

MATHEMATICAL MODEL AND NORMAL EQUATIONS

Example data set:

Completely Randomized Design

An experiment with 3 treatments and 2 replications could be installed as a Completely Randomized Design.

| | | |
|----------|----------|----------|
| 3 | 9 | 1 |
| t_1r_1 | t_3r_2 | t_1r_2 |
| 6 | 5 | 6 |
| t_2r_2 | t_2r_1 | t_3r_1 |

Mathematical model:

Each observation can be described by an equation that is made up of three components. This is the mathematical model that is expressed in terms of sample statistics:

$$Y_{ij} = \bar{Y}_{..} + T_i + e_{ij}$$

where Y_{ij} is an observation

$\bar{Y}_{..}$ is the general mean about which the observations are presumed to be varying

T_i is the applied treatment effect

e_{ij} is the residual component representing all other sources of variation that influence the observation – is generally referred to as the "experimental error" or "residual component".

The data from this experiment would be recorded in a table for statistical analysis.

| | T1 | T2 | T3 | Total |
|--------|----|----|----|-------|
| | 3 | 5 | 6 | |
| | 1 | 6 | 9 | |
| Totals | 4 | 11 | 15 | 30 |

Analysis of Variance

| Source of Variation | df |
|---------------------|----|
| Between treatment | 2 |
| Within treatment | 3 |
| Total | 5 |

Randomized Complete Block Design

The experiment could be installed as a Randomized Complete Block Design with 2 blocks.

| | | | |
|-------|---------------|---------------|---------------|
| b_1 | 3 t_1b_1 | 6 t_3b_1 | 9 t_2b_1 |
| b_2 | 1 t_2b_2 | 5 t_1b_2 | 6 t_3b_2 |

MATHEMATICAL MODEL:

Expressed in sample statistics the mathematical model is the following:

$$Y_{ij} = \bar{Y}_{..} + T_i + B_j + e_{ij}$$

Where all the statistics are the same as described above except for the new term:

B_j is the component for block or environmental effects

The data would be recorded in a table for statistical analysis.

| | T1 | T2 | T3 | B Totals |
|----------|----|----|----|----------|
| B1 | 3 | 9 | 6 | 18 |
| B2 | 5 | 1 | 6 | 12 |
| T Totals | 8 | 10 | 12 | 30 |
| T Means | 4 | 5 | 6 | 5 |

Analysis of Variance:

| Source of Var | df |
|---------------|----|
| Blocks | 1 |
| Trts | 2 |
| Error | 2 |
| Total | 5 |

The normal equation of a randomized complete block design:

$$\Sigma Y_{ij} = \Sigma (\bar{Y}_{..} + T_i + B_j)$$

Where the symbols are defined as stated above.

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