

Problem Set 4

1. Consider a binomial distribution with n trials and a success probability of $1/3$. Let X denote the number of successes. What is the average of X^2 ?
2. Suppose that X has a uniform distribution on $[5,10]$. Calculate:
 - a. $P(X < 5)$
 - b. $P(6 < X < 9)$
 - c. $P(X < 9.5)$
 - d. $P(X > 7)$
3. MENSA requires that its members have an IQ above 131.5. IQ's have a Normal distribution with mean 100 and standard deviation of 15. Solve the following:
 - a. What is the probability that a randomly selected person can get into MENSA?
 - b. If 10 people are selected at random from the population, what is the probability that their mean IQ will be above 131.5?
 - c. Suppose that the mean IQ of these 10 people is indeed above 131.5. Does this mean that all 10 of these people can join MENSA?
 - d. Suppose that we choose 100 people from the population. Of these, how many can we expect to join MENSA?
4. Suppose that X has a standard normal distribution. Calculate:
 - a. $P(X < 1)$
 - b. $P(-2 < X < 1)$
 - c. $P(X > -0.5)$
 - d. $P(X < -0.5 \cup X > 0.5)$
 - e. $P(X < 0.5 \cup X > -0.5)$
5. Suppose that X has a standard normal distribution. Calculate the:
 - a. 75th percentile
 - b. 45th percentile
 - c. 25th percentile
 - d. 2.5th percentile
 - e. 99th percentile

NOTE: the x th percentile is a number such that $x\%$ of the observations are to the left of that number.

6. A doorway is 72 inches high. Heights are normally distributed with a mean of 69 inches and a standard deviation of 2.8 inches.
- What is the probability that somebody cannot make it through the doorway without ducking?
 - If 50 people walk through the doorway, what is the expected number of "duckers?"
 - What is the probability that the number of "duckers" is less than 10?
 - What is the probability that the number of "duckers" is between 5 and 8?
- HINT: Use the binomial approximation of the normal distribution.
7. Suppose that X has a normal distribution with mean 10 and standard deviation 5. Calculate the z-scores for:
- $X = 14$
 - $X = 2$
 - $X = 11$
 - $X = 0.5$
 - $X = 1$
8. Suppose that X has a normal distribution with mean 10 and standard deviation 5. Calculate:
- $P(X < 14)$
 - $P(2 < X < 11)$
 - $P(X > 0.5)$
 - $P(X < 1 \cup X > 11)$
 - $P(X = 0.5)$
9. Suppose that X has a normal distribution with mean 10 and standard deviation 5. Calculate the:
- 75th percentile
 - 45th percentile
 - 25th percentile
 - 2.5th percentile
 - 99th percentile
10. Recall that heights are normally distributed with a mean of 69 inches and a standard deviation of 2.8 inches. Suppose that 130 people enroll in a class and let \bar{x} denote the mean height in this class. Calculate:
- $P(\bar{x} < 75)$
 - $P(\bar{x} < 60)$
 - $P(\bar{x} > 80)$