Introduction to Statistics

Professor Timothy Halliday

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Question 1

Consider a binomial distribution with 300 trials and a success probability of $1/3$. Let X denote the number of successes. What is the average of X^2?

Question 1 Solution

The RV can be approximated by a normal with

$$\mu = 100$$
$$\sigma^2 = 300 * \frac{1}{3} * \frac{2}{3} = 66.67$$

Therefore, we have that

$$E\left[X^2\right] = 66.77 + 100^2 = 10066.67$$

Question 2

Suppose that X has a uniform distribution on [5,10]. Calculate:

1. $P(X<5)$

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1. $P(X<5)$
2. $P(6<X<9)$

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1. $P(X<5)$
2. $P(6<X<9)$
3. $P(X<9.5)$
4. $P(X>7)$

Question 2 Solution

1. $P(X < 5) = 0$

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2. $P(6 < X < 9) = \frac{9-6}{10-5} = \frac{3}{5} = 0.6$

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2. $P(6 < X < 9) = \frac{9-6}{10-5} = \frac{3}{5} = 0.6$
3. $P(X < 9.5) = \frac{9.5-5}{10-5} = \frac{4.5}{5} = 0.9$

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2. $P(6 < X < 9) = \frac{9-6}{10-5} = \frac{3}{5} = 0.6$
3. $P(X < 9.5) = \frac{9.5-5}{10-5} = \frac{4.5}{5} = 0.9$
4. $P(X > 7) = \frac{10-7}{10-5} = \frac{3}{5} = 0.6$

Question 3

MENSA requires that its members have an IQ above 131.5. IQ's have a Normal distribution with mean 100 and standard deviation of 15. Solve the following:

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1. What is the probability that a randomly selected person can get into MENSA?
2. If 10 people are selected at random from the population, what is the probability that their mean IQ will be above 131.5?

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MENSA requires that its members have an IQ above 131.5. IQ's have a Normal distribution with mean 100 and standard deviation of 15. Solve the following:

1. What is the probability that a randomly selected person can get into MENSA?
2. If 10 people are selected at random from the population, what is the probability that their mean IQ will be above 131.5?
3. Suppose that the mean IQ of these 10 people is indeed above 131.5. Does this mean that all 10 of these people can join MENSA?

Question 3

MENSA requires that its members have an IQ above 131.5. IQ's have a Normal distribution with mean 100 and standard deviation of 15. Solve the following:

1. What is the probability that a randomly selected person can get into MENSA?
2. If 10 people are selected at random from the population, what is the probability that their mean IQ will be above 131.5?
3. Suppose that the mean IQ of these 10 people is indeed above 131.5. Does this mean that all 10 of these people can join MENSA?
4. Suppose that we choose 100 people from the population. Of these, how many can we expect to

Question 3 Solution

1. $P\left(X > 131.5\right) = P\left(Z > \frac{131.5-100}{15}\right) = P\left(Z > 2.1\right) = 1 - 0.9821 = 0.0179$

Question 3 Solution

1. $P(X > 131.5) = P\left(Z > \frac{131.5-100}{15}\right) = P(Z > 2.1) = 1 - 0.9821 = 0.0179$
2. $\overline{x} \sim N(100, 22.5) \Rightarrow P(\overline{x} > 131.5) = P\left(Z > \frac{131.5-100}{4.7437}\right) = P(Z > 6.64) \approx 0$

Question 3 Solution

3. No. For example, consider the sample

$$\left(\underbrace{135, ..., 135}_{\times 9}, 131\right).$$

The mean is above 131.5 but the 10th guy cannot join.

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4. The probability of a success (i.e. joining MENSA) is 1.79%. This is just a binomial RV and so

$$\mu = 0.0179 * 100 = 1.79$$

Question 4

Suppose that X has a standard normal distribution. Calculate:

1. $P(X < 1)$

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1. $P(X < 1)$
2. $P(-2 < X < 1)$

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4. $P(X < -0.5 \cup X > 0.5)$

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Suppose that X has a standard normal distribution. Calculate:

1. $P(X < 1)$
2. $P(-2 < X < 1)$
3. $P(X > -0.5)$
4. $P(X < -0.5 \cup X > 0.5)$
5. $P(X < 0.5 \cup X > -0.5)$

Question 4 Solution

1. $P(X < 1) = 0.8413$

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2. $P(-2 < X < 1) = P(X < 1) - P(X < -2) = 0.8413 - 0.0228 = 0.8185$

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2. $P(-2 < X < 1) = P(X < 1) - P(X < -2) = 0.8413 - 0.0228 = 0.8185$
3. $P(X > -0.5) = 1 - 0.3085 = 0.6915$

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3. $P(X > -0.5) = 1 - 0.3085 = 0.6915$
4. $P(X < -0.5 \cup X > 0.5) = 2 * P(X < -0.5) = 2 * 0.3085 = 0.617$

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5. $P(X < 0.5 \cup X > -0.5) = 1$

Question 5

Suppose that X has a standard normal distribution. Calculate the:

1. 75th percentile

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1. 75th percentile
2. 45th percentile

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3. 25th percentile

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1. 75th percentile
2. 45th percentile
3. 25th percentile
4. 2.5th percentile

Question 5

Suppose that X has a standard normal distribution. Calculate the:

1. 75th percentile
2. 45th percentile
3. 25th percentile
4. 2.5th percentile
5. 99th percentile

Question 5 Solution

1. 0.675

Question 5 Solution

1. 0.675
2. -0.125

Question 5 Solution

1. 0.675
2. -0.125
3. -0.675

Question 5 Solution

1. 0.675
2. -0.125
3. -0.675
4. -1.96

Question 5 Solution

1. 0.675
2. -0.125
3. -0.675
4. -1.96
5. 2.325

Question 6

A doorway is 72 inches high. Heights are normally distributed with a mean of 69 inches and a standard deviation of 2.8 inches.

a. What is the probability that somebody cannot make it through the doorway without ducking?

b. If 50 people walk through the doorway, what is the expected number of "duckers?"

c. What is the probability that the number of "duckers" is less than 10?

d. What is the probability that the number of "duckers" is between 5 and 8?

Question 6 Solution

1. $X \sim N(69, 7.84) \Rightarrow P(X > 72) = P\left(Z > \frac{72-69}{2.8}\right) = P(Z > 1.0714) = 1 - 0.8577 = 0.1423$

Question 6 Solution

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2. This is a binomial RV with success rate 0.1423. $\mu = 0.1423 * 50 = 7.115$

Question 6 Solution

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2. This is a binomial RV with success rate 0.1423. $\mu = 0.1423 * 50 = 7.115$
3. Let Y denote the number of "duckers." Then $Y \sim N(7.115, 6.1025) \Rightarrow P(Y < 10) = P\left(Z < \frac{10-7.115}{\sqrt{6.1025}}\right) = P(Z < 1.1679) = 0.8790$

Question 6 Solution

1. $X \sim N(69, 7.84) \Rightarrow P(X > 72) = P\left(Z > \frac{72-69}{2.8}\right) = P(Z > 1.0714) = 1 - 0.8577 = 0.1423$
2. This is a binomial RV with success rate 0.1423. $\mu = 0.1423 * 50 = 7.115$
3. Let Y denote the number of "duckers." Then $Y \sim N(7.115, 6.1025) \Rightarrow P(Y < 10) = P\left(Z < \frac{10-7.115}{\sqrt{6.1025}}\right) = P(Z < 1.1679) = 0.8790$
4. $P(5 < Y < 8) = P(Y < 8) - P(Y < 5) = P\left(Z < \frac{8-7.115}{\sqrt{6.1025}}\right) - P\left(Z < \frac{5-7.115}{\sqrt{6.1025}}\right) = P(Z < 0.35825) - P(Z < -0.85616) = 0.6406 - 0.1949 = 0.4457$

Question 7

Suppose that X has a normal distribution with mean 10 and standard deviation 5. Calculate the z-scores for:

a. $X = 14$

b. $X = 2$

c. $X = 11$

d. $X = 0.5$

e. $X = 1$

Question 7 Solution

1. $\frac{14-10}{5} = 0.8$

Question 7 Solution

1. $\frac{14-10}{5} = 0.8$
2. $\frac{2-10}{5} = -1.6$

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2. $\frac{2-10}{5} = -1.6$
3. $\frac{11-10}{5} = 0.2$
4. $\frac{0.5-10}{5} = -1.9$

Question 7 Solution

1. $\frac{14-10}{5} = 0.8$
2. $\frac{2-10}{5} = -1.6$
3. $\frac{11-10}{5} = 0.2$
4. $\frac{0.5-10}{5} = -1.9$
5. $\frac{1-10}{5} = -1.8$

Question 8

Suppose that X has a normal distribution with mean 10 and standard deviation 5. Calculate:

a. $P(X < 14)$

b. $P(2 < X < 11)$

c. $P(X > 0.5)$

d. $P(X < 1 \cup X > 11)$

e. $P(X = 0.5)$

Question 8 Solution

1. $P(X < 14) = P(Z < 0.8) = 0.7881$

Question 8 Solution

1. $P(X < 14) = P(Z < 0.8) = 0.7881$
2. $P(2 < X < 11) = P(X < 11) - P(X < 2) = P(Z < 0.2) - P(Z < -1.6) = 0.5793 - 0.0548 = 0.5245$

Question 8 Solution

1. $P(X < 14) = P(Z < 0.8) = 0.7881$
2. $P(2 < X < 11) = P(X < 11) - P(X < 2) = P(Z < 0.2) - P(Z < -1.6) = 0.5793 - 0.0548 = 0.5245$
3. $P(X > 0.5) = P(Z < 1.9) = 0.9713$

Question 8 Solution

1. $P(X < 14) = P(Z < 0.8) = 0.7881$
2. $P(2 < X < 11) = P(X < 11) - P(X < 2) = P(Z < 0.2) - P(Z < -1.6) = 0.5793 - 0.0548 = 0.5245$
3. $P(X > 0.5) = P(Z < 1.9) = 0.9713$
4. $P(X < 1) + P(X > 11) = P(Z < -1.8) + P(Z > 0.2) = 0.0359 + 0.4207 = 0.4566$

Question 8 Solution

1. $P(X < 14) = P(Z < 0.8) = 0.7881$
2. $P(2 < X < 11) = P(X < 11) - P(X < 2) = P(Z < 0.2) - P(Z < -1.6) = 0.5793 - 0.0548 = 0.5245$
3. $P(X > 0.5) = P(Z < 1.9) = 0.9713$
4. $P(X < 1) + P(X > 11) = P(Z < -1.8) + P(Z > 0.2) = 0.0359 + 0.4207 = 0.4566$
5. 0

Question 9

Suppose that X has a normal distribution with mean 10 and standard deviation 5 . Calculate the:

a. 75th percentile
b. 45th percentile
c. 25th percentile
d. 2.5th percentile
e. 99th percentile

Question 9 Solution

1. $0.675 = \frac{X-10}{5} \Leftrightarrow X = 0.675 * 5 + 10 = 13.375$

Question 9 Solution

1. $0.675 = \frac{X-10}{5} \Leftrightarrow X = 0.675 * 5 + 10 = 13.375$
2. $-0.125 = \frac{X-10}{5} \Leftrightarrow X = -0.125 * 5 + 10 = 9.375$

Question 9 Solution

1. $0.675 = \frac{X-10}{5} \Leftrightarrow X = 0.675 * 5 + 10 = 13.375$
2. $-0.125 = \frac{X-10}{5} \Leftrightarrow X = -0.125 * 5 + 10 = 9.375$
3. $-0.675 = \frac{X-10}{5} \Leftrightarrow X = -0.675 * 5 + 10 = 6.625$

Question 9 Solution

1. $0.675 = \frac{X-10}{5} \Leftrightarrow X = 0.675 * 5 + 10 = 13.375$
2. $-0.125 = \frac{X-10}{5} \Leftrightarrow X = -0.125 * 5 + 10 = 9.375$
3. $-0.675 = \frac{X-10}{5} \Leftrightarrow X = -0.675 * 5 + 10 = 6.625$
4. $-1.96 = \frac{X-10}{5} \Leftrightarrow X = -1.96 * 5 + 10 = 0.2$

Question 9 Solution

1. $0.675 = \frac{X-10}{5} \Leftrightarrow X = 0.675 * 5 + 10 = 13.375$
2. $-0.125 = \frac{X-10}{5} \Leftrightarrow X = -0.125 * 5 + 10 = 9.375$
3. $-0.675 = \frac{X-10}{5} \Leftrightarrow X = -0.675 * 5 + 10 = 6.625$
4. $-1.96 = \frac{X-10}{5} \Leftrightarrow X = -1.96 * 5 + 10 = 0.2$
5. $2.325 = \frac{X-10}{5} \Leftrightarrow X = 2.325 * 5 + 10 = 21.625$

Question 10

Recall that heights are normally distributed with a mean of 69 inches and a standard deviation of 2.8 inches. Suppose that 130 people enroll in a class and let x denote the mean height in this class. Calculate:

a. $P(\overline{x} < 75)$

b. $P(x < 60)$

c. $P(x > 80)$

Question 10 Solution

1. $\bar{x} \sim N\left(69, \underbrace{\frac{2.8^2}{130}}_{0.06}\right) \Rightarrow P(\bar{x} < 75) =$
$P\left(Z < \frac{75-69}{\sqrt{0.06}}\right) = P\left(Z < 24.495\right) = 1$

Question 10 Solution

1. $\bar{x} \sim N\left(69, \underbrace{\frac{2.8^2}{130}}_{0.06}\right) \Rightarrow P(\bar{x} < 75) =$
   $P\left(Z < \frac{75-69}{\sqrt{0.06}}\right) = P(Z < 24.495) = 1$
2. $P(x < 60) = P\left(Z < \frac{60-69}{\sqrt{0.06}}\right) =$
   $P(Z < -36.74) = 0$

Question 10 Solution

1. $\bar{x} \sim N\left(69, \underbrace{\frac{2.8^2}{130}}_{0.06}\right) \Rightarrow P(\bar{x} < 75) =$
   $P\left(Z < \frac{75-69}{\sqrt{0.06}}\right) = P(Z < 24.495) = 1$
2. $P(x < 60) = P\left(Z < \frac{60-69}{\sqrt{0.06}}\right) =$
   $P(Z < -36.74) = 0$
3. $P(x > 80) = 0$