QUESTIONS

1. Consider the differential equation
   \[ \ddot{x} + 3\dot{x} + 2x = 0. \]
   (i) Show that the set of solutions to this differential equation is a vector space over the reals.
   (ii) Find the dimension of this vector space.

2. Consider Figure 3.1 in the textbook.
   (i) Find the transformation \( P \) relating the two basis’ \( \{q_1, i_2\} \) and \( \{q_2, i_1\} \).
   (ii) Find the representation \( \beta_1 \) of \( x \) with respect to the basis \( \{q_1, i_2\} \).
   (iii) Using your results in (i)-(ii), find the representation \( \beta_2 \) of \( x \) with respect to the basis \( \{q_2, i_1\} \).

3. (i) Show that
   \[
   e_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
   \]
   are linearly independent.
   (ii) A linear operator \( L \) maps \((\mathbb{R}^3, \mathbb{R})\) to \((\mathbb{R}^3, \mathbb{R})\). It is known that
   \[
   L(e_1) = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \quad L(e_2) = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \quad L(e_3) = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}.
   \]
   Find \( A \), the matrix representation of \( L \), with respect to the basis \( \{e_i\}_{i=1}^3 \).

4. Textbook Problem 3.5.


7. Consider any \( m \times n \) real matrix \( A \) with \( \rho(A) = n \). Show that \( \det(A^T A) \neq 0 \) where \( A^T \) is the transpose of \( A \).

8. Diagonalize the matrices \( A_1, A_2, A_3 \) given in Textbook Problem 3.13 by appropriate similarity transformations.