

Introduction to Statistics

Professor Timothy Halliday

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Question 1

Which of these numbers cannot be a probability?

- a) -0.00001
- b) 0.5
- c) 1.001
- d) 0
- e) 1
- f) 20%

Question 1 Solution

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Question 2

There are 5 red balls and 3 yellow balls in a box. They are exactly the same except the different colors. If a ball from this box is selected at random, what is the probability that this ball is yellow?

Question 2 Solution

$$\frac{3}{8}$$

Question 3

- There are 5 balls in a box. They are exactly the same except the different colors—red and white. If the probability to select a ball that is red randomly is $\frac{2}{5}$
- i How many red balls and white balls in the box?
 - ii What is the probability to select two balls that are both red randomly? (Assume that we are sampling with replacement).
 - iii What is the probability to select two balls that are both red randomly? (Assume that we are sampling without replacement).

Question 3 Solution

i 2 are red and 3 are white.

ii $\frac{2}{5} * \frac{2}{5} = 0.16$

iii $\frac{2}{5} * \frac{1}{4} = 0.1$

Question 4

The blood groups of 400 people are distributed as follows: 100 have type A blood, 130 have B blood type, 140 have O blood type and 30 have type AB blood. If a person from this group is selected at random, what is the probability that this person has O blood type?

Question 4 Solution

$$\frac{140}{400} = 0.35$$

Question 5

A dice is rolled, find the probability that an odd number is obtained.

Question 5 Solution

$$\frac{1}{2}$$

Digression: Independence

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- Then A and B are **independent events**.
- When A and B are independent then

$$P(A \cap B) = P(A) P(B).$$

Question 6

Consider two independent events A and B . Let $P(A) = 0.2$ and $P(B) = 0.5$. What is $P(\bar{A} \cap \bar{B})$? Are these independent events?

Question 6 Solution

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$$P(\bar{A} \cap \bar{B}) = 1 - P(A) - P(B) + P(A)P(B) = 1 - 0.2 - 0.5 + 0.2 * 0.5 = 0.4$$

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- So, $P(\bar{A} \cap \bar{B}) = P(\bar{A}) * P(\bar{B}) = 0.4$.
- Therefore, \bar{A} and \bar{B} are independent.

Question 7

If Sue eats Wheaties for breakfast she has a 90% of getting an A on this quiz, but if she does not eat Wheaties then she only has a 50% chance of getting an A. If Sue eats Wheaties 60% of the mornings of a quiz, what is the probability that she gets an A?

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- $P(A \cap \overline{W}) = P(A|\overline{W}) P(\overline{W}) = 0.5 * 0.4 = 0.2$
- Therefore, $P(A) = 0.54 + 0.2 = 0.74$.

Question 8

Suppose that I shoot a rifle at a target and hit it 25% of the time. Suppose that I fire 10 shots and the probabilities of hitting the target each time are independent. What is the probability of hitting it at least once?

Question 8 Solution

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- Let A be the event where I hit the target at least once.
- Then \bar{A} is the event where I miss all 10 times.
- So, $P(A) = 1 - P(\bar{A}) = 1 - \frac{3}{4}^{10} = 0.94369$.

Question 9

Suppose that your friend has 4 coins. Half are fair, but half turn up "heads" 75% of the time. Suppose that you randomly choose one of these coins and flip it 3 times and get 3 heads. What is the probability that you chose a fair coin?

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- But, $P(HHH \cap F) = P(HHH|F) P(F) = \left(\frac{1}{2}\right)^4 = 0.0625$

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- And, $P(HHH) = P(HHH \cap F) + P(HHH \cap \bar{F}) = \left(\frac{1}{2}\right)^4 + \left(\frac{3}{4}\right)^3 * \left(\frac{1}{2}\right) = 0.27344$

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- But, $P(HHH \cap F) = P(HHH|F) P(F) = \left(\frac{1}{2}\right)^4 = 0.0625$
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- Therefore, $P(F|HHH) = \frac{0.0625}{0.27344} = 0.22857$.

Question 10

Suppose that $P(A) = 0.25$ and $P(B) = 0.5$. If $P(A \cup B) = 0.70$ what is $P(A \cap B)$?

Question 10 Solution

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

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- So, $0.7 = 0.25 + 0.5 - P(A \cap B)$

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- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- So, $0.7 = 0.25 + 0.5 - P(A \cap B)$
- And, $P(A \cap B) = 0.05$