\sum_{j=1}^{J} \beta^{j-1} u(c_{j,t+j-1}) - I_i DIS(i),
$$

where

$$I_i = \begin{cases} 
1 & \text{if } s_i = c \\
0 & \text{if } s_i = h 
\end{cases}
$$

subject to the conditional budget constraints (3.2)-(3.4). Here DIS($i$) is the ability-related disutility cost for individual $i$. We have DIS($i$) $\geq$ 0 and DIS'($i$) < 0. If an individual chooses go to college, she has to bear this idiosyncratic disutility cost. Notice that the disutility cost DIS($i$) does not enter into the budget constraints, therefore everyone will have the same life-time utility derived from physical consumption. I use UTIL$^c$ to denote the discounted life-time utility for college graduates, UTIL$^h$ denotes the discounted life-time utility derived from physical consumption for high school graduates. UTIL$^c$ – UTIL$^h$ represents the utility gain from attending college. Obviously individual $i$ will choose go to college if DIS($i$) < [UTIL$^c$ – UTIL$^h$], not go if DIS($i$) > [UTIL$^c$ – UTIL$^h$], indifferent if DIS($i$) = [UTIL$^c$ – UTIL$^h$].

Notice that since the borrowing constraint does not exist, the model implies

$$
\text{UTIL}^c_t - \text{UTIL}^h_t \geq 0 \text{ iff } \text{NPV}_t \geq 0,
$$

where

$$
\text{NPV}_t = \sum_{j=1}^{J} \frac{w^c_{j,t+j-1} - w^h_{j,t+j-1}}{\prod_{i=2}^{j} (1 + r_{t+i-1})} - \sum_{j=1}^{4} \frac{\text{tuition}_{t+j-1}}{\prod_{i=2}^{j} (1 + r_{t+i-1})}.
$$

Here NPV stands for the net present value of higher education. Since $w^c_{j,t+j-1} = 0, \forall j = 1, \ldots, 4$, students never work when they stay in college, we can further decompose NPV into
The basic intuition of this model can also be seen from Figure 3.4. In this figure, x-axis measures the ability. I rank people from zero to one. Disutility cost \( DIS(i) \) is a decreasing function of ability index \( i \). The cut-off ability (or indifference level) \( i^* \) is determined by

\[
DIS(i^*) = [UTIL_c - UTIL^h].
\]

Hence people with ability \( i < i^* \) will choose not to go to college, while people with \( i > i^* \) will choose to go. In this case, enrollment rate is equal to the probability when \( i > i^* \). Now if skill premium increases, so does the NPV, therefore, the utility gain form schooling \( VD = UTIL_c - UTIL^h \) increases accordingly, this will decrease the cut-off point to \( i^{**} \). Since \( Pr(i > i^{**}) > Pr(i > i^*) \), more people go to college, enrollment rate increases too.

### 3.2.5 Gender Difference

Next I will introduce the gender difference into the model. Obviously male and female face different life time wage profiles \( \{w_{j,t}^{s,g}\}_{j=1}^{J} \), here \( s \) stands for the education attainments, \( s = h \) or \( c \). \( g \) stands for the gender, \( g = m \) for the male, \( g = f \) stands for the female. Furthermore, I assume there is gender difference in the magnitude of disutility cost, but not the distribution. In other words, male and female share the same distribution of the ability, there is no gender difference in for example the distribution of non-cognitive skills. But we do have the gender difference in the initial level of non-cognitive skills. For simplicity and
the calibration purpose, I assume the ability-related disutility cost takes the form

\[ DIS^g(i) = b^g \left( \frac{1}{i} - 1 \right) \tag{3.7} \]

For the lowest ability individual \((i = 0)\), \(DIS(i) = \infty\), so he or she will never go to college. On the other hand, for the highest ability individual \((i = 1)\), \(DIS(i) = 0\), if \(NPV > 0\), he or she for sure will choose go to college. This functional form hence guarantees enrollment rate is an interior point between zero and one. I also assume the ability has an uniform distribution between zero and one for both sexes, which means in this hyperbola form, the gender difference only reflects on the interception term \(b^g\). In this way, I rule out the fundamental gender difference in the non-cognitive skills which is the major topic in Jacob (2002) and Cho (2005), and restrict my emphasis on the gender difference in the “economic” variable: college wage premium. Certainly this does not mean gender difference in the non-cognitive skills is not important. Just because I want to fully understand the explanation power of the gender differences in the skill premium on the gender differences in the college attendance, therefore, I have to isolate this effect as much as I can.

### 3.3 Data

I use 1950, 1960 Census and March CPS 1962-2003 data to construct the data counterparts in the model. I choose the sample restrictions to follow those used in Eckstein and Nagypal (2004) except that I do further restriction to only include high school graduates (HSG hereafter) between age 18 and 65 and college graduates (CG) between age 22 to 65 in my sample. As in their paper, I restrict my attention to the full-time full-year (FTFY) workers. The wage here is the annualized wage and salary earnings. And I use the Personal Consumption Expenditure deflator from NIPA to convert all wages in terms of constant 2002 dollars.

#### 3.3.1 Cohort-Specific Wage Premium

First of all, let’s look at the trend of the aggregate skill premium by gender \(\frac{w^c,g}{w^h,g}\). The trend of the aggregate skill premium is well known as a “N” shape. Starting from early 1950s, it went up until around 1970, decreased for the 1970 decade, then increased dramatically since 1980. (See Katz (1999) Figure 4, Krusell, Ohanian, Rios-Rull and Violante (2000) Figure 3, Acemoglu (2003) Figure 1). Trend of the aggregate gender-specific skill premium more or less follows that of the aggregate one. Figure 3.5 shows the male skill premium increased slightly during 1950s, then decreased until 1963, and increased again until 1969. The whole 1970s was the depressed decade. The trend reversed in 1981, the skill premium increased.
from 1.43 to the peak 1.91 in 2001, the average annual growth rate is around 1.45%. The female skill premium follows the male one, except the dip in the 1970s is deeper. Then starting from 1978, it increased from 1.35 to 1.75 in 2002, the average annual growth rate is around 1.08%.

Figure 3.6 shows the trend of gender wage gap by education. HSG gender wage gap $\frac{w_{h,f}}{w_{m,f}}$ was moderately stable from 1949 to 1979, the gap was 0.60 in 1949, 0.58 in 1979. Starting from 1979, it began to decrease, reaching around 0.72 in 2002. CG gender wage gap $\frac{w_{c,f}}{w_{m,f}}$ exhibits the similar pattern. Starting 0.57 in 1949, it fluctuated in the range from 0.60 to 0.54, reached the bottom level 0.538 in 1978, and turned to narrow since then. It reached the peak 0.69 in 1996. In 2002, the gap was around 0.68. To summarize, the gender wage gaps were fairly stable or even a little divergent till late 1970s, then began a rapid convergence.

Combining Figure 3.2 and 3.5, we can get a rough idea how the gender-specific skill premium affects the enrollment gender gap. In Figure 3.7, I put the enrollment rate of male HSG and the male skill premium together. We can see that they move together quite closely, especially for the downward part until 1963, and the increasing part from the early 1980s. From 1963 to 1976, the trend of enrollment rate is much more volatile than that of skill premium. A popular explanation is due to the Vietnam War. Male HSGs wanted to use the student deferments to avoid the draft, hence the enrollment rate of males increased unusually. (From 1963 to 1968, it increased from 52.27% to 63.18%. The next time enrollment rate exceeds this peak is almost 30 years later, 1997!) The draft ended in 1972. As the effect of the draft phased out, we see enrollment rate went down to the earth from 63.18% in 1968 to 47.24% in 1976.

In the female counterpart Figure 3.8, we don’t observe this comovement pattern before 1976, actually they went into different directions. The female enrollment rate kept increasing from 34.61% in 1955 to 50.33% in 1976, but the female skill premium went down during this period. After 1976, two trends converged, both have increased dramatically since the later 1970s.

Figure 3.7 and 3.8 shed some light on the motivation of trying to use the gender differences in the skill premium to explain the college gender gap as we observe in the data. Rapid increase of gender-specific skill premium after 1980 implies for males and females, the future wage differentials between CG and HSG increase. According to the model in Section 3.2, we would expect the enrollment rates of both sexes increase too. On the other hand, the narrowing wage gender gap since the late 1970s means the increase of female earnings is relatively more than that of males. The expected gain from schooling is higher for females, hence we should expect the increase of female enrollment rate is more than that of males, which is exactly the phenomenon shown in the data as in Figure 3.2.
But people do not make the educational decision based on the aggregate skill premium. When an individual graduates from high school, she forms her own expected life time wage profile specific to the cohort she is in. For example, a 18 year old female HSG in 1970 faces the choice to go to college or not. The criteria she uses is her expectations about if she chooses to go to college, how much she is going to earn when she is 22, graduated from college, when she is 23, 24, and so on. On the other hand, if she does not choose to go to college, how much she is going to earn this year, next year when she is 19, the year after next year when she is 20, and so on. Each cohort will have its own expected wage profiles, and these profiles might be different if the trend of the skill premium changes over time. The question is how can we get the data of these cohort-specific wage profiles.

Fortunately since CPS is a repeated cross-sectional data set, I can use so-called “pesudo-cohort construction method” to construct them as I used in Chapter 1.\(^1\) For example, the 1962 cohort (18 year old HSG in 1962)’s life time (18-65 year old) male HSG wage profile \(\{w_{1961+j}^{h,m}\}_{j=1}^{48}\) is constructed as follows: I take 18 year old male HSGs in 1962, calculate their mean wage, then 19 year old male HSGs in 1963, calculate the mean wage, next 20 year old HSGs on 1964, 21 year old HSGs in 1965, and so on, until 58 year old HSGs in 2002 which is the end year of CPS dataset. Later I will explain how to predict the mean HSG wage after age 58 to complete the life time wage profile for this cohort. See Figure 1.7 for the construction.

I do the similar thing to construct the 1962 cohort’s male CG wage profile \(\{w_{1961+j}^{c,m}\}_{j=1}^{48}\). But I start from 1966 because if someone from 1962 cohort chooses to go to college, he needs to spend four years in college, and graduates in 1966 and starts to earn CG wage since that year. Therefore, I take 22 year old male CGs in 1966, calculate their mean wage, follow the procedure above again. See Figure 1.8 for the construction.

Similarly, we can also construct the 1962 cohort’s HSG and CG wage profiles for females \(\{w_{1961+j}^{s,f}\}_{j=1}^{48}\), \(s = h\) or \(c\). Due to the large sample size of the CPS data, I can view these profiles as the expected wage profiles which a representative agent in that cohort has. I apply this procedure to March CPS for 1962-2002 period and 1960 Census data and obtain the original data sequences of cohort-specific gender different HSG and CG life-time wage profiles for 1955-2002 cohorts.

However, due to the time range of the CPS data, I do not have a complete life-time wage profile for any cohort. For example, some cohorts miss the later age data points (cohorts after 1961), some miss the early age data points (cohort 1955), some miss both (cohorts between 1956 and 1960). I use the econometric method to predict the mean wage at that

\(^1\) It is pseudo-cohort because CPS is not a panel dataset. It does not track people over life time. Heckman, Lochner and Todd (2003) use the similar method to estimate the cohort-based return to schooling. They also face the missing data problem as I mention in this section, and they use the Mincer equation to extrapolate.
specific age to extrapolate the missing data. I predict them by either second or third order polynomial specification, or conditional Mincer equation as following

\[
\begin{align*}
\log[HSG_{g,h}(age)] &= \beta_{g,h}^0 + \beta_{g,h}^1 \text{experience}_h + \beta_{g,h}^2 \text{experience}_h^2 + \epsilon_{g,h}, \text{ experience}_h = \text{age}-18 \\
\log[CG_{g,c}(age)] &= \beta_{g,c}^0 + \beta_{g,c}^1 \text{experience}_c + \beta_{g,c}^2 \text{experience}_c^2 + \epsilon_{g,c}, \text{ experience}_c = \text{age}-22
\end{align*}
\]

The criteria basically is the goodness of fitness, I also check with the neighborhood cohorts to make sure the predicted value is reasonable. The “rule of thumb” of hump-shaped profile applies here to help make choice too. For example, if an estimation gives me an exponential trend of life time wage profile, I cannot take it because I believe in two stylized facts about the age-earning profiles: (1) Earnings rise over time, but at a decreasing rate; (2) Earnings increase faster for more educated workers (See Borjas (2005) page 266), which implies CG should have higher and steeper hump-shaped (or increasing but concave) wage profile than that of HSG. This criteria implies for most of cohorts I use second order polynomial method to extrapolate (essentially it is similar to the Mincer equation). My extrapolation work stops at 1979 cohort because after this cohort, lack of data point creates trouble, hence I do not have reliable prediction.\(^2\) Eventually I obtain complete cohort-specific gender different HSG and CG life time wage profiles from 1955 to 1979 cohort. 1950s cohorts have the best data quality because they have least missing data points.

These cohort-specific wage profiles will provide the information needed in the first two terms in equation (3.6). To calculate the net present value of higher education over time, I also need the information about the tuitions, the direct cost of college-going. In Figure 1.5 I report the real tuition, fee, room and board (TFRB) per student charged by an average four-year institution. TFRB increased over time except the 1970s when it became stable. Starting from 1980, real TFRB raised dramatically. I will use the TFRB data for the tuition payments in equation (3.2).

### 3.3.2 Cross-Section Wage Premium

To reconcile with the empirical literature, also to test which assumption (perfect foresight vs. using current information for expectations) gives me better explanation power, I obtain the cross-section wage premium for each cohort from CPS 1962-2003. I assume that individuals in each birth cohort (18 years old HSG) form the expectation about their cohort-specific age-earning profiles based on the current information they possess at age 18. In this way, the cohort-specific wage premium and the cross-section wage premium (at age 18) are the same. For example, the 1962 cohort’s life time (18-65 year old) male HSG wage

\(^2\)Heckman, Lochner and Todd (2003) also notice this problem and stop in 1983 for their cohort-based estimates.
profile \( w_{j,1961+j}^{h,m} \) is the cross-sectional (with respect to age) male HSG wage in 1962.
\( \{w_{j,1962}^{h,m}\}_{j=1}^{48} \). An advantage of this specification is that I do not need to worry about the missing data problem as it appears in the cohort-specific wage premium. But the disadvantage is it does not capture the change in the wage patterns. If wage is increasing over time, cross-section age-earning profile would exhibit a clear hump shape, but cohort-specific age-earning profile should be higher and steeper, and probably there is no late age drop as shown in Figure 3.9.

### 3.3.3 NPV and IRR of Higher Education

For all of the data obtained above, I use three period moving average method to reduce the noise of the data. Now by using these filtered data, given the interest rate is constant at 6\%, I can calculate the NPV of higher education by sex according to equation (3.6). The results are shown in Figure 3.10 and 3.11.

For men, Figure 3.10 shows NPV calculated by using the cohort-specific wage profiles (NPV-Cohort) is higher than the NPV calculated by using the cross-section wage profiles (NPV-Crosssection). The reason is exactly what I explain in Figure 3.9. Since the skill premium increases over most of time, therefore the cohort-specific wage profiles are much higher than cross-section ones. Both NPVs decreased until 1963, which is consistent with the downward part of enrollment rate. NPV-Cohort then kept decreasing until 1968, and started to increase dramatically since 1970. The bottom of NPV-Cohort was around 1970 because the forward-looking 1970 cohort would experience the longest skill premium depression (recall Figure 3.5, skill premium turned to decrease around 1970). Later cohorts, however, would enjoy higher benefit of rising skill premium. On the other hand, NPV-Crosssection turned to increase from 1963 to 1973, then decreased and kept stable, and started to increase again dramatically since 1981. Compare with the enrollment rate data, we could say NPV-Crosssection seems to be more consistent with the trend of enrollment rate.

For women, Figure 3.11 shows a different story. As in the male case, NPV-Cohort is higher than NPV-Crosssection. From 1955 to 1970, although there were fluctuations, the trend was increasing. Since 1970, it started to increase dramatically. On the other hand, NPV-Crosssection inherits the trend of the female skill premium as shown in Figure 3.5. It increased from 1963 to 1971, since then decreased quite deeply, and starting from 1978, turned to increase dramatically.

I also use another measurement of return to schooling to show how the evolution of the skill premium is going to affect the return of higher education. It is the interest rate to make NPV in equation (3.6) equal to zero. I call it Internal Return Rate to Schooling (IRR). When the actual market interest rate \( r_t \) is less than IRR, investment in higher education
is profitable, otherwise, it is not. Higher NPV means higher IRR, hence more likely the human capital investment in higher education is profitable. Therefore, it is not surprising to see in Figure 3.12 and 3.13 the trend of IRR by sex follows that of NPV in Figure 3.10 and 3.11. In both figures, it seems that IRR-Crosssection captures the trend of enrollment rate pretty well.\(^3\) In both cases, IRR-Cohort is higher than IRR-Cross since the forward-looking prediction in cohort-specific estimates gives us higher NPV. And they are very close to each other around 1970 because the depression of the skill premium in the 1970s affects the forward-looking prediction of the 1970 cohort most deeply, it decreased the NPV and IRR. On the other hand, pre-depression cross-sectional estimate in 1970 does not take into account the future decreasing of skill premium. Another thing that needs to be mentioned is for IRR-Crosssection. Most time male’s is higher than that of female, which reflects that in most years (especially after 1975), male skill premium is higher than female one. Since the tuition is the same for both sexes, this means male NPV is much higher than female’s, so does the IRR.

### 3.4 Calibration

From the trend of NPV and IRR, I can infer the trend of the enrollment rate. But still it is not the enrollment rate itself. One would like to see if I feed the data of wage profiles and tuitions into the model above, what is the enrollment rate that the model will generate. Then comparing it with the actual data, one can conclude how many percentage of the change in enrollment rate over time can be explained by the change in the wage profiles that male and female face differently.

The calibration strategy I employ here is to adopt the common values that are widely used in literature for the preference and discount factor parameters. For the model-specific disutility parameter \(b^g\) as in equation (3.7), I will calibrate it to match the enrollment rate data of the initial cohort, and then use this parameter value to predict the enrollment rate thereafter.

The value of discount factor \(\beta\) I take is 0.99, which is quite close the the one used in Auerbach and Kotlikoff (1987) for a representative agent, life cycle model with certain lifetimes (they use 0.9852). The value of preference parameter \(\sigma\) is 2, which is taken from

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\(^3\)Heckman, Lochner and Todd (2003) also calculate the IRR of the higher education (Figure 7b in their paper) by using the similar two methods as mine, but only for the white men. Comparing their results to mine in Figure 3.12, they are quite different. On the cross-sectional data side, both show the dramatic increase since 1980. But HLT figure shows IRR went down from 1964 to 1980, while my figure actually increased from 1963 to around 1970, then decreased in 1974 and became stable since then. Their magnitude is also higher than mine. On the cohort-based data side, their IRR started to grow since 1962, which is much earlier than mine (around 1972). It would be interesting to understand why there is such a difference in these two calculations.
Ímrohoğlu, Ímrohoğlu, and Joines (1995), and it is also widely used in the life cycle literature. Since this is a partial equilibrium model, all the prices are given, I take the interest rate $r_t = 6\%$ for the consistence with the NPV calculation.

I calibrate the model based on two alternative expectations hypotheses. One is the perfect foresight or rational expectations (RE). I assume people rationally expect their future wage profiles, which I am going to use the cohort-specific wage profiles as a proxy. Another is naive expectations (NE) in which people just use the current information to predict future when they make decision. In this case, the future age-earning profile is the current cross-section age-earning profile. I want to evaluate these two hypotheses based on their performances on the college enrollment rate.

Due to the availability of the data, cohort-specific wage profiles begin from 1955 cohort, end in 1979, while cross-section data are from 1961 to 2002 cohort. As mentioned above, I calibrate disutility parameter $b^g, g = m$ or $f$ to match the enrollment rate data of the initial cohort (which is 1955 for rational expectations (RE), 1961 for the naive expectations (NE)). In Table 3.1, I show the values for each case.

<table>
<thead>
<tr>
<th>Value</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.575</td>
<td>male, RE</td>
</tr>
<tr>
<td>5.238</td>
<td>female, RE</td>
</tr>
<tr>
<td>1.619</td>
<td>male, NE</td>
</tr>
<tr>
<td>1.7</td>
<td>female, NE</td>
</tr>
</tbody>
</table>

Table 3.1: Parameter Values of the Disutility Scale Factor

Obviously the disutility cost of schooling is higher for females than males. Given the parameter values as above, the model generates the time series of enrollment rate. I show the results in Figure 3.14 and 3.15.

For male, in Figure 3.14, starting from 59.17% in 1955, Rational Expectations model (Model-RE) is consistent with the data fairly well until 1969, then they diverge. RE model implies the enrollment rate starts to go up since 1970. But data seem to be more consistent with the Naive Expectations model (Model-NE). It goes down and hovers around 50% until 1980. NE model is consistent with the data fairly well until 1981. Then starting from 1981, the model predicts that the enrollment rate increases from 48.4% to 75.8% in 2002, while the data are from 54.8% to 62.1%. Model overpredicts the rising of enrollment rate since 1980.

Turn to females, in Figure 3.15, the interesting thing is now RE model matches data amazingly well. Starting from 34.7% in 1955, the RE model predicts that enrollment rate reaches 50.8% in 1979, compare to the data 48.4%. On the other hand, NE model is quite out of tone. Especially before 1983, model-generated enrollment rate fluctuates a lot. It
backs to track since 1983, increases from 49.2% in 1983 to 76.6% in 2002, while the data increases from 53.4% to 68.4%.

Both figures show that the effect of the gender-specific college wage premium on the gender-specific college enrollment rate is positive and significant, especially the drastic increase of enrollment rate for both sexes after 1980 is clearly driven by the rising college wage premium. The conclusion that Averett and Burton (1996) draw is partially right, we cannot ignore the effect of the college wage premium on female college attendance.

3.5 Gender Differences in Employment and Labor Participation Rate

It is well known that the gender gap in labor force participation rate (LFP)\(^4\) has been largely closed in the recent three decades, so does the employment rate (EMP) gender gap. As shown in Figure 3.16, in 1962, the gender gap in LFP is 48.8%, in EMP is 45.9%. After 41 years, in 2003 these gaps has been narrowed to 12.6% and 11%. As more and more women have being liberalized from housework and went on the job market to earn more than before, could that provide a stronger incentive for women to go to college? Also, in the model and calibration above, I actually assume full labor force participation and employment rate, everybody can find a job, there is no unemployment. To make the model and calibration more realistic, I should take into account the chance that people will not be in the labor force, or even they are in, but get chance to become unemployed.

Since unemployment rate in the US does not show a secular trend over time, the trend of EMP follows that of LFP very closely, therefore I will focus on the employment rate hereafter. I treat EMP as a probability that an individual gets employed to earn wage income. Now in the model, every individual faces the schooling choice. If she chooses to go to college, after four years, she will have this probability to find a job and earn CG wage. She also has a chance to not be employed hence earn nothing. Therefore instead of certain CG wage sequence, she actually face an expected CG wage sequence. The budget constraints of an individual with sex $g$ born at time $t$ who chooses $s = c$ becomes

\[
\begin{align*}
    c_{j,t+j-1} + & tuition_{j,t+j-1} + a_{j,t+j-1} & \leq (1 + r_{t+j-1}) a_{j-1,t+j-2} & \forall j = 1, 2, 3, 4 & \quad (3.8) \\
    c_{j,t+j-1} + & a_{j,t+j-1} & \leq (1 + r_{t+j-1}) a_{j-1,t+j-2} + emp_{t+j-1}^{c^g} w_{j,t+j-1}^{c^g} & \forall j = 5, ..., J \\
    c_{j,t+j-1} & \geq 0, a_{0,t-1} = 0, a_{J,t+J-1} & \geq 0. & \quad (3.9)
\end{align*}
\]

\(^4\)The labor force participation rate is the fraction of total number of people age 18-65 and in the labor force divided by the population of that age group. The employment rate is the fraction of total number of people age 18-65 who are employed divided by the population of that age group. Hence we have EMP+Unemployment Rate=LFP.
The employment rate of CG with sex $g$ at time $t + j - 1$ is given by $emp_{t+j-1}^{c,g}$. Similarly, the budget constraints of an individual with sex $g$ born at time $t$ who chooses to go on the job market becomes

$$
c_{j,t+j-1} + a_{j,t+j-1} \leq (1 + r_{t+j-1})a_{j-1,t+j-2} + emp_{t+j-1}^{h,g}w_{j,t+j-1}^h \quad \forall j = 1, \ldots, J \tag{3.10}
$$

$$
c_{j,t+j} \geq 0, a_{0,t} = 0, a_{J,t+J-1} \geq 0.
$$

Through this way, we introduce the gender difference in employment rate gap into the model. From 1960 Census and 1962-2003 CPS data, I obtain the employment rate by education and sex $\{emp_t^{s,g}\}_{t=1960}^{2003}$, it is shown in Figure 3.17. From this figure, it seems clear that control for education, the gender EMP gaps $\frac{emp^{c,m}}{emp^{c,f}}$ are narrowing. But control for gender, the EMP education gaps $\frac{emp^{c,g}}{emp^{h,g}}$ show a different pattern. Male EMP education gap $\frac{emp^{c,m}}{emp^{h,m}}$ is widening over time. While female EMP education gap $\frac{emp^{c,f}}{emp^{h,f}}$ is quite stable or even slightly narrowing.

One would also like to see what are the EMP-adjusted NPV and IRS of higher education for both sexes. Keeping interest rate equal to 6% as before, I calculate NPV and IRS in two ways. One is that I assume rational expectations. Hence I feed in cohort-specific wage profiles and cohort-specific expected EMP $\{emp_t^{s,g}\}_{t=1960}^{2003}$. In contrast, for naive expectations case, I feed in current time (age 18 at time $t$) cross-section wage profiles and current $emp_t^{s,g}$ for expected ones. It is not surprising to see NPV goes down for both sexes since the expected wages go down. For IRR, female EMP-adjusted IRR is higher than non-adjusted one under both methods. This is because LFP and EMP increase over time for female, this effect reinforces the increasing skill premium effect to make college even more attractive for women.

To see whether the inclusion of employment rate will contribute to a better model performance, I calibrate the model again by using all of the EMP-adjusted data. The disutility parameter $b^g$ is recalibrated to match the enrollment rate of the initial cohort. Other parameter values are as same as in Section 3.4. The results are shown in Figure 3.18 and 3.19. For male, the degree of fitness decreases for both RE and NE case. Female one is even worse, especially under NE hypothesis, the model-generated enrollment rates are quite below the actual ones, and they are always lower than the male ones. Hence the model cannot generate the cross-over as observed in the data in this circumstance.

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5I interpolate the 1961 data as the mean value between 1960 and 1962 data.

6I extrapolate the EMP data from 1955 to 1959 based on the third order polynomial estimation. We also need to predict future EMP for late cohorts’ expectation. For example, 1979 cohort will need the information about EMP up to 2026 to form the life-time expected wage profiles. For simplicity, I use the 2003 data for all of the subsequent years.
3.6 Conclusion

This paper documents the dramatic changes in the gender gap of college educational attainment over the past four decades, and shows the pattern of this gender gap reversed after later 1970s. Although there are some empirical literature already link this phenomenon to the narrowing wage gender gap, gender difference in non-cognitive skills or the dispersion of the wage profiles, none of them establish a life cycle model and ask the quantitative question to what extent changes in the college gender gap can be accounted for by the changes in the gender differences in cohort-based college wage premium, which is exactly the question this chapter tries to answer.

In order to investigate the gender differences in college enrollment rate, a discrete time overlapping generations model is established. An individual faces the discrete choice of college-going in the first period, and the decision is based on the cost-benefit analysis implied by standard human capital investment theory. In this model, gender difference reflects in two aspects: different age-earning profiles, and different disutility scale factor. This model is then calibrated to generate the enrollment rate which is compared with the actual data.

There are two expectations hypotheses are tested in this paper. One is the perfect foresight. I assume people rationally expect their future wage profiles, which I use the cohort-specific wage profiles constructed from Census and CPS dataset to proxy. Another is naive expectations, people just use the current information to predict future earnings when they make decision. In this circumstance, the expected age-earning profile is as same as the current cross-section age-earning profile. Based on these two hypotheses, the calibration results show for men, the naive expectations prediction matches the data better. While for women, the rational expectations prediction captures fairly well the trend of enrollment rate up to 1979. For both sexes, the naive expectations prediction captures (even overshoot) the dramatic increase of enrollment rate since 1980. Hence the trend of gender skill premium does explain the trend of college gender gap to a significant extent.
Figure 3.1: Total Fall Enrollment of Degree-Granting Higher Education Institution
Figure 3.2: College Enrollment Rate of Recent HSG: 1955-2002

Figure 3.3: College Enrollment Rate of Recent HSG: Smoothed Data
Figure 3.4: Ability and Schooling Choice
Figure 3.5: Skill Premium by Sex

Figure 3.6: Gender Wage Gap by Education

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Figure 3.7: Skill Premium and College Enrollment Rate: Male

Figure 3.8: Skill Premium and College Enrollment Rate: Female
Figure 3.9: Life Cycle Wage Profile: Cross Section vs. Cohort View
Figure 3.10: NPV of Return to Schooling: Male

Figure 3.11: NPV of Return to Schooling: Female
Figure 3.12: Internal Rate of Return to Schooling: Male

Figure 3.13: Internal Rate of Return to Schooling: Female
Figure 3.14: College Enrollment Rate of Male: Model vs. Data
Figure 3.15: College Enrollment Rate of Female: Model vs. Data
Figure 3.16: Labor Force Participation and Employment Rate by Sex

Figure 3.17: Employment Rate by Sex and Education
Figure 3.18: College Enrollment Rate of Male: EMP-Adjusted Model vs. Data

Figure 3.19: College Enrollment Rate of Male: EMP-Adjusted Model vs. Data
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