

# EE 650 Homework #8 due Wednesday, 11/19/08

## QUESTIONS

1. Given the LTI system

$$\dot{x} = \underbrace{\begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix}}_A x + \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix}}_B u$$
$$y = \underbrace{\begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix}}_C x,$$

- (i) Check observability using the observability matrix
  - (ii) Check observability using the PBH test
  - (iii) Check observability by obtaining the limiting observability gramian.
  - (iv) If the system is not completely observable, bring it to an observability canonical form.
  - (iv) Determine if the system is detectable.
2. Consider an LTI system  $\dot{x} = Ax$ ,  $y = Cx$  with some initial state  $x(0)$ .
- (i) Show that if  $(A, C)$  is completely observable, then

$$y(t) = 0, \text{ for all } t \geq 0 \Rightarrow x(t) = 0, \text{ for all } t \geq 0$$

- (ii) Show that if  $(A, C)$  is detectable, then

$$y(t) = 0, \text{ for all } t \geq 0 \Rightarrow \lim_{t \rightarrow \infty} x(t) = 0$$

3. Find an irreducible realization of the transfer function matrix

$$\hat{G}(s) = c(sI - A)^{-1}b + d$$

when

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 0.5 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad c = [1 \quad 2 \quad 2], \quad d = 0.$$

4. Find the degree of an irreducible realization of the transfer function matrix

$$H(s) = \begin{bmatrix} 0 & \frac{1}{s} \\ \frac{1}{s^2} & \frac{1}{s} \end{bmatrix}$$

and find an irreducible realization.