QUESTIONS


2. Consider the following matrix $A$ and let $\bar{A}$ denote a Jordan canonical form representation of $A$. Find the distinct eigenvalues of $A$ and their algebraic and geometric multiplicities. How many Jordan blocks would $\bar{A}$ have for each distinct eigenvalue of $(A$ or $\bar{A})$? Find chains of generalized eigenvectors (with maximum number of linearly independent generalized eigenvectors in each chain) for each distinct eigenvalue. What is an appropriate similarity transformation that will give us $\bar{A}$? How does $\bar{A}$ look like? Hint: You can use the MATLAB commands `rank` and `null` (however, keep in mind that some MATLAB commands are not always reliable).

$$
A = \begin{bmatrix}
-1 & 0 & -1 & 1 & 1 & 3 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 2 & -1 & -1 & -6 & 0 \\
-2 & 0 & -1 & 2 & 1 & 3 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
-1 & -1 & 0 & 1 & 2 & 4 & 1
\end{bmatrix}
$$

3. (OPTIONAL FOR EXTRA CREDIT)

   (i) Show that any matrix norm satisfies

   $$
   ||A|| \geq \max |\lambda(A)|,
   $$

   where $\max |\lambda(A)|$ is the largest eigenvalue of $A$ in magnitude.

   (ii) Show that the eigenvalues of $A^*A$ are always nonnegative reals.

   (iii) Show that the matrix norm

   $$
   ||A||_2 = \sup_{x \neq 0} \frac{||Ax||_2}{||x||_2}
   $$

   satisfies

   $$
   ||A||_2 \geq \sqrt{\max \lambda(A^*A)}.
   $$

4. (OPTIONAL FOR EXTRA CREDIT) Consider the vector space $L_2([-1, 1], \mathbb{R})$ of measurable, real-valued, and square integrable functions defined on $[-1, 1]$ with the inner product

   $$
   < x, y > = \int_{-1}^{1} x(t)y(t)dt.
   $$

   Find an orthonormal set of vectors $\{q^1, q^2, q^3\}$ such that $\text{span}\{q^1, q^2, q^3\} = \text{span}\{1, t, t^2\}$.


6. Use the matrix exponential series to evaluate $e^{At}$ for

   (i) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

   (ii) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$