EE452—Homework # 5

1. Consider the transfer function model $G_5(s)$ for the double-mass spring system in Figure A.8 of your textbook (page 697) with $u$ as the input and $y$ as the output. Set $m = M = k = b = 1$.

   (a) Draw an all-integrator block diagram for this system.
   (b) By labelling the output of each integrator as a state, obtain an state-space representation for this system.
   (c) Find a discrete-time state-space representation for the system below assuming that the sampling frequency is 1 Hz.
   (d) Find a discrete-time transfer function representation for the system below assuming that the sampling frequency is 1 Hz.

\[
\begin{array}{c}
\text{D/A (ZOH)} \\
\text{G}_5(s) \\
\text{A/D}
\end{array}
\]

2. Repeat Problem 1 with $G_5(s) = \frac{a-b}{(s-a)(s-b)}$. Do not assume particular values for the constants $a$ and $b$. (Hint: Expand $G_5(s)$ to partial fractions first.)

3. Transform the discrete-time state-space representation you obtained in Problem 2 to a different one.

4. Obtain the control and observer canonical forms for $\frac{2z^2+3z+1}{3z^3-2z^2-z-1}$.

5. (a) Consider the discrete-time state-space system you obtained in Problem 2 with $a = -1$, $b = -2$ and initial state vector $x_0 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. Find the response of this system due to a step input, i.e., $u_k = 1$, for all $k \geq 0$.
   (b) Verify your answer by solving the state-space equations recursively in MATLAB for $0 \leq k \leq 10$. Attach your code and a single plot where the solution you obtained in the first part and the numerical solution you obtained in MATLAB are shown.