

Equilibrium Efficiency Improvement in MIMO Interference Systems: A Decentralized Stream Control Approach

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Abstract—We consider a multi-link MIMO interference system in which each link wishes to maximize its own mutual information by choosing its own signal vector, which leads to a multi-player game. We show the existence of a Nash equilibrium and obtain sufficient conditions for the uniqueness of equilibrium. We consider two decentralized link adjustment algorithms called best-response process (a.k.a. iterative water-filling) and gradient-play (an autonomous and non-cooperative version of the well-known gradient ascent algorithm). Under our uniqueness conditions, we establish the convergence of these algorithms to the unique equilibrium provided that the links use some *inertia*.

To improve the efficiency of an equilibrium with respect to the total mutual information by imposing limits on the number of independent data streams, we present a stream control approach using linear transformation of the link covariance matrices. We then show how to decentralize our stream control approach by allowing the links to negotiate the limits on the number of independent data streams that they are willing to impose upon themselves. To achieve this, we introduce a variation of a learning algorithm called “adaptive play” that has desirable convergence properties in potential games with reduced computation.

Index Terms—MIMO systems, cochannel interference, ad-hoc networks, game theory, Nash equilibrium, distributed decision-making.

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) wireless links exploit the spatial dimension by using antenna arrays at both ends of a link to transmit multiple parallel streams in the same time and frequency channel [1], [2]. With antenna arrays at both ends, signals transmitted and received by array elements at different physical locations possibly go through different paths, thus, can be separated by array processing algorithms. Depending on the channel scattering conditions, such spatial multiplexing can yield large gains in capacity of wireless systems.

Recently, factors such as increased number of links and reduced cell sizes in wireless networks have spurred interest

in interference management in MIMO systems [3]–[6]. In particular, an additional level of spatial multiplexing among multiple MIMO links has been considered in [7]–[10]. With this scheme, multiple links, each with different transmitter-receiver pairs, are allowed to transmit in a given range possibly through multiple streams per link. Such a network of interfering MIMO links can yield high spectral efficiency with optimized signaling that uses the spatial processing capabilities at both ends of the links according to the channel and interference conditions. Adaptive beamforming done at the transmitter for each stream using the knowledge of the channel state information (CSI) not only affects the efficiency in directing the transmitted power towards the designated receiver but also shapes the interference emitted to the other links. Accordingly, the capacity of a link is dependent on all transmissions in the range.

Finding the capacity of the interference channel is in general an open problem. For tractability, one can model the interference as colored noise and use a whitening transform [11] to find the capacity in the presence of external interference. With such a modeling assumption, the problem of maximizing the total mutual information of a network with interfering MIMO links (in general a non-convex optimization problem) is studied in [7], [9], [10]. The variants of the well-known gradient-ascent method used in [10] are centralized in essence and require all CSI between each pair of transmitting and receiving nodes. Furthermore, such centralized methods tend to be computationally prohibitive in large-scale networks.

The need for optimizing the link parameters with local information and reasonable computational burden motivates a decentralized approach. The set of parameters for each link includes the number of streams to be transmitted, array weights for each stream and power distribution among the streams. A decentralized iterative water-filling method was presented in [7], where each link attempts to selfishly maximize its own mutual information based on the knowledge of its own channel matrix and the covariance of the total interference and the noise at its own receiver. In this approach, the link parameters very often converge to a Nash equilibrium of the underlying multi-player game. Although such an equilibrium does not necessarily maximize the total mutual information [9], [10], it can be obtained using a decentralized algorithm with much less computational effort and without global information.

To bridge the gap in performance between optimal and equilibrium link parameters, a stream control approach is

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presented in [9], where it was shown that imposing limits on the number of independent data streams may improve the efficiency of an equilibrium substantially. However, in [9], although the computation of an equilibrium is done using a decentralized algorithm, choosing the best combination of limits on the number of independent data streams required a brute-force enumeration approach that may not be suitable to large-scale networks.

In this paper, we propose a two-level hierarchical approach to realize the benefits of using stream control, namely system-wide efficiency improvement, without centralized coordination. At the higher level of the hierarchy, each link negotiates, with the rest of the links, a limit on the number of streams that it is willing to impose upon itself. The purpose of each link at the higher-level negotiations is to improve the efficiency of an equilibrium solution with respect to the total mutual information by agreeing to a limit on the number of data streams that it will use. Therefore, the link behavior at the higher level of the hierarchy is expected to be more cooperative towards improving the system-wide performance. We introduce a variant of a learning algorithm called “adaptive play” [12] to allow the higher-level negotiations to proceed with minimal information exchange and without centralized coordination. At the lower level of the hierarchy, however, each link selfishly pursues the maximization of its own mutual information subject to the limit on the number of data streams to which it agreed at the higher level. Our analysis and simulation results confirm that the system-wide performance improvement at an equilibrium can be achieved with decentralized stream control via link negotiations.

Other somewhat related previous work that is worth mentioning here include [13]–[15], where users with single antennas allocate their powers over frequency-selective channels in a competitive way. We emphasize that the model in these studies are different from ours, since the user decision parameters in [13]–[15] are only the transmitted powers over the spectrum - in the form of scalar functions or vector variables - rather than covariance matrices of the transmitted signals. The paper [13] characterizes the Nash equilibrium in the Gaussian interference channel where two selfish users choose their power spectra to maximize their own rates. In [14], the inefficiency of Nash equilibrium obtained using iterative waterfilling in a two-user power control game is illustrated based on simulations. Finally, in [15], the authors show that the inefficiency of Nash equilibrium in a spectrum sharing game can be reduced by considering the repeated game equilibria.

The rest of this paper is organized as follows. Section II presents the system model, whereas, Section III introduces a multi-link game and the notion of Nash equilibrium. Two decentralized algorithms that allow the links to adjust their signal vectors towards an equilibrium are presented in Section IV. In Section V, we discuss the details of our stream control approach using linear transformation of link covariance matrices. Section VI is devoted to the details of decentralized stream control via link negotiations. Simulation results confirming the benefits of decentralized stream control are presented in Section VII. Finally, some concluding remarks are presented in Section VIII.

II. SYSTEM MODEL

We consider an L -link communication system where each link is associated with a transmitter-receiver pair. Each transmitter and receiver are equipped with N_t and N_r antennas, respectively. We assume link k , for $k = 1, \dots, L$, transmits a complex signal vector \mathbf{x}_k of dimension N_t . Consequently, a complex baseband signal vector of dimension N_r denoted by \mathbf{y}_k is received at the k -th receiver. The received signal vectors are related to the transmitted signal vectors by

$$\mathbf{y}_k = \sqrt{\rho_k} \mathbf{H}_{k,k} \mathbf{x}_k + \sum_{l=1, l \neq k}^L \sqrt{\eta_{k,l}} \mathbf{H}_{k,l} \mathbf{x}_l + \mathbf{n}_k,$$

where

- ρ_k is the noise-normalized total transmit power, referred to as signal-to-noise ratio (SNR), of the k -th link,
- $\eta_{k,l}$ is the signal-to-interference ratio (INR), defined analogously to SNR, corresponding to the interference caused by the l -th link at the k -th receiver,
- $\mathbf{H}_{k,l}$ is the complex channel matrix of dimension $N_r \times N_t$ for the link between the l -th transmitter and the k -th receiver,
- \mathbf{n}_k denotes the zero-mean circularly symmetric complex Gaussian noise vector at the k -th receiver. We assume that $E[\mathbf{n}_k \mathbf{n}_k^\dagger] = \mathbf{I}$, where $E[\cdot]$ and \dagger denote the expectation and the conjugate transpose operations, respectively, and \mathbf{I} denotes an identity matrix of proper dimension.

We now consider a scenario in which the k -th link wishes to maximize its own mutual information $I(\mathbf{x}_k; \mathbf{y}_k)$ between the input and the output of the channel characterized by $\mathbf{H}_{k,k}$ by choosing the distribution of \mathbf{x}_k , independently of the other links, subject to the power constraint

$$E[\mathbf{x}_k^\dagger \mathbf{x}_k] \leq 1. \quad (1)$$

The k -th link, not knowing the distributions of the signal vectors chosen by the other links, models the total interference $\sum_{l=1, l \neq k}^L \sqrt{\eta_{k,l}} \mathbf{H}_{k,l} \mathbf{x}_l$ at its receiver as a zero-mean circularly symmetric complex Gaussian noise vector. Under the modeling assumptions delineated above, it is known that the k -th link’s mutual information $I(\mathbf{x}_k; \mathbf{y}_k)$ is maximized only by a zero-mean circularly symmetric complex Gaussian distribution satisfying the power constraint (1); see [1]. Note that if all links make the same modeling assumptions then the best distributions that can be chosen by the links would be mutually consistent with their modeling assumptions. We will assume that this is the case, and note from [1] that the mutual information of each link can now be written as

$$I(\mathbf{x}_k; \mathbf{y}_k) = \log_2 \det(\mathbf{I} + \rho_k \mathbf{R}_k^{-1/2} \mathbf{H}_{k,k} \mathbf{Q}_k \mathbf{H}_{k,k}^\dagger \mathbf{R}_k^{-1/2}), \quad (2)$$

$k = 1, \dots, L$, where $\mathbf{Q}_k := E[\mathbf{x}_k \mathbf{x}_k^\dagger]$ is a Hermitian positive-semi definite matrix satisfying $\text{trace}(\mathbf{Q}_k) = 1$, and

$$\mathbf{R}_k := \mathbf{I} + \sum_{l=1, l \neq k}^L \eta_{k,l} \mathbf{H}_{k,l} \mathbf{Q}_l \mathbf{H}_{k,l}^\dagger$$

is the covariance matrix of the total interference and noise at the k -th receiver. We will furthermore assume that both the transmitter and receiver nodes of the k -th link have the knowledge of the SNR level ρ_k and the whitened channel

matrix $\mathbf{R}_k^{-1/2}\mathbf{H}_{k,k}$ once the other links choose their signal vectors. Now, from the perspective of each link, the problem of maximizing its own mutual information amounts to choosing an appropriate covariance matrix in the presence of the other links that also want to maximize their own mutual information.

III. A MULTI-LINK GAME AND NASH EQUILIBRIUM

The setup introduced in the previous section leads us to an L -link game with the utility functions

$$\begin{aligned} U_k(\mathbf{Q}) &= \log_2 \det(\mathbf{I} + \rho_k \mathbf{R}_k^{-1/2} \mathbf{H}_{k,k} \mathbf{Q}_k \mathbf{H}_{k,k}^\dagger \mathbf{R}_k^{-1/2}) \\ \mathbf{R}_k(\mathbf{Q}) &= \mathbf{I} + \sum_{l=1, l \neq k}^L \eta_{k,l} \mathbf{H}_{k,l} \mathbf{Q}_l \mathbf{H}_{k,l}^\dagger, \end{aligned} \quad (3)$$

where \mathbf{Q} denotes the profile of the covariance matrices of all transmitters, i.e., $\mathbf{Q} := \{\mathbf{Q}_1, \dots, \mathbf{Q}_L\}$. The covariance matrix of each link belongs to a common strategy space, denoted by $\Phi^{N_t \times N_t}$, where, for any integer $r > 0$,

$$\Phi^{r \times r} := \{\mathbf{F} : \mathbf{F} \text{ is } r \times r \text{ Hermitian positive-semi-definite and } \text{trace}(\mathbf{F}) = 1\}. \quad (4)$$

A selfish link in such a strategic engagement would not be satisfied with its choice unless its utility is maximized given the choices of the other links. A steady state situation in which all link utilities are mutually maximized is called a Nash equilibrium¹. For a more precise definition of equilibrium, let \mathbf{Q}_{-k} denote the profile of the covariance choices of the links *other than* the k -th link, i.e., $\mathbf{Q}_{-k} = \{\mathbf{Q}_1, \dots, \mathbf{Q}_{k-1}, \mathbf{Q}_{k+1}, \dots, \mathbf{Q}_L\}$. With this notation, we will sometimes write $U_k(\mathbf{Q})$ as $U_k(\mathbf{Q}_k, \mathbf{Q}_{-k})$. A profile of covariance choices $\mathbf{Q}^* = \{\mathbf{Q}_1^*, \dots, \mathbf{Q}_L^*\}$ is called an equilibrium if, for all $k = 1, \dots, L$,

$$U_k(\mathbf{Q}_k^*, \mathbf{Q}_{-k}^*) = \max_{\mathbf{Q}_k \in \Phi^{N_t \times N_t}} U_k(\mathbf{Q}_k, \mathbf{Q}_{-k}^*).$$

An equilibrium represents a steady-state situation in which no link has an incentive to unilaterally change its choice. As such, equilibrium is a particularly useful notion when it is not practical to obtain and/or implement a system-wide optimal solution. For example, an equilibrium can emerge out of local optimizations performed by autonomous links in an ad-hoc wireless network without centralized coordination. We now address the issue of existence and uniqueness of an equilibrium using the standard results on concave games; see [16].

Proposition 3.1: A multi-link game characterized by the utility functions (3) and the common strategy space $\Phi^{N_t \times N_t}$ (4) for each link has an equilibrium solution. Furthermore, such a game has a unique equilibrium whenever

- 1) $\max_{k,l \in \{1, \dots, L\}} \eta_{k,l} < \bar{\eta}$ for a sufficiently small $\bar{\eta} > 0$ and
- 2) $\text{rank}(\rho_k \mathbf{H}_{k,k}) = N_t$ for all $k \in \{1, \dots, L\}$.

Proof: See Appendix I.

We should point out that in general multiple equilibria may exist, and different initial conditions used by the links in their adjustments of the covariance matrices may lead to different equilibria. In the next section, we present two

decentralized algorithms for link adjustment and establish their convergence to an equilibrium under the uniqueness conditions of Proposition 3.1.

IV. DECENTRALIZED ALGORITHMS FOR LINK ADJUSTMENT

The algorithms presented in this section allow each link to iteratively adjust its covariance matrix in response to the adjustments made by the other links. The k -th link always knows its SNR ρ_k and its own channel matrix $\mathbf{H}_{k,k}$. However, the covariance matrix \mathbf{R}_k of the total interference and the noise at its receiver depends on the choices \mathbf{Q}_{-k} made by the other links, and therefore, the k -th link's most recent measurement of \mathbf{R}_k at any step corresponds to the choices of the other links made at the previous step. Based on this, the k -th link adjusts $\mathbf{Q}_k \in \Phi^{N_t \times N_t}$ at any step to improve its own utility in response to its last measurement of \mathbf{R}_k . There are at least two ways for each link to make such adjustments, which lead to two different decentralized algorithms.

A. Best-response Process

We define the best response mapping of the k -th link as

$$\begin{aligned} BR_k(\mathbf{Q}_{-k}) &= \{\mathbf{Q}_k \in \Phi^{N_t \times N_t} : \\ &U_k(\mathbf{Q}_k, \mathbf{Q}_{-k}) = \max_{\tilde{\mathbf{Q}}_k \in \Phi^{N_t \times N_t}} U_k(\tilde{\mathbf{Q}}_k, \mathbf{Q}_{-k})\}. \end{aligned}$$

Given \mathbf{Q}_{-k} , the best response $BR_k(\mathbf{Q}_{-k})$ for the k -th link can be obtained by water-filling [1]. Best-response process can now be given as:

- 1) Initialize $\mathbf{Q}_1(0) \in \Phi^{N_t \times N_t}, \dots, \mathbf{Q}_L(0) \in \Phi^{N_t \times N_t}$.
- 2) For all $k = 1, \dots, L$, iteratively update:

$$\mathbf{Q}_k(t+1) \in (1 - \alpha_k(t))\mathbf{Q}_k(t) + \alpha_k(t)BR_k(\mathbf{R}_k(t)),$$

where $0 \leq \alpha_k(t) \leq 1$ is a parameter that represents the k -th link's willingness to optimize (in other words, $1 - \alpha_k(t)$ is the k -th link's inertia) at step t .

One can also obtain a useful variation of best-response process by occasionally allowing some links not to update, however, each link must not stop updating. The case where $\alpha(t) = 1$, for all $t \geq 0$, has been presented in [7] with simulation results indicating convergence in most cases. The use of inertia is helpful in enabling convergence since it prevents the links from overreacting. Indeed, our simulation results indicate that best-response process is always convergent when the inertia is increased at a suitable rate. Furthermore, best-response process can provably converge when INR levels are sufficiently small.

Proposition 4.1: Assume that the uniqueness conditions given in Proposition 3.1 hold true. If the parameters $\alpha_k(t)$ are chosen to satisfy 1) $\lim_{t \rightarrow \infty} \alpha_k(t) = 0$ and 2) $\sum_{t=0}^{\infty} \alpha_k(t) = +\infty, \forall k \in \{1, \dots, L\}$, then best-response process converges to the unique equilibrium of the game.

Proof: See Appendix II.

Example 4.1: The conditions on the parameters $\alpha_k(t)$ given in Proposition 4.1 are satisfied for $\alpha_k(t) = 1/t$.

¹We will henceforth refer to a Nash equilibrium simply as an equilibrium.

B. Gradient-play

Gradient-play is similar to the well-known gradient-ascent dynamics. Each link computes the gradient of its utility with respect to its own covariance matrix, and updates its covariance matrix in its utility ascent direction. The subtlety is that since the links update simultaneously they may end up decreasing their utility as in best-response process. Gradient-play process is given as:

- 1) Initialize $\mathbf{Q}_1(0) \in \Phi^{N_t \times N_t}, \dots, \mathbf{Q}_L(0) \in \Phi^{N_t \times N_t}$.
- 2) For all $k = 1, \dots, L$, iteratively update:

$$\mathbf{Q}_k(t+1) = (1 - \alpha_k(t))\mathbf{Q}_k(t) + \alpha_k(t)\Pi_{\Phi^{N_t \times N_t}} [\mathbf{Q}_k(t) + \gamma_k(t)\nabla_{\mathbf{Q}_k} U_k(\mathbf{Q}(t))],$$

where

- Given an $N_t \times N_t$ complex matrix \mathbf{F} , $\Pi_{\Phi^{N_t \times N_t}}[\mathbf{F}]$ is the matrix in $\Phi^{N_t \times N_t}$ that is at minimum distance from \mathbf{F} , i.e., $\operatorname{argmin}_{\mathbf{E} \in \Phi^{N_t \times N_t}} \operatorname{trace}(\mathbf{E} - \mathbf{F})(\mathbf{E} - \mathbf{F})^\dagger$.
- $\gamma_k(t) \geq 0$ is the k -th link's step size at step t ,
- $\nabla_{\mathbf{Q}_k} U_k := \rho_k \mathbf{H}_{k,k}^\dagger (\mathbf{R}_k + \rho_k \mathbf{H}_{k,k} \mathbf{Q}_k \mathbf{H}_{k,k}^\dagger)^{-1} \mathbf{H}_{k,k}$,
- $0 \leq \alpha_k(t) \leq 1$ is a parameter that represents the k -th link's willingness to optimize at step k .

For an $N_t \times N_t$ Hermitian matrix \mathbf{F} , $\Pi_{\Phi^{N_t \times N_t}}[\mathbf{F}]$ can be obtained as follows:

- Write \mathbf{F} as $\mathbf{F} = \mathbf{V} \operatorname{diag}\{\lambda_1, \dots, \lambda_{N_t}\} \mathbf{V}^\dagger$ where $\mathbf{V} \mathbf{V}^\dagger = \mathbf{I}$ and $\operatorname{diag}\{\lambda_1, \dots, \lambda_{N_t}\}$ denotes a diagonal matrix with the diagonal entries $\lambda_1, \dots, \lambda_{N_t}$.
- Choose $\mu \in \mathbb{R}$ such that $\sum_{i=1}^{N_t} \max\{\lambda_i + \mu, 0\} = 1$.
- Set $\rho_i := \max\{\lambda_i + \mu, 0\}$ and

$$\Pi_{\Phi^{N_t \times N_t}}[\mathbf{F}] := \mathbf{V} \operatorname{diag}\{\rho_1, \dots, \rho_{N_t}\} \mathbf{V}^\dagger.$$

In our simulation results, gradient-play process exhibited a consistently convergent behavior. Gradient-play can also be proven to be convergent for small INR levels if the parameters $\gamma_k(t)$ and $\alpha_k(t)$ are chosen properly.

Proposition 4.2: Assume that the uniqueness conditions given in Proposition 3.1 hold true. If the parameters $\alpha_k(t)$ and $\gamma_k(t)$ are chosen to satisfy 1) $\lim_{t \rightarrow \infty} \alpha_k(t) = 0$ and 2) $\sum_{t=0}^{\infty} \alpha_k(t) = +\infty, \forall k \in \{1, \dots, L\}$, and 3) $\lim_{t \rightarrow \infty} \gamma_k(t) = \bar{\gamma}, \forall k \in \{1, \dots, L\}$, for a sufficiently small $\bar{\gamma} > 0$, then gradient-play process converges to the unique equilibrium of the game.

Proof: See Appendix III.

The algorithms presented above allow links to adjust their covariance matrices towards an equilibrium. However, it is well-known that a Nash equilibrium may be an inferior solution with respect to a system-wide objective, see [17], such as the total mutual information $\sum_{l=1}^L U_l(\mathbf{Q})$. Such inefficiency of an equilibrium in the context of a MIMO interference system has been recognized in [9] and a stream control approach has been proposed to limit the degradation caused by the selfish nature of the links on the system-wide performance. The main idea behind stream control is to impose a limit on the number independent data streams that can be used by a link.

In the approach taken in [9], a set of rank limitations are imposed on the link covariance matrices to limit the number of independent data streams. This leads to link strategy spaces that are non-convex. For example, if a link with two transmitting antennas is constrained to use at most one independent

data stream, then it will have to choose a covariance matrix from $\Phi^{2 \times 2}$ with rank at most 1. Clearly, a convex combination of two such rank deficient matrices may have full rank. A consequence of the non-convexity of the link strategy spaces is that we cannot use the standard results on concave games, [16], to establish the existence of an equilibrium with stream control. Non-existence of an equilibrium implies that the link utilities can never be mutually optimized, which would lead to a situation in which the links adjust their covariance matrices in an everlasting fashion. Another consequence of the non-convexity of the link strategy spaces is that, adjustment of the covariance matrices may become more complicated, even if an equilibrium exists. For example, a link employing inertia in its adjustment process, as in best-response and gradient-play processes, may generate a covariance matrix that violates its rank constraint.

In this paper, to circumvent the issues stemming from non-convexity of link strategy spaces, we will follow an alternative approach to stream control whose details are presented next.

V. STREAM CONTROL WITH SELECTION MATRICES

Here, instead of imposing rank limitations on the link covariance matrices, we will require the k -th link to pre-process its signal vector \mathbf{x}_k according to

$$\mathbf{x}_k = \mathbf{W}_k \mathbf{s}_k,$$

where \mathbf{W}_k is a complex stream selection matrix of dimension $N_t \times r_k$ for some integer $r_k > 0$ and \mathbf{s}_k is a complex signal vector of dimension r_k . To ensure that the signal vectors \mathbf{x}_k and \mathbf{s}_k have the same power, we will also require $\mathbf{W}_k^\dagger \mathbf{W}_k = \mathbf{I}$. Note that when \mathbf{x}_k is chosen as above the covariance matrix of \mathbf{x}_k can have rank at most r_k , and hence, the number of independent data streams transmitted by the k -th link can be at most r_k . Thus, using stream selection matrices \mathbf{W}_k allows us to control the number of independent data streams.

We incorporate stream control into our system model by replacing the channel matrices $\mathbf{H}_{k,l}$ with $\mathbf{H}_{k,l} \mathbf{W}_l$. Accordingly, the links adapt to the modified channel matrices $\mathbf{H}_{k,l} \mathbf{W}_l$, for all $k, l \in \{1, \dots, L\}$, by adjusting the covariance matrices of their modified signal vectors \mathbf{s}_k by using, for instance, best-response or gradient-play process. Note that the covariance matrix of \mathbf{s}_k belongs to the convex set $\Phi^{r_k \times r_k}$, which is now the k -th link's strategy space. This allows us to use Proposition 3.1 to establish the existence of an equilibrium with stream control. Also, in light of Proposition 3.1, small enough INR levels and $\operatorname{rank}(\rho_k \mathbf{H}_{k,k} \mathbf{W}_k) = r_k$, for all $k \in \{1, \dots, L\}$, are sufficient for the uniqueness of an equilibrium with stream control.

Let us now discuss how to choose the stream selection matrices $\mathbf{W}_k, k = 1, \dots, L$, to improve the efficiency of an equilibrium with respect to the total sum of the link utilities. Ideally, one would like to choose \mathbf{W}_k in such a way that equilibrium solutions yield as high total mutual information as possible. This would lead to an optimization problem of the following form:

$$\max_{\mathbf{W}} \sum_{k=1}^L U_k(\mathbf{Q}^*(\mathbf{W})),$$

where $\mathbf{W} = \{\mathbf{W}_1, \dots, \mathbf{W}_L\}$ and $\mathbf{Q}^*(\mathbf{W})$ is an equilibrium² corresponding to \mathbf{W} . However, we do not know any reasonable way of carrying out such an optimization, and hence we will not pursue this direction. Instead, we will follow a heuristic approach to choose the stream selection matrices.

To understand our heuristic way of choosing the stream selection matrices, let \mathbf{Q}^* be an equilibrium in the case of no stream control. By definition, all link utilities are maximized at \mathbf{Q}^* , and hence the k -th link's equilibrium covariance matrix \mathbf{Q}_k^* satisfies $\mathbf{Q}_k^* = \mathbf{V}_k \mathbf{\Lambda}_k \mathbf{V}_k^\dagger$, where $\mathbf{V}_k = [\mathbf{v}_k^1, \dots, \mathbf{v}_k^{N_t}]$ is a unitary matrix whose columns are eigenvectors of $\mathbf{H}_{k,k}^\dagger \mathbf{R}_k^{-1} \mathbf{H}_{k,k} \Big|_{\mathbf{Q}_k^*}$, and $\mathbf{\Lambda}_k \in \Phi^{N_t \times N_t}$ is a diagonal matrix of power levels $\lambda_k^1 \geq \dots \geq \lambda_k^{N_t}$ given by "water-filling" [1]. This means that we can interpret $\mathbf{\Lambda} = \{\mathbf{\Lambda}_1, \dots, \mathbf{\Lambda}_L\}$ as an equilibrium of the game with stream control where the stream selection matrices are chosen as \mathbf{V}_k , i.e., $\mathbf{W}_k = \mathbf{V}_k$, for all $k \in \{1, \dots, L\}$. In this case, the k -th link's equilibrium signal vector \mathbf{s}_k can be generated as N_t independent data streams with power levels $\lambda_k^1, \dots, \lambda_k^{N_t}$.

Clearly, the particular choice of the stream selection matrices above would not reduce the number of independent data streams; however, it suggests a sensible way of stream control by choosing \mathbf{W}_k as

$$\mathbf{W}_k = [\mathbf{v}_k^1, \dots, \mathbf{v}_k^{r_k}], \quad (5)$$

where $r_k \leq N_t$ is the desired limit on the independent data streams transmitted by the k -th link. The rationale behind this choice is to limit the k -th link's signal vector to the subspace spanned by r_k directions to which it assigns the highest power levels at \mathbf{Q}^* . We should emphasize, however, that an equilibrium in the case of stream control via (5) is *not* necessarily a collection of diagonal covariance matrices where the diagonal entries of the k -th covariance matrix are $\lambda_k^1, \dots, \lambda_k^{r_k}$. The links need to adapt to the stream selection matrices (5) by adjusting their covariance matrices towards a new equilibrium using a decentralized algorithm. This new equilibrium *may* yield a higher total mutual information than \mathbf{Q}^* does. Next, we illustrate the possible efficiency improvement through a simple example.

Example 5.1: Consider a 2 link interference system with $\mathbf{H}_{11} = \text{diag}\{2, 1\}$, $\mathbf{H}_{22} = \text{diag}\{1, 2\}$, $\mathbf{H}_{12} = \mathbf{H}_{21} = \mathbf{I}$, and $\rho_1 = \rho_2 = 13$ dB, $\eta_{12} = \eta_{21} = 10$ dB. For this system, best-response process consistently leads to the following equilibrium in the case of no stream control: $\mathbf{Q}^* = \{\text{diag}\{0.8, 0.2\}, \text{diag}\{0.2, 0.8\}\}$, at which the link utilities are $U_1(\mathbf{Q}^*) = U_2(\mathbf{Q}^*) = 5.9$. Suppose that we want to limit the number of independent data streams for each link to 1. Our heuristic explained above leads us to choose the stream selection matrices as $\mathbf{W}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{W}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Best-response process now leads to a new equilibrium $\mathbf{Q}^{*(1)} = \{1, 1\}$, at which the link utilities are $U_1(\mathbf{Q}^{*(1)}) = U_2(\mathbf{Q}^{*(1)}) = 6.3$.

Therefore, a *brute-force* version of our stream control mechanism can be given as follows:

- 1) Let the links adjust their covariance matrices towards

an equilibrium \mathbf{Q}^* with no stream control using a decentralized algorithm.

- 2) For each possible combination of independent data stream limits (r_1, \dots, r_L) , with $1 \leq r_k \leq N_t$, set the stream selection matrices as in (5) and let the links adjust their choices towards a new equilibrium.
- 3) Choose the set of stream selection matrices that yield the highest total mutual information.

The stream control mechanism given above requires a global search over all feasible combinations of limits (r_1, \dots, r_L) on the number of independent data streams, and hence it can be computationally demanding. To alleviate this issue as well as for complete decentralization, we will present a decentralized version of our stream control approach in Section VI. We will also illustrate through simulations that our heuristic way of choosing the stream selection matrices consistently improve equilibrium efficiency; see Section VII.

VI. DECENTRALIZED STREAM CONTROL VIA LINK NEGOTIATION

To achieve the decentralization of the process of imposing limits on the number of independent data streams, we introduce a multi-player finite game with the common strategy space $\Sigma := \{1, \dots, N_t\}$ for each player. In this higher-level game, each link, say the k -th link, chooses a number $r_k \in \Sigma$ which represents the limit on the number of independent data streams that it agrees to impose upon itself. Let $r := (r_1, \dots, r_L)$ and let $\mathbf{Q}^{*(r)}$ be an equilibrium of the lower-level game introduced in Section V with stream control where the limits on the number of independent data streams are given by r and the stream selection matrices are chosen according to (5). Then, the k -th link's utility in the higher-level game is given as $V_k(r) = \sum_{l=1}^L U_l(\mathbf{Q}^{*(r)})$ which is the total mutual information at $\mathbf{Q}^{*(r)}$. Note that the higher-level game may not be well defined when the lower-level game has multiple equilibria, however, we will ignore this issue since our simulation results suggest that the link utilities have similar values at different equilibria of a given lower-level game.

The higher-level game introduced above belongs to a class of games called *identical interest games* since each link's utility function is identical. If the k -th link causes interference only to a subset of the other links, then we can restrict the k -th link's utility in the higher-level game to

$$V_k(r) = \sum_{l \in \mathcal{N}_k} U_l(\mathbf{Q}^{*(r)}), \quad (6)$$

where $\mathcal{N}_k := \{l \in \{1, \dots, L\} : l \neq k, \eta_{l,k} > 0\} \cup \{k\}$ represents the k -th link's neighborhood. In this case, the link utilities in the higher-level game may not be identical, however, they still belong to a specific class of games, called *potential games*, which includes all identical interest games. The defining property of a potential game in our context is that the total mutual information

$$V(r) = \sum_{l=1}^L U_l(\mathbf{Q}^{*(r)}) \quad (7)$$

²In the case of multiple equilibria, the difficulty arises as to which equilibrium is referred to by $\mathbf{Q}^*(\mathbf{W})$.

(at lower-level game equilibrium $\mathbf{Q}^{*(r)}$) serves as a potential function, which means that, $\forall k \in \{1, \dots, L\}$, $\forall r'_k \in \Sigma$, $\forall r''_k \in \Sigma$, and $\forall r_{-k} \in \Sigma^{L-1}$,

$$V_k(r'_k, r_{-k}) - V_k(r''_k, r_{-k}) = V(r'_k, r_{-k}) - V(r''_k, r_{-k}),$$

where we use the notation (r'_k, r_{-k}) to denote a profile of choices where the k -th link chooses $r'_k \in \Sigma$ and the other links choose $r_{-k} \in \Sigma^{L-1}$.

For potential games, there is a known decentralized negotiation method called *adaptive play*, see [12], that leads the links to a choice that maximizes their common utility with arbitrarily high probability. One issue with adaptive play that makes it impractical for our purposes is that adaptive play requires one link at any step to calculate its utility for every possible choice in Σ for fixed choices of the other links. This means that at every step of the higher-level negotiations the links need to run a decentralized algorithm such as best-response or gradient-play process N_t times to find an equilibrium for each lower-level game, which can be computationally demanding. To reduce the computational burden of higher-level negotiations, we will introduce a variant of adaptive play as follows:

- 1) Initialize $r(0) := (N_t, \dots, N_t)$.
- 2) Iteratively update:
 - a) Set $r_j(t) = r_j(t-1)$ for all $j \in \{1, \dots, L\}$.
 - b) Choose the k -th link as the updating link with probability $1/L$.
 - c) Let the k -th link choose $m_k \in \Sigma$ with probability $1/N_t$.
 - d) For some smoothing factor $\tau > 0$, set $r_k(t) = m_k$ with probability

$$\frac{e^{V_k(m_k, r_{-k}(t-1))/\tau}}{e^{V_k(m_k, r_{-k}(t-1))/\tau} + e^{V_k(r_k(t-1), r_{-k}(t-1))/\tau}}.$$

In this variant of adaptive play, the links at any step need to find an equilibrium of only one lower-level game, therefore, the computational burden on the links is now significantly reduced. The above algorithm requires each link to know at any step whether it is given the choice of updating its limit on the number of independent data streams. The links can determine the updating link at any step of the higher-level negotiations in a decentralized fashion. The only additional information that needs to be exchanged between the links at any step is the total mutual information at the new equilibrium of the lower-level game obtained at that step. Such information would be available to each link if each link broadcasts its own mutual information at the new equilibrium of the lower-level game obtained at any step.

The long-term behavior of the link negotiations described above is characterized by the following proposition.

Proposition 6.1: Assume that the link utilities in the higher-level game are well defined and given by (6). Let $r(t)$ be the profile of choices at step t generated by the variant of adaptive play introduced above. Then,

$$\lim_{\tau \downarrow 0} \lim_{t \rightarrow \infty} P(r(t) \in \operatorname{argmax}_{r \in \Sigma^L} V(r)) = 1,$$

where $V(r)$ is the total mutual information (at lower-level game equilibrium corresponding to $r \in \Sigma^L$) given by (7).

Proof: The proof follows along the similar lines of the proof of Theorem 6.1 in [12]. First, the link negotiations induce a stationary Markov chain on the finite state space Σ^L . For any fixed $\tau > 0$, the transition probability from state $r' \in \Sigma^L$ to state $r'' \in \Sigma^L$ is given by $P^\tau(r''|r') = \frac{1}{LN_t} \frac{1}{1 + e^{(V(r') - V(r''))/\tau}}$, provided that r' and r'' differ in exactly one link position, i.e., $r'_k \neq r''_k$ for some $k \in \{1, \dots, L\}$ and $r'_l = r''_l$ for all $l \neq k$. Clearly, if r' and r'' differ in more than one link position, then $P^\tau(r''|r') = 0$. Moreover, we always have $P^\tau(r'|r') > 0$. Therefore, for any fixed $\tau > 0$, the induced Markov chain is irreducible and aperiodic.

By direct verification of the balance equations

$$\mu^\tau(r') P^\tau(r''|r') = \mu^\tau(r'') P^\tau(r'|r''),$$

for all $r' \in \Sigma^L$ and $r'' \in \Sigma^L$, we conclude that the stationary distribution of the induced Markov chain is $\mu^\tau(r) = \frac{e^{V(r)/\tau}}{\sum_{r \in \Sigma^L} e^{V(r)/\tau}}$, for any $r \in \Sigma^L$. Hence, from irreducibility and aperiodicity, we must have $\lim_{t \rightarrow \infty} P(r(t) = r) = \mu^\tau(r)$, for any $r \in \Sigma^L$. Now, taking the limit as $\tau \downarrow 0$ implies the desired result.

In light of the above result, one can choose small values for the smoothing factor $\tau > 0$ to guarantee that the probability of negotiated limits $r(t)$ on the number of independent data streams being optimal for large t is arbitrarily high. However, when $\tau > 0$ is small, it may take long time for $r(t)$ to tend to an optimal combination because of lack of exploration in Σ^L . Therefore, it may be desirable to decrease τ gradually to a small number during the negotiations to allow initial exploration. In fact, we believe that one can obtain convergence, in probability, of the negotiated limits $r(t)$ to an optimal combination if τ is decreased sufficiently slowly as in simulated annealing [18]. In our simulations, choosing τ inversely proportional to t^2 during the negotiations typically resulted in much faster convergence of the negotiated limits $r(t)$ to a near optimal combination than the brute-force enumeration, as will be illustrated in the next Section.

VII. SIMULATION RESULTS

In this section, we numerically illustrate the benefits of stream control in terms of average per-link mutual information. We also analyze the decentralized algorithms (with and without stream control) in terms of their convergence properties.

Average per-link mutual information with brute-force stream control: We assume flat Rayleigh-fading channels between each node pair. In addition, we assume that $\rho_k = \rho$ and $\eta_{k,l} = \eta$ for all $k, l \in \{1, \dots, L\}$, $k \neq l$. We consider all (ρ, η) pairs where $\rho \in \{0, 5, 10, 15, 20\}$ dB and $\eta \in \{-5, 0, 5, 10, 15, 20\}$ dB. For each (ρ, η) pair considered, we perform 1000 simulation runs with (i) the centralized gradient-ascent algorithm [10], (ii) best response process (with $\alpha_k(t) = 1/t^{0.2}$), (iii) gradient-play process (with $\alpha_k(t) = 1/t^{0.5}$ and $\gamma_k(t) = 0.01 + 1/t^{0.5}$), and (iv) best response process (with $\alpha_k(t) = 1/t^{0.2}$) with brute-force stream control where the stream selection matrices are chosen using our heuristic approach. In each run, we randomly generate the channel matrices where the real and imaginary parts of each entry of each channel matrix are independently generated according to zero-mean Gaussian distribution with variance $1/2$.

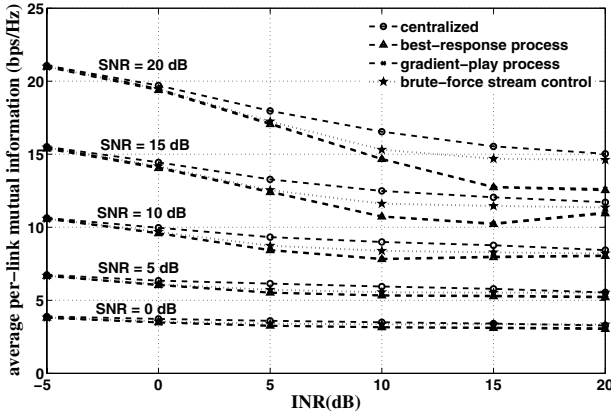


Fig. 1. Average per-link mutual information for $L = 2$ and $N_t = N_r = 4$.

For each (ρ, η) pair, we compute the per-link mutual information averaged over 1000 runs with each of the four methods mentioned above. Figure 1 shows the average per-link mutual information for a 2 link system with 4 antennas at each node. Let us focus on the top-four curves in Figure 1 which correspond to the case where $\rho = 20$ dB. In this case, the highest average per-link mutual information is obtained using the centralized gradient-ascent algorithm, which is used here as a reference to compare the performance of the decentralized algorithms presented in this paper. The centralized gradient-ascent algorithm typically converges to a solution that locally maximizes the total mutual information. As expected, best-response and gradient-play processes yield lower average per-link mutual information than the centralized gradient-ascent algorithm because of inefficiency of equilibria. Without stream control, the gap between the average per-link mutual information obtained by the centralized algorithm and the decentralized algorithms is up to 22%. The use of brute-force stream control coupled with best response process consistently improves the performance by improving efficiency of equilibria. For instance, the performance improvement provided by brute-force stream control for the cases where $\eta \in \{15, 20\}$ dB (and $\rho = 20$ dB) is approximately 15%. As seen in Figure 1, for other SNR levels, i.e., $\rho \in \{0, 5, 10, 15\}$ dB, the decentralized algorithms with and without stream control yield similar performance relative to that obtained by the centralized algorithm.

Average per-link mutual information with stream control via link negotiations: We consider a 3 link system with 4 antennas at each node to compare the performance improvement provided by stream control via link negotiations and brute-force approaches coupled with best response process. The simulation setup is otherwise very similar to the previous 2 link scenario. As seen in Figure 2, the average per-link mutual information obtained via link negotiations is almost identical to that obtained by brute-force stream control, with maximum 2% difference, for all (ρ, η) pairs.

Convergence characteristics of the decentralized algorithms: We observed that the rate of convergence of gradient-play process is very similar to that of best-response process, and that the use of inertia generally speeds up the convergence

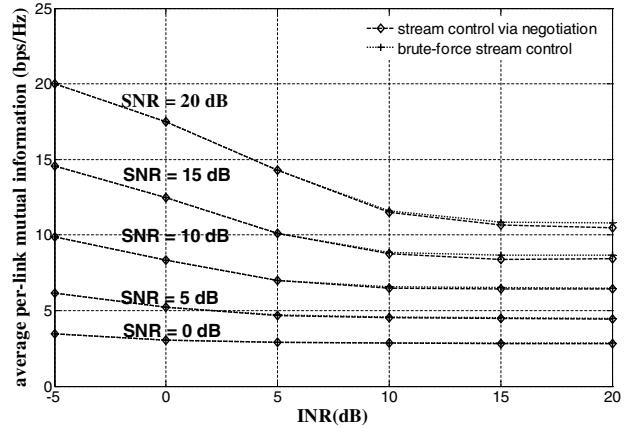


Fig. 2. Average per-link mutual information for $L = 3$ and $N_t = N_r = 4$.

by preventing the links from overreacting to each other's adjustments. However, we refrain from making definitive statements regarding convergence rate, since the parameters used in the algorithms as well as the channel parameters can greatly change the behavior of the decentralized algorithms.

Now, we discuss the rate of convergence of best-response process (with $\alpha_k(t) = 1/t^{0.2}$) on four different scenarios where $L \in \{2, 4, 6, 10\}$ and $N_r = N_t = 4$. For each scenario, we performed 1000 simulation runs where, in each run, the channel matrices are randomly generated as before. The number of iterations (averaged over 1000 runs) required for convergence were found to be 6.87, 7.58, 7.9, 8.55 for the scenarios $L = 2, L = 4, L = 6, L = 10$, respectively. In all 4000 simulation runs, best response process converged and the maximum number of iterations required for convergence was 155. Furthermore, in more than 98% of all simulation runs, best response process converged in less than 20 iterations.

We will also briefly discuss the convergence properties of link negotiations. Figure 3 shows the evolution of the negotiated limits on the number of independent data streams during link negotiations for a 5-link system with $N_t = N_r = 6$, when $\rho = 15$ dB and $\eta = 7.8$ dB. For the same setup, Figure 4 shows the evolution of the total mutual information during link negotiations. As seen in Figure 3 and 4, the negotiated limits settle at an optimal combination while the total mutual information monotonically increasing during the link negotiations. Note that the updating link switches to a different choice only if this would increase equilibrium efficiency. The link negotiations in this particular case took less than 100 iterations to settle at an optimal combination while the brute-force approach requires a search over 7^5 different rank combinations. This numerically verifies that stream control via link negotiations is a practical approach to improve efficiency of equilibria.

VIII. CONCLUSIONS

We have studied a problem of equilibrium efficiency improvement in MIMO interference networks within the framework of multi-player games. We established the existence of an equilibrium and obtained sufficient conditions for the

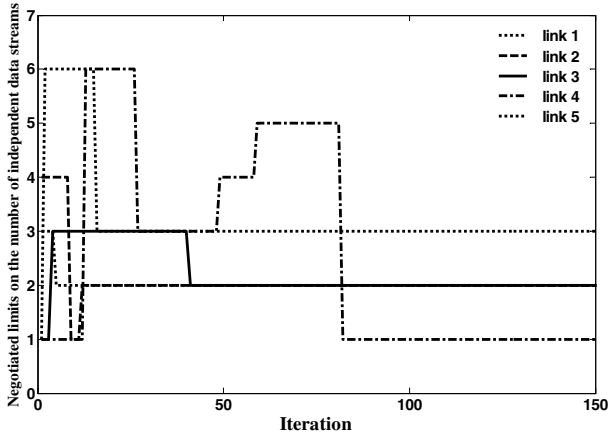


Fig. 3. Evolution of the negotiated limits on the number of independent data streams for $L = 5$, $N_t = N_r = 6$, $\rho = 15$ dB, $\eta = 7.8$ dB.

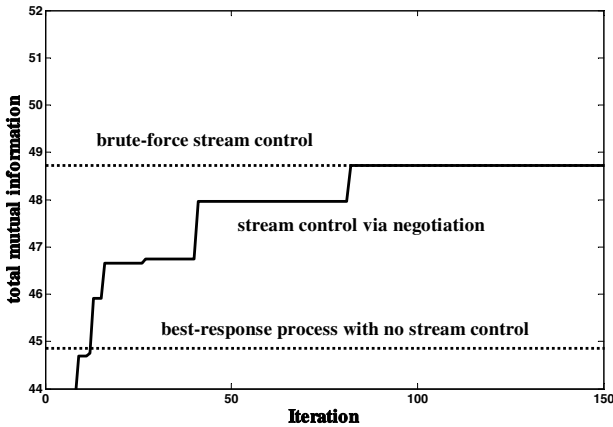


Fig. 4. Evolution of the total mutual information during the link negotiations for $L = 5$, $N_t = N_r = 6$, $\rho = 15$ dB, $\eta = 7.8$ dB.

uniqueness of equilibrium. We also established the convergence of best-response process (iterative water-filling) and gradient-play with inertia. For equilibrium efficiency improvement, we have introduced a stream control mechanism to impose limits on the number of independent data streams using a two-level hierarchical approach. We also introduced a learning algorithm that can be used by the links to negotiate such limits at the higher-level of our hierarchy with minimal information exchange and computational burden. We showed that the learning algorithm introduced in this paper leads to the best combination of limits on the number of independent data streams with arbitrarily high probability. Finally, our simulation results numerically verified that the decentralized stream control approach we presented substantially improves the efficiency of equilibrium solutions.

APPENDIX I
PROOF OF PROPOSITION 3.1

We first summarize some standard results on concave n -person games from [16]. Loosely speaking, in a concave n -person game, each user chooses its decision parameters

from a closed, bounded, convex subset of a Euclidian space, and receives a real-valued utility where each user’s utility function is (i) continuous in all user decision parameters, and (ii) concave in its own decision parameters when the decision parameters of all other users are fixed. Theorem 1 in [16] states that an equilibrium exists for every concave n -person game. Moreover, Theorem 2 in [16] establishes the uniqueness of the equilibrium when the user utility functions satisfy an additional technical condition called diagonal strict concavity. We now use these results from [16] to prove Proposition 3.1.

We will regard $\Phi^{N_t \times N_t}$ as a subset of the ν -dimensional Euclidian vector space \mathbb{R}^ν over the real numbers, where $\nu := 2(N_t)^2$. Accordingly, we will regard $U_k(\cdot)$ as a mapping from $\Phi := (\Phi^{N_t \times N_t})^L \subset \mathbb{R}^{\nu L}$ to \mathbb{R} .

The existence follows from Theorem 1 in [16] by noting that (i) $U_k(\mathbf{Q}_k, \mathbf{Q}_{-k})$ is continuous in \mathbf{Q} and concave in \mathbf{Q}_k for each fixed \mathbf{Q}_{-k} on its domain Φ , for all $k \in \{1, \dots, L\}$; see [1], and (ii) $\Phi^{N_t \times N_t}$ is convex, closed, and bounded.

To show the uniqueness, we will use the notation $\vec{\mathbf{Q}}_k$ to denote the vector in $\mathbb{R}^{\nu k}$ obtained by stacking the columns of the real and imaginary parts of $\mathbf{Q}_k \in \Phi^{N_t \times N_t}$. Moreover, for $\mathbf{Q} = \{\mathbf{Q}_1, \dots, \mathbf{Q}_L\} \in \Phi$, we will use the notation $\vec{\mathbf{Q}}$ to denote the vector obtained by stacking $\vec{\mathbf{Q}}_1, \dots, \vec{\mathbf{Q}}_L$, and $\vec{\mathbf{Q}}_{-k}$ is defined analogously. Now, let $\nabla_{\vec{\mathbf{Q}}_k} U_k(\mathbf{Q}) \in \mathbb{R}^\nu$ denote the gradient of U_k with respect to $\vec{\mathbf{Q}}_k$ at \mathbf{Q} , which is given as

$$\nabla_{\vec{\mathbf{Q}}_k} U_k(\mathbf{Q}) = \overrightarrow{\rho_k \mathbf{H}_{k,k}^\dagger (\mathbf{R}_k + \rho_k \mathbf{H}_{k,k} \mathbf{Q}_k \mathbf{H}_{k,k}^\dagger)^{-1} \mathbf{H}_{k,k}} \quad (8)$$

Let $\mathbf{G}_{k,l}(\mathbf{Q}) \in \mathbb{R}^{\nu \times \nu}$ be the Jacobian matrix of partial derivatives of $\nabla_{\vec{\mathbf{Q}}_k} U_k(\mathbf{Q})$ with respect to $\vec{\mathbf{Q}}_l$, and let

$$\mathbf{G}(\mathbf{Q}) := \begin{bmatrix} \mathbf{G}_{11}(\mathbf{Q}) & \dots & \mathbf{G}_{1L}(\mathbf{Q}) \\ \vdots & \ddots & \vdots \\ \mathbf{G}_{L1}(\mathbf{Q}) & \dots & \mathbf{G}_{LL}(\mathbf{Q}) \end{bmatrix}.$$

For any $N_t \times N_t$ Hermitian matrix \mathbf{Z} , the second-differential of U_k with respect to $\vec{\mathbf{Q}}_k$ at any $\mathbf{Q} \in \Phi$ is given as $(\vec{\mathbf{Z}})^T \mathbf{G}_{k,k}(\mathbf{Q}) \vec{\mathbf{Z}}$, which in turn equals

$$\left. \frac{\partial^2}{\partial t^2} U_k(\mathbf{Q}_k + t\mathbf{Z}, \mathbf{Q}_{-k}) \right|_{t=0} = -\text{trace} \left(\rho_k \mathbf{H}_{k,k}^\dagger (\mathbf{R}_k + \rho_k \mathbf{H}_{k,k} \mathbf{Q}_k \mathbf{H}_{k,k}^\dagger)^{-1} \mathbf{H}_{k,k} \mathbf{Z} \right)^2,$$

where $(\cdot)^T$ denotes the transpose. Therefore, $\mathbf{G}_{k,k}(\mathbf{Q})$ is negative definite with respect to Hermitian matrices at any $\mathbf{Q} \in \Phi$ provided that $\text{rank}(\rho_k \mathbf{H}_{k,k}) = N_t$.

Let η be the vector formed by stacking $\eta_{k,l}$ for all $k, l \in \{1, \dots, L\}$. In the case where $\eta = 0$, $\mathbf{G}(\mathbf{Q}) + \mathbf{G}^T(\mathbf{Q})$ is a block diagonal matrix with diagonal blocks $2\mathbf{G}_{k,k}(\mathbf{Q})$. Hence, $\mathbf{G}(\mathbf{Q})$ is negative-definite with respect to collections of Hermitian matrices at any $\mathbf{Q} \in \Phi$ when $\eta = 0$ and $\text{rank}(\rho_k \mathbf{H}_{k,k}) = N_t$ for all $k \in \{1, \dots, L\}$. Since \mathbf{G} is continuous as a function of η at $\eta = 0$, $\mathbf{G}(\mathbf{Q})$ must be negative definite with respect to collections of Hermitian matrices at any $\mathbf{Q} \in \Phi$ when η is sufficiently small and $\text{rank}(\rho_k \mathbf{H}_{k,k}) = N_t$ for all $k \in \{1, \dots, L\}$. This implies that, under the conditions of the second part of the proposition, for

some $\rho > 0$, we have

$$\begin{aligned} \left| \overrightarrow{(\mathbf{Q}^1)} - \overrightarrow{(\mathbf{Q}^0)} \right|^T [g(\mathbf{Q}^1) - g(\mathbf{Q}^0)] \\ \leq -\rho \left| \overrightarrow{(\mathbf{Q}^1)} - \overrightarrow{(\mathbf{Q}^0)} \right|^2, \end{aligned} \quad (9)$$

for all $\mathbf{Q}^0 \in \Phi$ and $\mathbf{Q}^1 \in \Phi$, where

$$g(\mathbf{Q}) := \begin{bmatrix} \nabla_{\overrightarrow{\mathbf{Q}}_1} U_1(\mathbf{Q}) \\ \vdots \\ \nabla_{\overrightarrow{\mathbf{Q}}_L} U_L(\mathbf{Q}) \end{bmatrix}, \quad (10)$$

and $|\cdot|$ denotes the standard Euclidian norm. This is sufficient for the diagonal strict concavity condition of Theorem 2 in [16] for the uniqueness of the equilibrium to be satisfied.

APPENDIX II

PROOF OF PROPOSITION 4.1

We use the well-known ODE (Ordinary Differential Equation) method [19] to associate best-response process with the continuous-time dynamics

$$\frac{d}{dt} \overrightarrow{\mathbf{Q}} = \overrightarrow{BR(\mathbf{Q})} - \overrightarrow{\mathbf{Q}}, \quad (11)$$

where the composite best response mapping $BR(\mathbf{Q})$ is defined as

$$BR(\mathbf{Q}) := \{BR_1(\mathbf{Q}_{-1}), \dots, BR_L(\mathbf{Q}_{-L})\}.$$

The standard results on the ODE method [19] implies that best response process is convergent provided that the above continuous-time dynamics are globally stable at the unique Nash equilibrium, which we will show below to prove Proposition 4.1.

Note that, under the hypothesis, $BR(\mathbf{Q})$ is single valued and $BR(\mathbf{Q}) : \Phi \mapsto \Phi$. Let $\mathbf{Q}^* = \{\mathbf{Q}_1^*, \dots, \mathbf{Q}_L^*\}$ denote the unique equilibrium of the game. We will first show that, for some $\kappa < 1$,

$$\left| \overrightarrow{BR(\mathbf{Q})} - \overrightarrow{BR(\mathbf{Q}^*)} \right| < \kappa \left| \overrightarrow{\mathbf{Q}} - \overrightarrow{\mathbf{Q}^*} \right|, \quad \forall \mathbf{Q} \in \Phi. \quad (12)$$

Recall (from the proof of Proposition 3.1) that the Hessian matrix $\mathbf{G}_{k,k}(\mathbf{Q})$ of U_k with respect to $\overrightarrow{\mathbf{Q}}_k$ is negative definite with respect to Hermitian matrices at any $\mathbf{Q} \in \Phi$. This implies (for instance, from Theorem 3.63 in [20]) that the following second order growth condition holds true for some $c > 0$: $\forall \mathbf{Q}_k \in \Phi^{N_t \times N_t}$,

$$U_k(\mathbf{Q}_k^*, \mathbf{Q}_{-k}^*) - U_k(\mathbf{Q}_k, \mathbf{Q}_{-k}^*) \geq c \left| \overrightarrow{\mathbf{Q}}_k^* - \overrightarrow{\mathbf{Q}}_k \right|^2. \quad (13)$$

Now, fix any $\mathbf{Q}^o \in \Phi$, and consider the function $\Delta U_k(\mathbf{Q}_k) : \Phi^{N_t \times N_t} \mapsto \mathbb{R}$ defined as $\Delta U_k(\mathbf{Q}_k) := U_k(\mathbf{Q}_k, \mathbf{Q}_{-k}^*) - U_k(\mathbf{Q}_k, \mathbf{Q}_{-k}^o)$. Clearly, ΔU_k is Lipschitz on $\Phi^{N_t \times N_t}$. To bound the Lipschitz constant of ΔU_k , we note that the gradient $\nabla_{\overrightarrow{\mathbf{Q}}_k} U_k(\mathbf{Q})$, given in (8), is Lipschitz on Φ . This implies that there exists a constant $\bar{k} > 0$ such that, for all $\mathbf{Q}_k \in \Phi^{N_t \times N_t}$,

$$\begin{aligned} \left| \frac{\partial \Delta U_k}{\partial \overrightarrow{\mathbf{Q}}_k} \right| &= \left| \nabla_{\overrightarrow{\mathbf{Q}}_k} U_k(\mathbf{Q}_k, \mathbf{Q}_{-k}^*) - \nabla_{\overrightarrow{\mathbf{Q}}_k} U_k(\mathbf{Q}_k, \mathbf{Q}_{-k}^o) \right| \\ &\leq \bar{k} \left| \overrightarrow{\mathbf{Q}}_{-k}^* - \overrightarrow{\mathbf{Q}}_{-k}^o \right|. \end{aligned}$$

Therefore, we can take the Lipschitz constant of ΔU_k as $\bar{k} \left| \overrightarrow{\mathbf{Q}}_{-k}^* - \overrightarrow{\mathbf{Q}}_{-k}^o \right|$. Now, from Proposition 4.32 in [20], the second order growth condition (13) and the Lipschitz continuity of ΔU_k imply

$$\left| \overrightarrow{BR_k(\mathbf{Q}_{-k}^*)} - \overrightarrow{BR_k(\mathbf{Q}_{-k}^o)} \right| \leq c^{-1} \bar{k} \left| \overrightarrow{\mathbf{Q}}_{-k}^* - \overrightarrow{\mathbf{Q}}_{-k}^o \right|.$$

Furthermore, for sufficiently small INR levels, 1) the Lipschitz constant $\bar{k} > 0$ can be taken as arbitrarily small due to the fact that the Jacobian matrix $\mathbf{G}_{k,l}$ of $\nabla_{\overrightarrow{\mathbf{Q}}_k} U_k$ with respect to $\overrightarrow{\mathbf{Q}}_l$ is arbitrarily small for $l \neq k$ at any $\mathbf{Q} \in \Phi$; see the proof of Proposition 3.1, and 2) the constant $c > 0$ can be taken as independent of the INR levels. This implies (12).

It is now straightforward to see that, because of (12), \mathbf{Q}^* is a globally exponentially stable equilibrium of (11). With this, the desired result follows from Theorem 6.9 in [19].

APPENDIX III

PROOF OF PROPOSITION 4.2

Using the ODE method [19], we associate gradient-play process with the continuous-time dynamics

$$\frac{d}{dt} \overrightarrow{\mathbf{Q}} = \Pi_{\Phi} \left[\overrightarrow{\mathbf{Q}} + \bar{\gamma} g(\mathbf{Q}) \right] - \overrightarrow{\mathbf{Q}},$$

where $g(\mathbf{Q})$ is as defined in (10). Since $g(\mathbf{Q})$ is strongly monotone due to (9), the unique equilibrium of the game is globally exponentially stable under the above continuous-time dynamics; see Theorem 5.2 in [21]. With this, the desired result follows from Theorem 6.9 in [19].

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