# EE 650 Homework #6 due Monday, 11/3/08

## QUESTIONS

1. Consider the linear system
$$\dot{x}(t) = \begin{bmatrix} -\alpha & 1 \\ -1 & -\alpha \end{bmatrix} x(t),$$
where $\alpha$ is a real scalar. For what values of $\alpha$ is the system
   (i) stable in the sense of Lyapunov,
   (ii) asymptotically stable,
   (iii) unstable ?

2. Consider the linear system
$$\begin{aligned} \dot{x}(t) &= \underbrace{\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u(t) \\ y(t) &= \underbrace{[-2 \;\; 2]}_{C} \; x(t). \end{aligned}$$
   (i) Is the system BIBO stable ?
   (ii) Is the system totally stable ?
   (ii) Find the stable and unstable subspaces?

3. For the LTI system described by
$$\dot{x}(t) = \underbrace{\begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}}_{A} x(t),$$
investigate asymptotic stability using the Lyapunov equation
$$A^T P + PA = -Q \quad \text{with } Q = I.$$
You can use the *lyap* function in MATLAB; however be careful with the transposes.

4. Given the system
$$\dot{x}(t) = \underbrace{\begin{bmatrix} -1 & 4 \\ 1 & -1 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{B} u(t),$$
   (i) Show that the open-loop system (i.e., the system with $u(t) = 0$ for all $t \geq 0$) is unstable.
   (ii) We now apply the control $\alpha(x_1 - x_2)$, where $\alpha$ is a scalar. For what value(s) of $\alpha$, if any, does the closed-loop system (the system with $u = \alpha(x_1 - x_2)$) is stable and has both eigenvalues at $-3$ ? (Recall that for a matrix $F$, we have $\sum_i \lambda_i = \text{trace}\{F\}$ and $\prod_i \lambda_i = \det F$, where $\lambda_i$ are the eigenvalues of $F$.)

5. Study the stability of the equilibrium state $x_e = 0$ for the nonlinear system
$$\begin{aligned} \dot{x}_1 &= -2x_1 + x_2 \\ \dot{x}_2 &= \sin x_1 - x_2 - x_2^3 \end{aligned}$$
using the function $V(x) = \frac{1}{2}||x||_2^2 = \frac{1}{2}(x_1^2 + x_2^2)$ as a candidate Lyapunov equation.

6. Consider the nonlinear system

$$\begin{aligned}\dot{x}_1 &= -x_1^2 + x_1 x_2 \\ \dot{x}_2 &= -2x_2^2 + x_2 - x_1 x_2 + 2.\end{aligned}$$

(i) Show that $x_e = [1 \;\; 1]^T$ is an equilibrium state.

(ii) Is $x_e$ the only equilibrium state ?

(iii) Is $x_e$ an (locally) asymptotically stable solution of the above nonlinear differential equations ?

7. You are given a LTV system:

$$\dot{x}(t) = \underbrace{\begin{bmatrix} -1 & e^{2t} \\ 0 & -1 \end{bmatrix}}_{A(t)} x(t).$$

(i) Obtain the eigenvalues of $A(t)$ for all $t \geq 0$. Based on this information, would you expect the system trajectories to be bounded for all $t \geq 0$.

(ii) Obtain a closed-form solution for $x(t)$ for all $t \geq 0$ if $x(0) = [0 \;\; 1]^T$.

(iii) Now, use the solution obtained in part (ii) to deduce whether the system states remain bounded for all $t \geq 0$. Any surprises?