

Introduction to Statistics

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Question 1

Suppose the premium of a home insurance policy is \$400 and in case of a fire the insurance company will pay out \$250,000. If X denotes the profit (or loss) of such a policy, X is a random variable. The probability of a fire $p=0.0002$. Then $X=-400$ with probability 0.9998 (when there is no fire). What is the expected value of X ?

Question 1 Solution

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- If there is no fire, $X = -400$ with $p = 0.9998$.
- Then

$$E[X] = 0.0002 * 249600 - 0.9998 * 400 = -350.00$$

Question 2

Suppose a die is tossed 10 times. What is the probability of getting exactly 3 fours?

Question 2 Solution

- Use the binomial distribution:

$$\begin{aligned}P(X = 3) &= \binom{10}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^7 \\&= \frac{10!}{3!7!} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^7 \\&= \frac{10 * 9 * 8}{3 * 2 * 1} * \left(\frac{1}{6}\right)^3 * \left(\frac{5}{6}\right)^7 = 0.155\end{aligned}$$

Question 3

Hospital records show that of patients suffering from a certain disease, 80% die of it. What is the probability that of 10 randomly selected patients, 4 will recover?

Question 3 Solution

- Use the binomial distribution:

$$\begin{aligned}P(X = 4) &= \binom{10}{4} (0.8)^6 (0.2)^4 \\&= \frac{10 * 9 * 8 * 7}{4 * 3 * 2 * 1} * (0.8)^6 * (0.2)^4 \\&= 0.08808\end{aligned}$$

Question 4

Forty sheets of aluminum alloy were examined for surface flaws. The frequency of the number of sheets with a given number of flaws per sheet was as follows:

Number of flaws	Frequency
0	8
1	6
2	10
3	4
4	8
5	2
6	2

What is the probability of finding a sheet chosen at random which contains 3 or more surface flaws?

Question 4 Solution

$$\begin{aligned}P(X \geq 3) &= P(X = 3) + P(X = 4) + \\ &\quad P(X = 5) + P(X = 6) \\ &= \frac{4}{40} + \frac{8}{40} + \frac{2}{40} + \frac{2}{40} \\ &= \frac{16}{40} = 0.4\end{aligned}$$

Question 5

Consider the probability density function below

x	1	2	3	4	5	6
$p(x)$	0.1	0.2	0.3	0.2	0.05	0.05

What is the mean and standard deviation for this distribution?

Question 5 Solution

- The mean is

$$\begin{aligned}\mu &= 0.1 * 1 + 0.2 * 2 + 0.3 * 3 + 0.2 * 4 + 0.05 * 5 \\ &\quad + 0.05 * 6 + 0.1 * 7 = 3.45\end{aligned}$$

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- The variance is

$$\begin{aligned}\sigma^2 &= 0.1 * (1 - 3.45)^2 + 0.2 * (2 - 3.45)^2 + \\ &\quad 0.3 * (3 - 3.45)^2 + 0.2 * (4 - 3.45)^2 + \\ &\quad 0.05 * (5 - 3.45)^2 + 0.05 * (6 - 3.45)^2 + \\ &\quad 0.1 * (7 - 3.45)^2 = 2.8475\end{aligned}$$

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- $\text{sd} = \sqrt{2.84575} = 1.6869$

Question 6

A random variable has mean of 1 and a variance of 10.
What is $E[X^2]$?

Question 6 Solution

- Recall that

$$\sigma^2 = E[X^2] - E[X]^2$$

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- So, we obtain that

$$E[X^2] = 10 + 1 = 11.$$

Question 7

Consider 10 coin flips and let X denote the number of heads. Compute the probabilities of X equaling 1, 5, and 8.

Question 7 Solution

$$P(X = 1) = \frac{10}{1} * \left(\frac{1}{2}\right)^{10} = 0.00976$$

$$P(X = 5) = \frac{10 * 9 * 8 * 7 * 6}{5 * 4 * 3 * 2 * 1} * \left(\frac{1}{2}\right)^{10} = 0.24609$$

$$P(X = 8) = \frac{10 * 9}{2 * 1} * \left(\frac{1}{2}\right)^{10} = 0.043945$$

Question 8

Your friend has a coin that turns up heads $2/3$ of the time. Consider flipping this coin 10 times and again define X to be the number of heads that you get out the 10 flips. What is the mean and variance of X ?

Question 8 Solution

- The mean of a binomial RV is

$$\begin{aligned}\mu &= n * p \\ &= 10 * \frac{2}{3} \\ &= 6.667\end{aligned}$$

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- The variance is

$$\begin{aligned}\sigma^2 &= np(1 - p) \\ &= 10 * \frac{2}{3} * \frac{1}{3} \\ &= 2.22\end{aligned}$$

Question 9

We flip a coin 10 times and we find that 8 of the flips are heads. If the coin were fair, would this be unusual? Now suppose that we flip the coin 100 times and we observe that 80 of the flips are heads. If the coin were fair, would this be unusual? What changed?

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- For $n = 100$
 - $\mu = 50$
 - $\sigma^2 = 100 * \frac{1}{4} = 25 \Rightarrow \sigma = 5$

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 - $\mu = 50$
 - $\sigma^2 = 100 * \frac{1}{4} = 25 \Rightarrow \sigma = 5$
 - $\mu \pm 2\sigma = [40, 60]$
- 8 out of 10 is not unusual but 80 out of 100 is.

Question 10

Consider two assets. Asset A costs \$10 today and will be either \$5 or \$15 tomorrow with equal probability. Asset B costs \$8 today and will be valued at either \$0 or \$20 tomorrow with equal probability. What are the means and variances of both assets?

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- Mean return

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- Variance of returns

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- Asset B

- Mean return

$$-8 * 0.5 + 12 * 0.5 = 2$$

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- Mean return

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- Variance of returns

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- Asset B

- Mean return

$$-8 * 0.5 + 12 * 0.5 = 2$$

- Variance of returns

$$(-8 - 2)^2 * 0.5 + (12 - 2)^2 * 0.5 = 100$$