SPLIT PLOT DESIGN

2 Main Plot Treatments (1, 2)
2 Sub Plot Treatments (A, B)
4 Blocks

Block 1	2 A	2 B	1 B	1 A
Block 2	1 B	1 A	2 B	2 A
Block 3	1 B	1 A	2 A	2 B
Block 4	2 A	2 B	1 A	1 B

Mathematical Model - Split Plot

$$X_{ijk} = \bar{X}_{...} + M_i + B_j + d_{ij} + S_k + (MS)_{ik} + e_{ijk}$$

Where X_{iik} = an observation
$\bar{X}_{...}$ = the experiment mean
M_i = the main plot treatment effect
B_j = the block effect
d_{ij} = the main plot error (error a)
S_k = the subplot treatment effect
$(MS)_{ik}$ = the main plot and subplot treatment interaction effect
e_{ijk} = the subplot error (error b)
i = a particular main plot treatment
j = a particular block
k = a particular subplot treatment

Analysis of Variance

Source of Variation	df	
Total	mrs - 1	15
Main Treatment	m - 1	1
Block	r - 1	3
Error a (Block x Main Trt)	(m - 1)(r - 1)	3
Main Plot Subtotal	mr - 1	(7)

Subplot Treatment	s - 1	1
Main x Subplot Trt	(m - 1)(s - 1)	1
Error b		6
(Block x Subplot Trt +	(s - 1)(r - 1) +	
Block x Main Trt x Subplot Trt)	(m - 1)(s - 1)(r - 1)	

Standard Errors for a Split Plot Design

Means Compared	Standard Error of a Mean
Main plot treatments M1 - M2	$\sqrt{\frac{Ea}{rs}}$
Subplot treatments S1 - S2	$\sqrt{\frac{Eb}{rm}}$
Subplot treatments for the same main plot treatment: S1M1 - S2M1	$\sqrt{\frac{Eb}{r}}$
Subplot treatments for different main plot treatments: S1M1 - S1M2 or S1M1 - S2M2	$\sqrt{\frac{(s-1)Eb + Ea}{rs}}$

M = main plot treatment
S = subplot treatment
m = number of main plot treatments
s = number of subplot treatments
r = number of replicates
Ea = MS Main Plot Error
Eb = MS Subplot Error

$$LSD = t_{.05,df}\sqrt{2}s_{\bar{x}}$$

F. E. Satterthwaite's weighted t value for split plots

$$t_{a,b} = \frac{(s-1)E_b t_b + E_a t_a}{(s-1)E_b + E_a}$$

Advantages:

1. Experimental units which are large by necessity or design may be utilized to compare subsidiary treatments.

2. Increased precision over a randomized complete block design is attained on the subplot treatments and the interaction between subplot and main plot treatments.

3. The overall precision of the split plot design relative to the randomized complete block design may be increased by designing the main plot treatments in a latin square design or in an incomplete latin square design.

Disadvantages:

1. The main plot treatments are measured with less precision than they are in a randomized complete block design.

2. When missing data occur, the analysis is more complex than for a randomized complete block design with missing data.

3. Different treatment comparisons have different basic error variances which make the analysis more complex than with the randomized complete block design, especially if some unusual type of comparison is being made.

Appropriate use of split-plot designs:

1. When the practical limit for plot size is much larger for one factor compared with the other, e.g., in an experiment to compare irrigation treatments and population densities; irrigation treatments require large plots and should, therefore, be assigned to the main plots while population density should be assigned to the subplots.

2. When greater precision is desired in one factor relative to the other e.g., if several varieties are being compared at different fertilizer levels and the factor of primary interest is the varieties, then it should be assigned to the subplots and fertilizer levels assigned to the main plots.

SPLIT BLOCK OR STRIP PLOT DESIGN

2 Column Treatments (1, 2)
2 Row Treatments (A, B)
4 Blocks

Block 1	2 A	1 A
	2 B	1 B
Block 2	1 B	2 B
	1 A	2 A
Block 3	1 A	2 A
	1 B	2 B
Block 4	2 B	1 B
	2 A	1 A

Mathematical Model - Split Block

$$X_{ijk} = \bar{X}_{...} + R_i + Bj + (RB)_{ij} + C_k + (CB)_{kj} + (RC)_{ik} + e_{ijk}$$

Where
X_{iik} = an observation
$\bar{X}_{...}$ = the experiment mean
R_i = the row treatment effect
B_j = the block effect
$(RB)_{ij}$ = the row plot error (error a)
C_k = the column treatment effect
$(CB)_{kj}$ = the column plot error (error b)
$(RC)_{ik}$ = the treatment interaction effect
e_{ijk} = the subplot error (error c)
i = a particular row treatment
j = a particular block
k = a particular column treatment

Analysis of Variance

Source of Variation	df	
Total	brc - 1	15
Block	b - 1	3
Row Trt	r - 1	1
Error a (Block x Row Trt)	(b - 1)(r - 1)	3
Column Treatment	c - 1	1
Error b (Block x Column Trt)	(b - 1)(c - 1)	3
Row Trt x Column Trt	(r - 1)(c - 1)	1
Error c (Block x Row Trt x Column Trt)	(b - 1)(r - 1)(c - 1)	3

SPLIT SPLIT PLOT DESIGN

2 Main Plot Treatments (M1, M2)
2 Sub Plot Treatments (S1, S2)
2 Sub Sub Plot Treatments (T1, T2)
4 Blocks

<table>
<tr><td>Block 1</td><td colspan="4">M2</td><td colspan="4">M1</td></tr>
<tr><td>Block 2</td><td colspan="2">M1
S2</td><td colspan="2">M1
S1</td><td colspan="2">M2
S2</td><td colspan="2">M2
S1</td></tr>
<tr><td>Block 3</td><td>M1
S1
T1</td><td>M1
S1
T2</td><td>M1
S2
T2</td><td>M1
S2
T1</td><td>M2
S2
T2</td><td>M2
S2
T1</td><td>M2
S1
T1</td><td>M2
S1
T2</td></tr>
<tr><td>Block 4</td><td>M2
S1
T2</td><td>M2
S1
T1</td><td>M2
S2
T2</td><td>M2
S2
T1</td><td>M1
S2
T1</td><td>M1
S2
T2</td><td>M1
S1
T1</td><td>M1
S1
T2</td></tr>
</table>

Mathematical Model - Split Split Plot

$$X_{ijk} = \bar{X}....+M_i + B_j + d_{ij} + S_k + (MS)_{ik} + f_{ikj} + T_l + (MT)_{il} + (ST)_{kl} + (MST)_{ikl} + e_{ijkl}$$

Where

Where	X_{iik}	= an observation
	$\bar{X}$	= the experiment mean
	M_i	= the main plot treatment effect
	B_j	= the block effect
	d_{ij}	= the main plot error (error a)
	S_k	= the subplot treatment effect
	$(MS)_{ik}$	= the treatment interaction effect
	f_{ikj}	= the subplot error (error b)
	T_l	= the sub subplot treatment effect
	$(MT)_{il}$	= the treatment interaction effect
	$(ST)_{kl}$	= the treatment interaction effect
	$(MST)_{ikl}$	= the treatment interaction effect
	e_{ijk}	= the sub subplot error (error c)
	i, k, l	= a particular treatment
	j	= a particular block

Analysis of Variance

Source of Variation	df	
Total	mrst - 1	31
Main Treatment (M)	m - 1	1
Block (B)	r - 1	3
Error a (Block x Main Trt)	(m - 1)(r - 1)	3
Subplot Treatment (S)	s - 1	1
Main x Sub Trt	(m - 1)(s - 1)	1
Error b	(s - 1)(r - 1) + (m - 1)(s - 1)(r - 1)	6
(Block x Sub Trt + Block x Main Trt x Sub Trt)		
Sub Subplot Trt (T)	t -1	1
Main x T	(m -1)(t -1)	1
S x T	(s -1)(t -1)	1
M x S x T	(m -1)(s -1)(t -1)	1
Error c (T x B + M x T x B + S x T x B + M x S x T x B)	(r -1) [(t-1)+(m-1)(t-1) +(s-1)(t-1)+(m-1)(s-1)(t-1)]	12

Standard Errors for Split Split Plot

Means Compared	Standard Error	t Values
M means	$\sqrt{\frac{Ea}{rst}}$	t_a
S means	$\sqrt{\frac{Eb}{rmt}}$	t_b
S means for same M	$\sqrt{\frac{Eb}{rt}}$	t_b
S means for different Ms	$\sqrt{\frac{(s-1)Eb + Ea}{rst}}$	t_b
T means	$\sqrt{\frac{Ec}{rms}}$	t_c
T means for same M	$\sqrt{\frac{Ec}{rs}}$	t_c
T means for same S	$\sqrt{\frac{Ec}{rm}}$	t_c
S means for same or different T	$\sqrt{\frac{(t-1)Ec + Eb}{rmt}}$	$t_{bc} = \frac{(t-1)Ect_c + Ebt_b}{(t-1)Ec + Eb}$
M means for same or different T	$\sqrt{\frac{(t-1)Ec + Ea}{rst}}$	$t_{ac} = \frac{(t-1)Ect_c + Eat_a}{(t-1)Ec + Ea}$
T means for same M and S	$\sqrt{\frac{Ec}{r}}$	t_c
S means for same M and same or different T	$\sqrt{\frac{(t-1)Ec + Eb}{rt}}$	$t_{bc} = \frac{(t-1)Ect_c + Ebt_b}{(t-1)Ec + Eb}$
M means for same or different S and T	$\sqrt{\frac{s(t-1)Ec + (s-1)Eb + Ea}{rst}}$	$t_{abc} = \frac{s(t-1)Ect_c + (s-1)Ebt_b + Eat_a}{s(t-1)Ec + (s-1)Eb + Ea}$

M = main plot treatment
S = subplot treatment
T = sub subplot treatment
m = number of main plot treatments
s = number of subplot treatments
t = number of sub subplot treatments

r = number of replicates
Ea = MS Main plot error
Eb = MS Subplot error
Ec = MS Sub sub plot error
t_a = tabular t with degrees of freedom for Ea
t_b = tabular t with degrees of freedom for Eb
t_c = tabular t with degrees of freedom for Ec