EE 650 Homework #3 due Monday, 9/29/08

QUESTIONS

1. Textbook Problem 3.13 with the matrix A_4.

2. Consider the following matrix A and let $\bar{A}$ denote a Jordan canonical form representation of A. Find the distinct eigenvalues of A and their algebraic and geometric multiplicities. How many Jordan blocks would $\bar{A}$ have for each distinct eigenvalue of (A or $\bar{A}$)? Find chains of generalized eigenvectors (with maximum number of linearly independent generalized eigenvectors in each chain) for each distinct eigenvalue. What is an appropriate similarity transformation that will give us $\bar{A}$? How does $\bar{A}$ look like? Hint: You can use the MATLAB commands *rank* and *null* (however, keep in mind that some MATLAB commands are not always reliable).

$$A = \begin{bmatrix} -1 & 0 & -1 & 1 & 1 & 3 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 2 & -1 & -1 & -6 & 0 \\ -2 & 0 & -1 & 2 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 1 & 2 & 4 & 1 \end{bmatrix}$$

3. (i) Show that any matrix norm satisfies

$$||A|| \geq \max|\lambda(A)|,$$

where $\max|\lambda(A)|$ is the largest eigenvalue of A in magnitude.

(ii) Show that the eigenvalues of A^*A are always nonnegative reals.

(iii) Show that the matrix norm

$$||A||_2 = \sup_{x\neq 0} \frac{||Ax||_2}{||x||_2}$$

satisfies

$$||A||_2 \geq \sqrt{\max \lambda(A^*A)}.$$

4. Consider the vector space $L_2([-1,1],\mathbb{R})$ of measurable, real-valued, and square integrable functions defined on $[-1,1]$ with the inner product

$$< x, y >= \int_{-1}^{1} x(t)y(t)dt.$$

Find an orthonormal set of vectors $\{q^1, q^2, q^3\}$ such that $\text{span}\{q^1, q^2, q^3\} = \text{span}\{1, t, t^2\}$.

5. Textbook Problem 3.21.

6. Use the matrix exponential series to evaluate e^{At} for

$$\text{(i)} \quad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad \text{(ii)} \quad A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

7. Textbook Problem 3.22.