Latin Square Designs

Latin square designs differ from randomized complete block designs in that the experimental units are grouped in blocks in two different ways, that is, by rows and columns. Therefore, two different sources of variation can be isolated. For example, an experiment installed on a slope that also has a gradient of soil texture running across the slope can be installed as a latin square with the rows isolating the slope and the columns isolating the soil texture gradient. A restriction in the assignment of treatments in a latin square is that each treatment can occur once and only once in each row and column so that each row and each column is like a complete block. A requirement of the latin square is that the number of treatments, rows, and number of replications, columns, must be equal; therefore, the total number of experimental units must be a perfect square. For example, if there are 4 treatments, there must be 4 replicates, or 4 rows and 4 columns. This is a 4x4 latin square which gives a total of 16 experimental units. Because of this restriction, latin square experiments can become large and unmanageable very readily. Also, if the number of treatments is too small, there are too few df for error so that the most common squares are in the range of 5x5 to 8x8. Since two sources of variation are isolated in rows and columns, the mean square for error will be smaller than for the same data analyzed as either a completely randomized or randomized complete block design. However, the df for error will also be smaller as shown in the analysis of variance table.

Advantages of latin square designs

1. Controls more variation than CR or RCB designs because of 2-way stratification. Results in a smaller mean square for error.
2. Simple analysis of data
3. Analysis is simple even with missing plots.

Disadvantages of latin square designs

1. Number of treatments is limited to the number of replicates which seldom exceeds 10.
2. If have less than 5 treatments, the df for controlling random variation is relatively large and the df for error is small.

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LATIN SQUARE DESIGN

An experiment with 4 treatments (A, B, C, D).

Columns

Rows	1	2	3	4
1	1 A	2 B	3 C	4 D
2	1 B	2 C	3 D	4 A
3	1 C	2 D	3 A	4 B
4	1 D	2 A	3 B	4 C

Number in upper left-hand corner are plot numbers for each row.
Letters are treatments

Analysis of Variance

Source of Variation		df
Rows	r-1	3
Columns	c-1	3
Trts	t-1	3
Error	(r-1) (c-2)	6
Total	(rt)-1	15

Mathematical Model

$$Y_{ijk} = \overline{Y}_{...} + T_i + R_j + C_k + e_{ijk}$$

Where: Y_{ijk} = an observation

$\overline{Y}_{...}$ = the experiment mean

T_i = the treatment effect

R_j = the row effect

C_k = the column effect

e_{ijk} = the residual or expt'l error

i = a particular treatment

j = a particular row

k = a particular column