

## CLASS 4

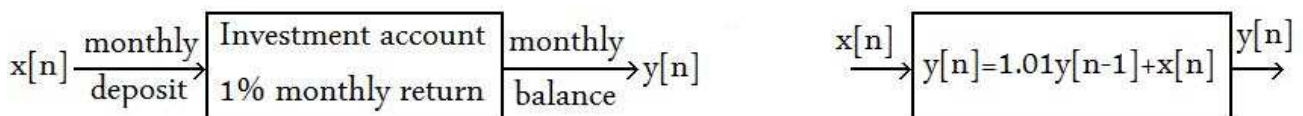
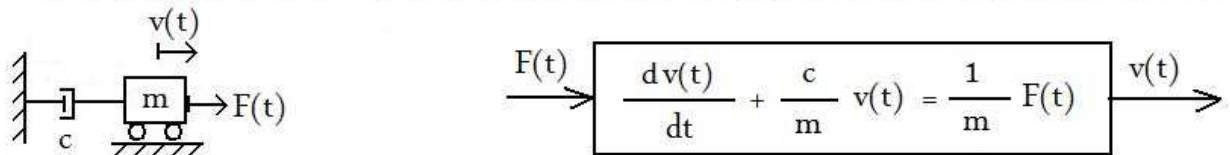
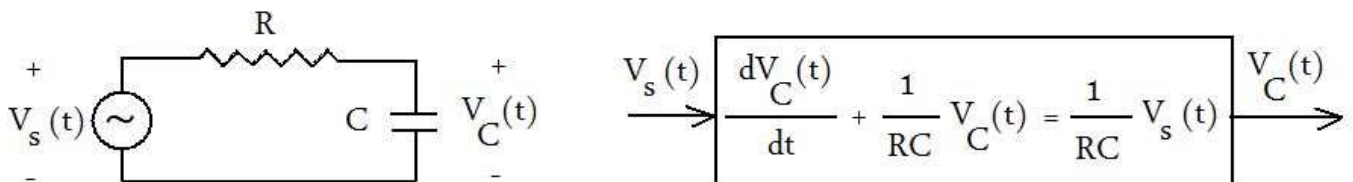
(Sections 1.5-1.6)

**Continuous-time and discrete-time systems**

- Physically, a system is an interconnection of components, devices, etc., such as a computer or an aircraft or a power plant.
- Conceptually, a *system* can be viewed as a black box which takes in an input signal  $x(t)$  (or  $x[n]$ ) and as a result generates an output signal  $y(t)$  (or  $y[n]$ ). A short-hand notation:  $x(t) \rightarrow y(t)$  or  $x[n] \rightarrow y[n]$ .
- A system is continuous-time (discrete-time) when its I/O signals are continuous-time (discrete-time).



- Examples:



Despite having different physical origins, these systems have similar mathematical representations.

- Hence, it makes sense to study a general mathematical representation that is common to many different applications.

## Basic System Properties

### Systems with and without memory:

- A system is called memoryless if the output at any time  $t$  (or  $n$ ) depends only on the input at time  $t$  (or  $n$ ); in other words, independent of the input at times before or after  $t$  (or  $n$ ).
- Examples of memoryless systems:

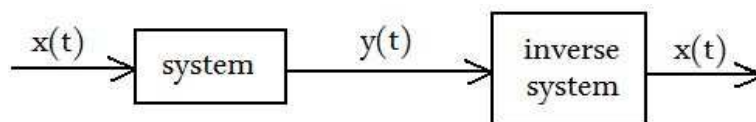
$$y(t) = Rx(t) \quad \text{or} \quad y[n] = (2x[n] - x^2[n])^2.$$

- Examples of systems with memory:

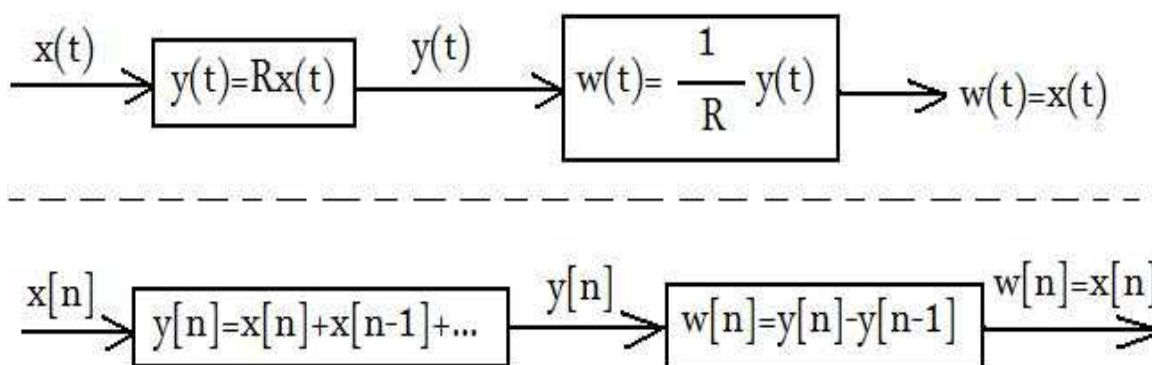
$$y(t) = \frac{1}{C} \int_{-\infty}^t x(\tau) d\tau \quad \text{or} \quad y[n] = x[n - 1].$$

### Invertibility and inverse systems:

- A system is called invertible if it produces distinct output signals for distinct input signals.
- If an invertible system produces the output  $y(t)$  for the input  $x(t)$ , then its inverse produces the output  $x(t)$  for the input  $y(t)$ :



- Examples of invertible systems: ( $R \neq 0$  below.)



- Examples of non-invertible systems:

$$y(t) = (x(t))^2 \quad \text{or} \quad y[n] = 0.$$

**Causality:**

- A system is called causal or non-anticipative if the output at any time  $t$  (or  $n$ ) depends only on the input at times  $t$  or before  $t$  (or  $n$  or before  $n$ ); in other words, independent of the input at times after  $t$  (or  $n$ ).
- All memoryless systems are causal.
- Physical systems where the time is the independent variable are causal.
- Non-causal systems may arise in applications where the independent variable is not the time such as in the image processing applications.
- Examples of causal systems:

$$y(t) = \frac{1}{C} \int_{-\infty}^t x(\tau) d\tau \quad \text{or} \quad y[n] = x[n - 1].$$

- Examples of non-causal systems:

$$y(t) = x(-t) \quad \text{or} \quad y[n] = \frac{1}{3}(x[n - 1] + x[n] + x[n + 1]).$$

### Stability:

- A system is called stable if it produces bounded outputs for all bounded inputs.

(A signal  $x(t)$  is bounded if, for some  $M < \infty$ ,  $|x(t)| \leq M$  for all  $t$ .)

- Stability in a physical system generally results from the presence of mechanisms that dissipate energy, such as the resistors in a circuit, friction in a mechanical system, etc.
- Examples of stable systems:
  - Consider the mass-damper example with  $c = m = 1$ :

$$\dot{v}(t) + v(t) = F(t).$$

If, for some  $M < \infty$ ,  $|F(t)| \leq M$  for all  $t$ , then

$$v(t) = \int_{-\infty}^t e^{-(t-\tau)} F(\tau) d\tau \quad \Rightarrow \quad |v(t)| \leq e^{-t} \int_{-\infty}^t e^{\tau} |F(\tau)| d\tau \leq M, \text{ for all } t.$$

- $y[n] = \frac{1}{3}(x[n-1] + x[n] + x[n+1]).$

- Examples of unstable systems:

- Pure integrator:

$$\dot{y}(t) = x(t).$$

- Investment account:

$$y[n] = 1.01y[n-1] + x[n].$$

### Time-invariance:

- A system is called time-invariant if the way it responds to inputs does not change over time:

$$x(t) \rightarrow y(t) \quad \Rightarrow \quad x(t - t_0) \rightarrow y(t - t_0), \quad \text{for any } t_0$$

$$x[n] \rightarrow y[n] \quad \Rightarrow \quad x[n - n_0] \rightarrow y[n - n_0], \quad \text{for any } n_0.$$

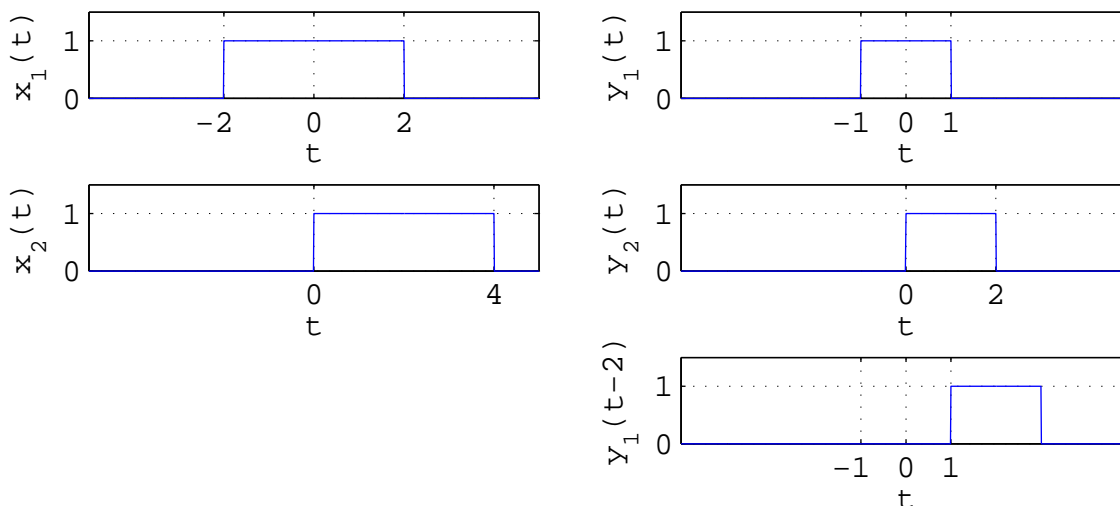
- Examples of time-invariant systems:

- The RC circuit considered earlier provided the values of  $R$  or  $C$  are constant.
- $y[n] = x[n - 1]$ .

- Examples of time-varying systems:

- The RC circuit considered earlier if the values of  $R$  or  $C$  change over time.
- $y(t) = x(2t)$  since

$$x(t) \rightarrow x(2t) \quad \text{but} \quad x(t - t_0) \rightarrow x(2t - t_0).$$



- Most physical systems are slowly time-varying due to aging, etc. Hence, they can be considered time-invariant for certain time periods in which its behavior does not change significantly.

### Linearity:

- A system is called linear if its I/O behavior satisfies the additivity and homogeneity properties:

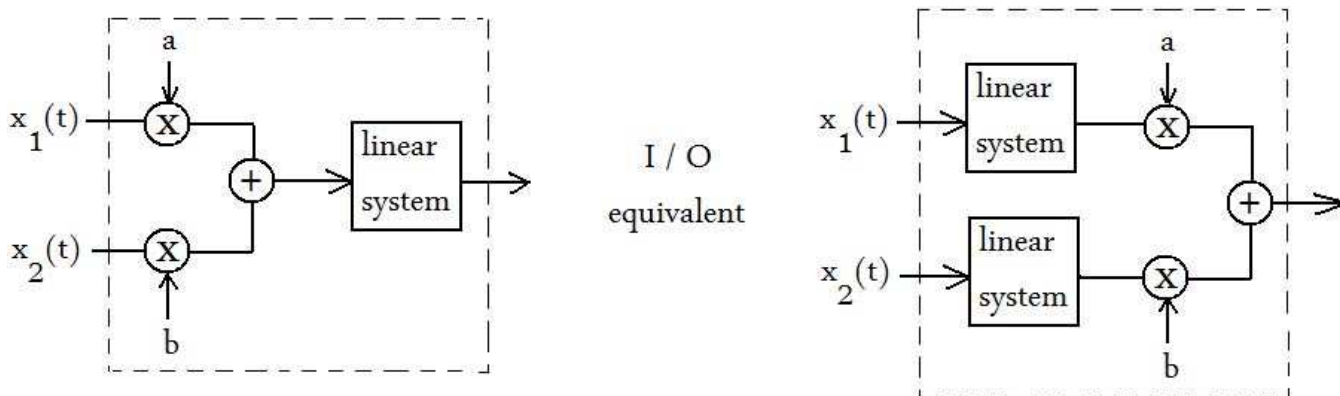
$$\left. \begin{array}{l} x_1(t) \rightarrow y_1(t) \\ x_2(t) \rightarrow y_2(t) \end{array} \right\} \Rightarrow \begin{array}{l} (x_1(t) + x_2(t)) \rightarrow (y_1(t) + y_2(t)) \\ (ax_1(t)) \rightarrow (ay_1(t)) \end{array}$$

for any complex constant  $a$ .

- Equivalently, a system is called linear if its I/O behavior satisfies the superposition property:

$$\left. \begin{array}{l} x_1(t) \rightarrow y_1(t) \\ x_2(t) \rightarrow y_2(t) \end{array} \right\} \Rightarrow (ax_1(t) + bx_2(t)) \rightarrow (ay_1(t) + by_2(t))$$

where any complex constants  $a$  and  $b$ .



- Definition of linearity is the same for discrete-time systems.
- For a linear system,

$$x(t) \equiv 0 \rightarrow y(t) \equiv 0 \quad \text{or} \quad x[n] \equiv 0 \rightarrow y[n] \equiv 0.$$

- Examples of linear systems:

- $y(t) = \int_{-\infty}^t x(\tau) d\tau$
- $y[n] = nx[n - 1]$ .

- Examples of non-linear systems:

- $y(t) = 2x(t) + 3$  since  $0 \rightarrow 3$ .
- $y[n] = \text{Re}(x[n])$  since  $1 \rightarrow 1$  but  $j.1 \rightarrow 0 \neq j.1$ .

- Many physical systems can be accurately modeled as linear system around an operating point.