

Steps in Covariance Analysis
(Randomized Complete Block Design)

1. Construct ANOVA tables as RCBD for X, independent variable or covariate, and for Y, dependent variable.
2. Check for treatment effect on X and on Y using F-test.
3. Calculate sums of cross-products
4. Construct Analysis of Covariance table including sums of squares for X and Y, and sums of cross-products. Include Trt+Err df, SSX, SP and SSY.
5. Calculate SSRegr (adj for trt) and SSDevRegr.
6. Calculate SSRegr (trt + err) and SSTrt (adj for regr).
7. Complete the Analysis of Covariance table and test MSRegr (adj for trt) and MSTrt (adj for regr) against MSDevRegr (the remaining error).
8. Adjust treatment means.

Example Problem-Covariance

Initial Weights (X) and Gains in Weight (Y) in Pounds For Bacon Pigs in a Feeding Trial
(Adapted from Steel and Torrie)

Pens (Blocks)	Rations							
	a1		a2		a3		Totals	
	X	Y	X	Y	X	Y	X	Y
1	38	9.52	39	8.51	48	9.11	125	27.14
2	35	12.34	38	9.95	37	8.5	110	30.79
3	41	11.27	46	8.43	42	8.9	129	28.60
4	48	10.56	40	8.86	42	9.51	130	28.93
5	43	10.42	40	9.20	40	8.76	123	28.38
Totals	205	54.11	203	44.95	209	44.78	617	143.84
Means	41.0	10.82	40.6	8.99	41.8	8.96	41.1	9.58

ANOVA FOR X

Pens	a1	a2	a3	Totals
(Blocks)	X	X	X	X
1	38	39	48	125
2	35	38	37	110
3	41	46	42	129
4	48	40	42	130
5	43	40	40	123
Totals	205	203	209	617
Means	41.0	40.6	41.8	41.1

SSTot = 205.733
 SSBlks = 85.733
 SSRation = 3.733

ANOVA TABLE

Source	df	SS	MS	F
Total	14	205.733		
Blocks	4	85.733	21.433	1.47
Rations	2	3.733	1.867	0.13
Error	8	116.267	14.533	

ANOVA FOR Y

Pens (Blocks)	a1 Y	a2 Y	a3 Y	Totals Y
1	9.52	8.51	9.11	27.14
2	12.34	9.95	8.50	30.79
3	11.27	8.43	8.90	28.60
4	10.56	8.86	9.51	28.93
5	10.42	9.20	8.76	28.38
Totals	54.11	44.95	44.78	143.84
Means	10.82	8.99	8.96	9.59

SSTot = 17.936

SSBlks = 2.315

SSRation = 11.399

ANOVA TABLE

Source	df	SS	MS	F
Total	14	17.936		
Blocks	4	2.315	0.579	1.096
Rations	2	11.399	5.699	10.798**
Error	8	4.223	0.528	

Calculation of Sums of Cross-Products

Correction Factor

$$C = \frac{\Sigma X \Sigma Y}{rb} = \frac{(617)(143.84)}{15} = \underline{5916.619}$$

Total Sum of Cross Products:

$$\begin{aligned} \text{SPTot} &= \Sigma xy - C = (38)(9.52) + (35)(12.34) + \dots + (40)(8.76) - C \\ &= 5906.24 - C = \underline{-10.3787} \end{aligned}$$

Sum of cross – products for Blocks:

$$\begin{aligned} \text{SPBlk} &= \frac{(\Sigma Bx) * (\Sigma By)}{r} - C \\ &= \frac{((125) * (27.14) + \dots + (123) * (28.38))}{3} - C = \underline{-9.806} \end{aligned}$$

Sum of cross – products for rations:

$$\begin{aligned} \text{SPRation} &= \frac{(\Sigma Rx) * (\Sigma Ry)}{b} - C \\ &= \frac{((205) * (54.11) + \dots + (209) * (44.78))}{5} - C = \underline{-1.335} \end{aligned}$$

Sum of cross – products for error:

$$\text{SPErr} = \text{SPTot} - \text{SPBlk} - \text{SPRation} = \underline{0.762}$$

Where R = ration total

B = block total

r = # rations

b = # blocks

ANALYSIS OF COVARIANCE

Sum of Squares and Products				
Source	df	SSX	SP	SSY
Total	14	205.733	-10.379	17.936
Blocks	4	85.733	-9.806	2.315
Rations	2	3.733	-1.335	11.399
Error	8	116.267	0.762	4.223
Ration+Err	10	120.00	-0.573	15.622

$$SS_{\text{Regr}} = r^2 \cdot SSY = \frac{SP^2}{SSX}$$

For Y, SS after removing Blocks is $SS_{\text{Trt+Err}} = 15.622$, with 10 df. This can be divided in 2 ways:

- 1) $SS_{\text{Trt}} + SS_{\text{Regr}} (\text{adj for trt}) + SS_{\text{DevRegr}} = 15.622$
- 2) $SS_{\text{Regr}} + SS_{\text{Trt}} (\text{adj for regr}) + SS_{\text{DevRegr}} = 15.622$

1. First adjust for Trt, then subdivide Error into Regr (adj for trt) and Dev from Regr.

$$SS_{\text{Trt}} = \underline{11.399}$$

$$SS_{\text{Err}} (\text{adj for trt}) = 4.223$$

$$SS_{\text{Regr}} (\text{adj for trt}) = \frac{SP_{\text{Err}}^2}{SSX_{\text{Err}}} = \frac{0.762^2}{116.267} = \underline{0.004994}$$

$$SS_{\text{DevRegr}} = SS_{\text{Err}} - SS_{\text{Regr}} = 4.223 - 0.004994 = \underline{4.218}$$

2. First adjust for Regr, then subdivide into Trt (adj for regr) and Dev from Regr

$$SS_{\text{Regr}} = \frac{(SP_{\text{Trt+Err}})^2}{SSX_{\text{Trt+Err}}} = \frac{-0.573^2}{120} = \underline{0.002736}$$

$$\begin{aligned} SS_{\text{Trt}} (\text{adj for regr}) &= SS_{\text{Trt+Err}} - SS_{\text{Regr}} - SS_{\text{DevRegr}} \\ &= 15.622 - 0.002736 - 4.218 = \underline{11.401} \end{aligned}$$

Note that Trt and Regr are not orthogonal, so the SS depends on which is calculated first. F-tests will be conducted on the adjusted values.

<u>Completion of ANOVA</u>				
Source	df	SS	MS	F
(SSTrt+Err)	(10)	(15.622)		
From 1) above				
Trt	2	11.399		
Regr (adj for trt)	1	0.004994	0.004994	0.0083
DevRegr	7	4.218		
From 2) above				
Regr	1	0.002736		
Trt (adj for regr)	2	11.401	5.7005	9.46*
DevRegr	7	4.218		

Note that df for SSDevRegr = df for Error - df for Regr

ADJUSTMENT OF TREATMENT MEANS

Adjusted Mean (Y_i) = $Y_i - b(X_i - \bar{X})$

$$b = \frac{SPErr}{SSXErr} = \frac{0.762}{116.267} = 0.006554 \text{ lb gain/lb initial wt}$$

Rations	Y_i	X_i	$X_i - \bar{X}$	$b(X_i - \bar{X})$	Y_i	
a1	10.82	41.0	-0.1	-0.00065	10.82	(10.819)
a2	8.99	40.6	-0.5	-0.00327	8.99	(8.987)
a3	8.96	41.8	0.7	0.00458	8.96	(8.955)

Summary of Interpretation of Covariance Analysis

Regression:

$P > 0.15$ Remove covariate from analysis

$P < 0.15$ Use covariance analysis

Treatment effects on:

X	Y		Conclusions
	Before Covariance	After Covariance	
ns	sig	ns	Treatment effects on Y exaggerated by X; interpret with caution.
ns	ns	sig	True treatment effects obscured by variation in X.
sig	sig	ns	Apparent treatment effect due to variation in X; interpret with caution.
sig	sig	sig	Treatment had significant effect on Y beyond that due to variation in X.

Using Covariance for Calculating Missing Data (Adapted from Steel and Torrie)

Covariance can be used to calculate missing data as shown below. This procedure gives an unbiased estimate of treatment and error sum of squares. It also leads to an unbiased test of treatment means. The analysis is convenient and simple.

Data are the mean ascorbic acid content of three 2 g samples of turnip greens in mg/100 g dry weight.

Treatment	Block (Day)										Total
	1		2		3		4		5		
	Y	X	Y	X	Y	X	Y	X	Y	X	
A	0	1	887	0	897	0	850	0	975	0	3609
B	857	0	1189	0	918	0	968	0	909	0	4841
C	917	0	1072	0	975	0	930	0	954	0	4848
Totals	1774	1	3148	0	2790	0	2748	0	2838	0	13298

Procedure:

1. Set $Y = 0$ for the missing plot.
2. Define a covariate as $X = 0$ for an observed Y , and $X = +1$ (or -1) for $Y = 0$.
3. Carry out the analysis of covariance to obtain the error sums of squares and products.
4. Compute $B = SP/SSX$ and change sign to estimate the missing value.

ANCOVA

Source	df	SSX	SP	SSY
Total	14	0.9333	-886.53	945.296
Block	4	0.2667	-295.20	359.823
Treatment	2	0.1333	-174.73	203.533
Error	8	0.5333	-426.60	381.940

Can now calculate b:

$$b = SP/SSX = (-426.60/0.5333) = -799.92 = -800$$

To change the sign: $-800 (-1) = 800$

This is the missing value. Redo the ANOVA with this value. Subtract 1 df from error value and total df.

Can calculate several missing data by introducing a new independent variable for each missing datum and using multiple covariance.