A Power Allocation Game in a Four Node Relay Network: An Upper Bound on the Worst-Case Equilibrium Efficiency

Ninoslav Marina*, Gürdal Arslan† and Aleksandar Kavčić
Department of Electrical Engineering
University of Hawaiʻi at Mānoa
2540 Dole Street, Honolulu, HI 96822

Abstract

We introduce a power allocation game in a four node relay network which consists of two source and two destination nodes. The sources employ a time sharing protocol such that in each discrete time instance one of the sources communicates with its destination while the other source aids this communication by acting as a relay. Each source uses some portion of its limited power for its own transmission and uses the remaining portion to aid the other source. The noncooperative solution, which is the Nash equilibrium of the game where each source tries to maximize its own rate, dictates each source to use all of its power for its own use, i.e., no relaying. This results in an inferior sum rate with respect to the optimum sum rate jointly maximized over all possible power allocations. The main contribution of this paper is to establish an upper bound on the worst-case equilibrium efficiency (a.k.a. the price of anarchy), defined as the ratio of the equilibrium sum rate to the optimal sum rate for the worst channel conditions. More specifically, we show that if the path loss coefficient is $\beta > 0$ and the received signals are corrupted by additive white Gaussian noise, then the worst case equilibrium efficiency is upper bounded by $(1/2)^{\beta}$. We also note that this upper bound can be extended to relay networks with more than two sources.

1. Introduction

In the past years there was an increased need for higher communication rates in all types of wireless networks. Wireless ad-hoc networks consist of a number of terminals or nodes, communicating with each other on a peer-to-peer basis, without the assistance of wired networks or planned infrastructure. In such systems, the communication between nodes might take place through several intermediate nodes. A limitation to wireless ad-hoc networks is that for large networks the capacity per node is inversely proportional to the square root of the total number of nodes and hence goes to zero as the total number of nodes increases [5]. A promising method to increase the network spectral efficiency is the node cooperation [16, 17, 10, 11, 12, 9, 7] by relaying [14, 15, 2, 6, 8]. The idea is that some nodes, while not active can offer their resources to other nodes, most often in their vicinity, and therefore helping them to achieve better performance, i.e., better transmission rate. In many situations it is not straightforward to "motivate" the nodes to cooperate. In this paper we compare the overall system performance when the nodes cooperate versus when they do not cooperate. The goal of the system designer might be to "encourage" the nodes to cooperate in order to increase the total system efficiency. In this paper, we show that there is significant room for performance improvement by creating a cooperative network.

The remainder of this paper is organized as follows. Section 2 describes the system model. Section 3 introduces a power allocation game. Section 4 presents the problem statement, whereas Section 5 presents the main result and its proof. Finally, Section 6 contains some concluding remarks.

2. System Model

We begin by considering a four-node network shown in Figure 1. The nodes 1 and 2, called the sources, are trying to communicate with the nodes 3 and 4, called the destinations, respectively; see [3]. We assume that the sources use a time-sharing protocol to communicate with their destinations. Certain windows of discrete time periods are devoted to one of the sources, say source $i \in \{1, 2\}$, in which source i tries to communicate with its destination while the other source acts as a relay. We will denote the portion of the time devoted to source 1 by $\tau_1 \in (0, 1)$, which implies, the

*N. Marina has been supported by the Swiss National Science Foundation Grant #PA002–117385.
†Research supported by NSF Grant #ECCS-0547692.

978-1-4244-2036-0/08/$25.00 (c)2008 IEEE.

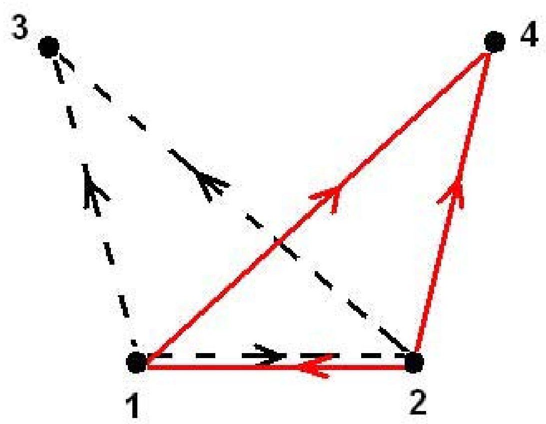

Figure 1. A four node network with two source nodes and two destination nodes.

portion of the time devoted to source 2 is $\tau_2 \triangleq 1 - \tau_1$. The average power per channel use is limited to $P_i > 0$ for the source i. Each source i uses some portion $k_i \in [0, 1]$ of its limited power to communicate with its own destination and uses the remaining portion $\bar{k}_i \triangleq 1 - k_i$ to act as a relay to help the other source communicate with its destination. Therefore, the average power used by the source i for its own transmission equals $k_i P_i/\tau_i$, whereas the average power used by the source i to help the other source equals $\bar{k}_i P_i/\bar{\tau}_i$ where $\bar{\tau}_i = 1 - \tau_i$.

In each period, we have a single relay channel whose various properties are discussed in [3, 2, 8]. The relay channel that arises when source 2 helps source 1, called the first relay channel, is described by

$$\begin{aligned} Y_3[n] &= h_{31}X_1[n] + h_{32}X_2[n] + Z_3[n] \\ Y_2[n] &= h_{21}X_1[n] + Z_2[n], \end{aligned} \tag{1}$$

whereas the relay channel that arises when source 1 helps source 2, called the second relay channel, is described by

$$\begin{aligned} Y_4[n] &= h_{42}X_2[n] + h_{41}X_1[n] + Z_4[n] \\ Y_1[n] &= h_{21}X_2[n] + Z_1[n], \end{aligned} \tag{2}$$

where $X_i[n]$ is the symbol transmitted by the source i at time n, $Y_i[n]$ is the symbol received by node i at time n, $Z_i[n]$ are additive white Gaussian noise with unit variance, and h_{ij} are the channel coefficients. We will assume that the magnitude of the channel coefficient satisfies $|h_{ij}|^2 = d_{ij}^{-\beta}$, where d_{ij} is the distance between the nodes i and j, and β is the *path loss coefficient* [13]. The path loss coefficient β takes values from 1.6 for confined line-of-site environments (such as tunnels and corridors) up to 6 in obstructed indoor environments. We will denote the capacity of the $i-$th relay channel per channel use by

$$C_i\big(k_i P_i/\tau_i, \bar{k}_{-i} P_{-i}/\bar{\tau}_{-i}\big),$$

where we use the notation $-i$ to refer to the source helping the source i, e.g., $k_{-1} = k_2$, $k_{-2} = k_1$, and the dependence of C_i on channel coefficients is suppressed.

3. A Power Allocation Game

We adopt a noncooperative game theoretic framework [1, 4] in which each source is viewed as an autonomous decision maker. Each source i decides the allocation k_i of its limited power between for its own use and for acting as a relay for the other source $-i$. Each source i makes its decision k_i in order to optimize its own utility function $U_i(k_i, k_{-i})$. We should emphasize that each source i in our setting is noncooperative in the sense that it is interested in selfishly optimizing its own utility function U_i. However, a network designer would typically be interested in the optimization of a network-level performance metric $U(k_1, k_2)$, for instance, the average network rate per channel use

$$U(k_1, k_2) = \sum_{i \in \{1,2\}} \tau_i C_i\big(k_i P_i/\tau_i, \bar{k}_{-i} P_{-i}/\bar{\tau}_{-i}\big). \tag{3}$$

Although chaotic behavior can emerge out of noncooperative interactions, it is also possible that a network-level performance metric $U(k_1, k_2)$ can be optimized through noncooperative players provided the individual utility functions $\{U_i\}_{i \in \{1,2\}}$ are properly aligned with $U(k_1, k_2)$. In fact, our ultimate goal (which is beyond the scope of this paper) is to carefully craft the individual utility functions in order to optimize a network-level performance measure such as the one given by (3)

Clearly, our viewpoint here deviates from that of direct optimization. Rather, we seek the optimization of a network-level objective through a set of autonomous sources each of which pursues the optimization of its own utility function [18]. The potential benefits of our autonomous operation approach in a general relay network include: 1) sources do not always need to depend on a centralized coordinator which may not exist in certain applications such as in ad-hoc networks, 2) the network (the number of nodes, the location of nodes, etc.,) may change over time, and our approach would provide a certain degree of adaptability to such variations since each source is only interested in optimizing its own utility function, 3) it would not require constant dissemination of information throughout the network, 4) if it is used as a centralized optimization tool, its computational burden would be quite light.

4. Problem Statement

The main question that will be addressed in this paper is whether some seemingly reasonable utilities can lead to efficient solutions. For this, we will first need to introduce the concept of Nash equilibrium from the theory of non-cooperative games. A profile of solutions $k^* = (k_1^*, k_2^*)$ constitutes a Nash equilibrium[1] if

$$U_i(k_i^*, k_{-i}^*) = \max_{k_i \in [0,1]} U_i(k_i, k_{-i}^*),\ \forall i \in \{1,2\}.$$

In other words, no source has an incentive to unilaterally deviate from k^*, i.e., it is person-by-person optimal. Given the individual utilities $\{U_i\}_{i\in\{1,2\}}$, an equilibrium may or may not exist, and when it exists it may or may not be unique.

In our context, we will expect that the sources will find their way towards an equilibrium using a learning algorithm [19]. An equilibrium will typically be inefficient with respect to the network designers objective U, that is, an equilibrium will typically not optimize U. A useful measure of efficiency of an equilibrium k^* with respect to U can be given as

$$\eta(k^*) \triangleq \frac{U(k^*)}{\max_{(k_1,k_2)\in[0,1]^2} U(k_1,k_2)}.$$

Now, an important consideration in designing the individual utilities $\{U_i\}_{i\in\{1,2\}}$ is to make a resulting equilibrium k^* efficient, i.e., $\eta(k^*) = 1$. Ideally, we would like all the equilibria to be fully efficient; however, this may not always be an easy task unless one obtains an optimal solution and retrofits the individual utilities into the optimal solution.

In the next section, we will investigate the efficiency of the unique equilibrium $\tilde{k} = (\tilde{k}_1, \tilde{k}_2) = (1,1)$ of the game characterized by the utilities

$$U_i(k_i, k_{-i}) = C_i\big(k_i P_i/\tau_i, \bar{k}_{-i} P_{-i}/\bar{\tau}_{-i}\big)$$

with respect to U given by (3). Note that, in this case, we have $U = \tau_1 U_1 + \tau_2 U_2$, and therefore one may expect that the average network rate would be optimized when each source optimizes its own average rate. However, we will show that the equilibria $\tilde{k} = (1,1)$ could be quite inefficient. More precisely, we will show that the worst-case efficiency of the equilibrium $\tilde{k} = (1,1)$ defined as

$$\eta_{\min} \triangleq \inf_G \frac{U(\tilde{k})}{\max_{(k_1,k_2)\in[0,1]^2} U(k_1,k_2)}, \qquad (4)$$

where G is the network geometry defined by

$$G = \big\{(d_{ij}) \ :\ d_{21} > 0, d_{31} > 0, d_{42} > 0,\ |d_{21} - d_{31}| \le d_{32} \le d_{21} + d_{31},\ |d_{21} - d_{42}| \le d_{41} \le d_{21} + d_{42}\big\},$$

and U is given by (3), can be quite small.

[1] We will henceforth refer to a Nash equilibrium simply as an equilibrium.

5. Main Result

Without loss of generality, we assume $P_2 \ge P_1$.

Proposition 1 For any fixed $P_2 \ge P_1 > 0$, $\tau_1 \in (0,1)$, and $\beta > 0$, the worst-case equilibrium efficiency $\eta_{\min}$ defined in (4) is upper bounded as

$$\eta_{\min} \le (\rho^*)^\beta < (1/2)^\beta,$$

where ρ^* is the unique solution to

$$P_1 \rho^{-\beta} = P_1 + P_2(1-\rho)^{-\beta}$$

on $(0,1)$.

Proof: At the unique equilibrium $\tilde{k} = (1,1)$, both relay channels reduce to regular point-to-point Gaussian channels. Therefore we have the average network rate at the equilibrium $\tilde{k}$ as

$$\begin{aligned} U(1,1) &= \sum_{i\in\{1,2\}} \tau_i C_i(P_i/\tau_i, 0) \\ &= \frac{\tau_1}{2}\log\left(1 + \frac{P_1}{\tau_1 d_{31}^\beta}\right) \\ &\quad + \frac{\tau_2}{2}\log\left(1 + \frac{P_2}{\tau_2 d_{42}^\beta}\right). \end{aligned}$$

The capacity of a single relay channel is not readily available, thus we will use the following lower bounds on C_i from [2, 8]:

$$C_i(k_i P_i/\tau_i, \bar{k}_{-i} P_{-i}/\tau_i) \ge C_i^{LB}(k_i P_i/\tau_i, \bar{k}_{-i} P_{-i}/\tau_i),$$

where

$$\begin{aligned} &C_1^{LB}(k_1 P_1/\tau_1, \bar{k}_2 P_2/\tau_1) = \\ &\frac{1}{2}\max\left\{\min\left\{\log\left(1 + \frac{k_1 P_1}{\tau_1 d_{21}^\beta}\right),\right.\right. \\ &\left.\log\left(1 + \frac{k_1 P_1}{\tau_1 d_{31}^\beta} + \frac{\bar{k}_2 P_2}{\tau_1 d_{32}^\beta}\right)\right\}, \\ &\left.\log\left(1 + \frac{k_1 P_1}{\tau_1 d_{31}^\beta} + \frac{\frac{k_1 P_1}{\tau_1 d_{21}^\beta}\frac{\bar{k}_2 P_2}{\tau_1 d_{32}^\beta}}{1 + \frac{k_1 P_1}{\tau_1 d_{21}^\beta} + \frac{k_1 P_1}{\tau_1 d_{31}^\beta} + \frac{\bar{k}_2 P_2}{\tau_1 d_{32}^\beta}}\right)\right\}, \end{aligned}$$

and $C_2^{LB}(k_2 P_2/\tau_2, \bar{k}_1 P_1/\tau_2)$ is obtained by replacing the indices $1, 2, 3$ with $2, 1, 4$, respectively, in the above expression (with $d_{12} = d_{21}$). This leads to the following lower bound U^{LB} on U:

$$U^{LB}(k_1,k_2) = \sum_{i\in\{1,2\}} \tau_i C_i^{LB}\big(k_i P_i/\tau_i, \bar{k}_{-i} P_{-i}/\tau_i\big),$$

which, in turn, leads to the following upper bound on $\eta_{\min}$:

$$\eta_{\min} \leq \inf_{G} \frac{U(1,1)}{\max_{(k_1,k_2)\in[0,1]^2} U^{LB}(k_1,k_2)}. \quad (5)$$

Now, for fixed (d_{21}, d_{31}, d_{42}), we observe that 1) $U(1,1)$ is independent of (d_{32}, d_{41}), and 2) $U^{LB}(k_1,k_2)$ is nonincreasing in d_{32} and d_{41}. This implies that, without loss of generality, the inf in (5) can be taken over

$$\{(d_{ij}) \;:\; d_{21} > 0, d_{31} > 0, d_{42} > 0, |d_{21} - d_{31}| = d_{32}, |d_{21} - d_{42}| = d_{41}\},$$

i.e., all four nodes are on a line.

We will further upper bound $\eta_{\min}$ by fixing (d_{21}, d_{31}) and letting $d_{42} \to \infty$ (and hence $d_{41} \to \infty$). This results in $C_2, C_2^{LB} \to 0$, and hence

$$\eta_{\min} \leq \inf_{G_1} \frac{C_1(P_1/\tau_1, 0)}{\max_{(k_1,k_2)\in[0,1]^2} C_1^{LB}(k_1 P_1/\tau_1, \bar{k}_2 P_2/\tau_1)}, \quad (6)$$

where G_1 is the new geometry defined by

$$G_1 = \{(d_{21}, d_{31}, d_{32}) : d_{21}, d_{31} > 0, d_{32} = |d_{21} - d_{31}|\}.$$

The max in the denominator in (6) is clearly achieved by $(1,0)$, therefore, we have

$$\eta_{\min} \leq \inf_{G_1} \frac{C_1(P_1/\tau_1, 0)}{C_1^{LB}(P_1/\tau_1, P_2/\tau_1)}.$$

Since $C_1(P_1/\tau_1, 0)$ does not depend on d_{21} or d_{32}, we can write

$$\eta_{\min} \leq \inf_{d_{31}>0} \frac{C_1(P_1/\tau_1, 0)}{\sup_{G_2(d_{31})} C_1^{LB}(P_1/\tau_1, P_2/\tau_1)}, \quad (7)$$

where

$$G_2(d_{31}) = \{(d_{21}, d_{32}) : d_{21} > 0, d_{32} = |d_{21} - d_{31}|\}.$$

We now focus on the sup in the denominator in (7). First, we claim that this sup must be achieved when $d_{21} \leq d_{31}$. To see this, assume $d_{21} \geq d_{31}$. In this case, we have

$$C_1^{LB}(P_1/\tau_1, P_2/\tau_1) = \frac{1}{2}\max\left\{\log\left(1+\frac{P_1}{\tau_1 d_{21}^\beta}\right), \log\left(1+\frac{P_1}{\tau_1 d_{31}^\beta} + \frac{\frac{P_1}{\tau_1 d_{21}^\beta}\frac{P_2}{\tau_1 d_{32}^\beta}}{1+\frac{P_1}{\tau_1 d_{21}^\beta}+\frac{P_1}{\tau_1 d_{31}^\beta}+\frac{P_2}{\tau_1 d_{32}^\beta}}\right)\right\},$$

which is nonincreasing in d_{21} on

$$\{(d_{21}, d_{32}) : d_{21} \geq d_{31}, d_{32} = d_{21} - d_{31}\},$$

for fixed $d_{31} > 0$. This means that we can ignore the case $d_{21} > d_{31}$. In other words, the sup in the denominator in (7) is achieved on

$$\{(d_{21}, d_{32}) : d_{21} > 0, d_{32} = d_{31} - d_{21} \geq 0\}.$$

Let $\rho \in [0,1]$ be such that $d_{21} = \rho d_{31}$ and $d_{32} = (1-\rho)d_{31}$. Let ρ^* be the unique solution to

$$P_1 \rho^{-\beta} = P_1 + P_2(1-\rho)^{-\beta}$$

on $(0,1)$. Note that, since $P_2 \geq P_1$, we have $\rho^* < 1/2$.

Next, we argue that the sup in the denominator in (7) is achieved when $\rho \geq \rho^*$. To see this, assume $\rho \leq \rho^*$. It is straightforward to see that if $\rho \leq \rho^*$, then

$$C_1^{LB}(P_1/\tau_1, P_2/\tau_1) = \frac{1}{2}\log\left(1+\frac{P_1}{\tau_1 d_{31}^\beta}+\frac{P_2}{\tau_1\big((1-\rho)d_{31}\big)^\beta}\right),$$

which is nondecreasing in ρ. This means that we can ignore the case $\rho < \rho^*$. Therefore, the sup in the denominator in (7) is achieved on

$$\{(d_{21}, d_{32}) : d_{21} = \rho d_{31}, d_{32} = (1-\rho)d_{31}, \rho \in [\rho^*, 1]\}.$$

It follows that, for fixed $d_{31} > 0$, we have

$$\sup_{G_2(d_{31})} C_1^{LB}(P_1/\tau_1, P_2/\tau_1) = \sup_{\rho\in[\rho^*,1]} \frac{1}{2}\log\left(1+\frac{1}{\tau_1 d_{31}^\beta}\max\left\{\frac{P_1}{\rho^\beta}, P_1 + \frac{\frac{P_1}{\rho^\beta}\frac{P_2}{(1-\rho)^\beta}}{\tau_1 d_{31}^\beta + \frac{P_1}{\rho^\beta} + P_1 + \frac{P_2}{(1-\rho)^\beta}}\right\}\right).$$

By ignoring the second term inside the max above, we obtain

$$\sup_{G_2(d_{31})} C_1^{LB}(P_1/\tau_1, P_2/\tau_1) \geq \frac{1}{2}\log\left(1+\frac{P_1}{\tau_1 d_{31}^\beta}\frac{1}{(\rho^*)^\beta}\right). \quad (8)$$

Finally, this implies

$$\eta_{\min} \leq \inf_{d_{31}>0} \frac{\log\left(1+\frac{P_1}{\tau_1 d_{31}^\beta}\right)}{\log\left(1+\frac{P_1}{\tau_1 d_{31}^\beta}\frac{1}{(\rho^*)^\beta}\right)} = (\rho^*)^\beta,$$

where the equality above follows from the fact that the term to which the inf is applied is nonincreasing in d_{31} on $(0,\infty)$. Recalling that $\rho^* < 1/2$ concludes the proof. □

Remark 1 As P_2/P_1 increases, the solution ρ^* to

$$P_1\rho^{-\beta} = P_1 + P_2(1-\rho)^{-\beta}$$

on $(0,1)$ decreases, and consequently the upper bound $(\rho^*)^\beta$ on the worst-case equilibrium efficiency $\eta_{\min}$ becomes significantly sharper than $(1/2)^\beta$.

Remark 2 In developing our upper bound $(\rho^*)^\beta$, we overestimated the right-hand-side of

$$\eta_{\min} \leq \inf_G \frac{U(1,1)}{\max_{(k_1,k_2)\in[0,1]^2} U^{LB}(k_1,k_2)}$$

on two occasions. First, we first let $d_{42} \to \infty$ for fixed d_{21}, d_{31} to obtain (6). This is intuitively appealing because, as $d_{42}, d_{41} \to \infty$, the source 2 totally wastes all of its power at the equilibrium $\tilde{k} = (1,1)$, whereas the optimum solution $(1,0)$ requires the source 2 use all of its power to help the source 1. We feel that this overestimation cannot be improved although a proof for this is lacking at the moment.

Second, we introduced the underestimation

$$\sup_{G_2(d_{31})} C_1^{LB}(P_1/\tau_1, P_2/\tau_1) \geq \frac{1}{2}\log\left(1 + \frac{P_1}{\tau_1 d_{31}^\beta}\frac{1}{(\rho^*)^\beta}\right)$$

in (8). Although this underestimation may be strict, it cannot be improved significantly because

$$\sup_{G_2(d_{31})} C_1^{LB}(P_1/\tau_1, P_2/\tau_1) \leq \frac{1}{2}\log\left(1 + \frac{P_1}{\tau_1 d_{31}^\beta}\left(1 + \frac{1}{(\rho^*)^\beta}\right)\right).$$

Remark 3 Our main result can be extended to general relay networks with n sources and n destinations. Here, each source uses some of its limited power for its own communication, and uses its remaining power to act as a relay for one other source. The sources use a time-sharing protocol, and consequently a single relay channel arises in each period. Let r_i be the source acting as a relay for source i. Assume that the ratio $P_{r_i}/P_i \geq 1$ is the greatest among all of the sources. We can upper bound the worst-case equilibrium efficiency by letting the distance between each source and its destination tend to ∞ except for the source i and its destination d_i. This effectively eliminates all the relay channels except the $i-$th one. Following along similar lines as in the proof of Proposition 1, one can obtain a similar upper bound ${\rho^*}^\beta$ on the worst-case equilibrium efficiency where $\rho^* < 1/2$ is the unique solution to

$$P_i\rho^{-\beta} = P_i + P_{r_i}(1-\rho)^{-\beta}$$

on $(0,1)$.

6. Conclusion

We introduced a power allocation game in a four node relay network where each source decides on its power allocation in order to optimize its own rate. We showed that the equilibrium emerging from such a noncooperative setting may lead to a significantly inferior sum rate compared to the optimal sum rate. Our main message is that, in a general relay network in which it is not feasible to require the sources to jointly maximize the sum rate, the sources should not (be allowed to) maximize their own individual rates, but rather they should optimize some carefully designed "utility functions" leading to a reasonably high sum rate. Designing such utility functions is a challenging future research topic. Other future research topics are to establish reasonably tight lower bounds on the worst-case equilibrium efficiency and to extend our results from an average-case point of view.

References

[1] T. Başar and G. J. Olsder. *Dynamic Noncooperative Game Theory*. SIAM, Philadelphia, PA, 1999.

[2] T. M. Cover and A. E. Gamal. Capacity theorems for the relay channel. *IEEE Transactions on Information Theory*, 25(5):572–584, September 1979.

[3] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. Wiley, New York, NY, 1991.

[4] D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, Cambridge, MA, 1991.

[5] P. Gupta and P. Kumar. The capacity of wireless networks. *IEEE Transactions on Information Theory*, 46(2):388–404, March 2000.

[6] A. Høst-Madsen. On the capacity of wireless relaying. In *Proceedings of the IEEE Vehicular Technology Conference VTC 2002-Fall*, pages 1333–1337, 2002.

[7] A. Høst-Madsen. Capacity bounds for cooperative diversity. *IEEE Transactions on Information Theory*, 52(4):1522–1544, April 2006.

[8] A. Høst-Madsen and J. Zhang. Capacity bounds and power allocation for wireless relay channels. *IEEE Transactions on Information Theory*, 51(6):2020–2040, June 2005.

[9] G. Kramer, M. Gastpar, and P. Gupta. Cooperative strategies and capacity theorems for relay networks. *IEEE Transactions on Information Theory*, 51(9):3037–3063, September 2005.

[10] J. Laneman, E. Martinian, G. Wornell, J. Apostolopoulos, and J. Wee. Comparing application-and physical-layer approaches to diversity on wireless channels. In *Proceedings of the IEEE International Conference on Communications ICC'03*, pages 2678– 2682, 2003.

[11] J. Laneman and G. Wornell. Distributed spacetime-coded protocols for exploiting cooperative diversity in wireless networks. *IEEE Transactions on Information Theory*, 49(10):2415–2425, October 2003.

[12] J. N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and

outage behavior. *IEEE Transactions on Information Theory*, 50(12):3062–3080, December 2004.

[13] T. S. Rappaport. *Wireless Communications, Principles and Practice*. Prentice Hall, Upper Saddle River, NJ, second edition, 1996.

[14] E. C. van der Meulen. *Transmission of information in a $T-$terminal discrete memoryless channel*. PhD thesis, Department of Statistics, University of California, Berkeley, 1968.

[15] E. C. van der Meulen. Three-terminal communication channels. *Advances in Applied Probability*, 3(1):120–154, Spring 1971.

[16] F. Willems. The discrete memoryless multi access channel with partially cooperating encoders. *IEEE Transactions on Information Theory*, 29(3):441–445, May 1983.

[17] F. Willems and E. van der Meulen. The discrete memoryless multiple-access channel with cribbing encoders. *IEEE Transactions on Information Theory*, 31(3):313–327, May 1985.

[18] D. H. Wolpert and K. Tumor. Optimal payoff functions for members of collectives. *Advances in Complex Systems*, 4(2&3):265–279, 2001.

[19] H. P. Young. *Strategic Learning and Its Limits*. SIAM, Oxford University Press Inc., 2004.