Fractional Factorials

One of the disadvantages of factorial experiments is that they can get large very quickly with several levels each of several factors. One technique for reducing the size of the factorial to more manageable levels is fractional replication.

Fractional replication is valuable in vary large experiments in which a single full replication would be too large for the available resources, or in which full replication gives more precision for estimating the main effects than is needed. For example, in a 2^6 factorial with 64 treatments, each main effect is averaged over 32 combinations containing each level of the factor. Often a fraction of the replication may be sufficient to obtain the desired precision in such experiments. A method for handling this proposed by Finney in 1945 allows investigators to handles 5 or more factors at a time in an experiment of practical size so that the investigator can discover quickly which factors are important in a particular study.

This technique involves using orthogonal contrasts to identify the treatment combinations used in estimating the factorial effects. The contrasts for the main effects and interactions for a 2x2x2 (2^3) arrangement of factors A (levels 0 and a), B (levels 0 and b) and C (levels 0 and c) are shown in the table below.

	Treatments							
Contrasts	0 (Control)	a	b	c	ab	ac	bc	abc
A	-1	+1	-1	-1	+1	+1	-1	+1
B	-1	-1	+1	-1	+1	-1	+1	+1
C	-1	-1	-1	+1	-1	+1	+1	+1
AB	+1	-1	-1	+1	+1	-1	-1	+1
AC	+1	-1	+1	-1	-1	+1	-1	+1
BC	+1	+1	-1	-1	-1	-1	+1	+1
ABC	-1	+1	+1	+1	-1	-1	-1	+1

The size of the experiment can be reduced by selecting a contrast for estimating the effect of a higher order interaction. The selected contrast is called the defining contrast. Only the treatments having the same sign in the defining contrast are selected for installation.

In our example, selecting the ABC contrast as the defining contrast and using only the + treatments would result in the 4 treatments a, b, c and abc being selected for the experiment. Thus only half of the 8 treatments would be included in the experiment.

When only a fraction of the treatments are selected for the experiment, some or all of the main effects and certain interactions can still be estimated, but not all possible effects.

Confounding:

The danger in using fractional replication is that it is easy to misinterpret the results. In our example, the selected treatment combinations are shown in the table below:

	Selected treatments			
Contrasts	a	b	c	abc
A	+1	-1	-1	+1
B	-1	+1	-1	+1
C	-1	-1	+1	+1
AB	-1	-1	+1	+1
AC	-1	+1	-1	+1
BC	+1	-1	-1	+1
ABC	+1	+1	+1	+1

With only 4 treatments installed, there will be 3 df for treatment that can be subdivided using the top 3 contrasts (main effect contrasts). Note that the main effect contrasts are orthogonal and independent, but that they are identical to the contrasts that would be used to estimate 2 factor interactions. The contrasts for the main effect of A and for the BC interaction are identical, meaning that the same contrast estimates both of these effects. The BC interaction and the A effects are called <u>aliases</u>, and are said to be <u>confounded</u>. If the A main effect was found to be significant, this may be due to one of 3 causes:

1. The A effect is significant.
2. The BC interaction is significant.
3. A combination of the above.

Note that the ABC contast is all +, this is because it is the defining contrast on which the factorial was split. Therefore, this effect cannot be estimated.

It is easy to misinterpret the results of a fractional replicate of this experiment. When fractional replication is used, it is important to be certain that the interactions with are aliases of the main effects or interactions of interest are not going to be significant, otherwise the results may be erroneously interpreted. The arrangement is all right for special situations, but it must be used with care.

In order to minimize the risks involved, it is important to know what the aliases are of the factorial effects of interest. A general rule is that in any 2^n system, the alias of any factorial effect is its generalized interaction with the defining contrast. A generalized interaction is interpreted by canceling any squared terms. With ABC as the defining contrast:

the alias of A is A x ABC = AABC = BC

the alias of B is B x ABC = ABBC = AC
the alias of C is C x ABC = ABCC = AB