EE 315 Notes

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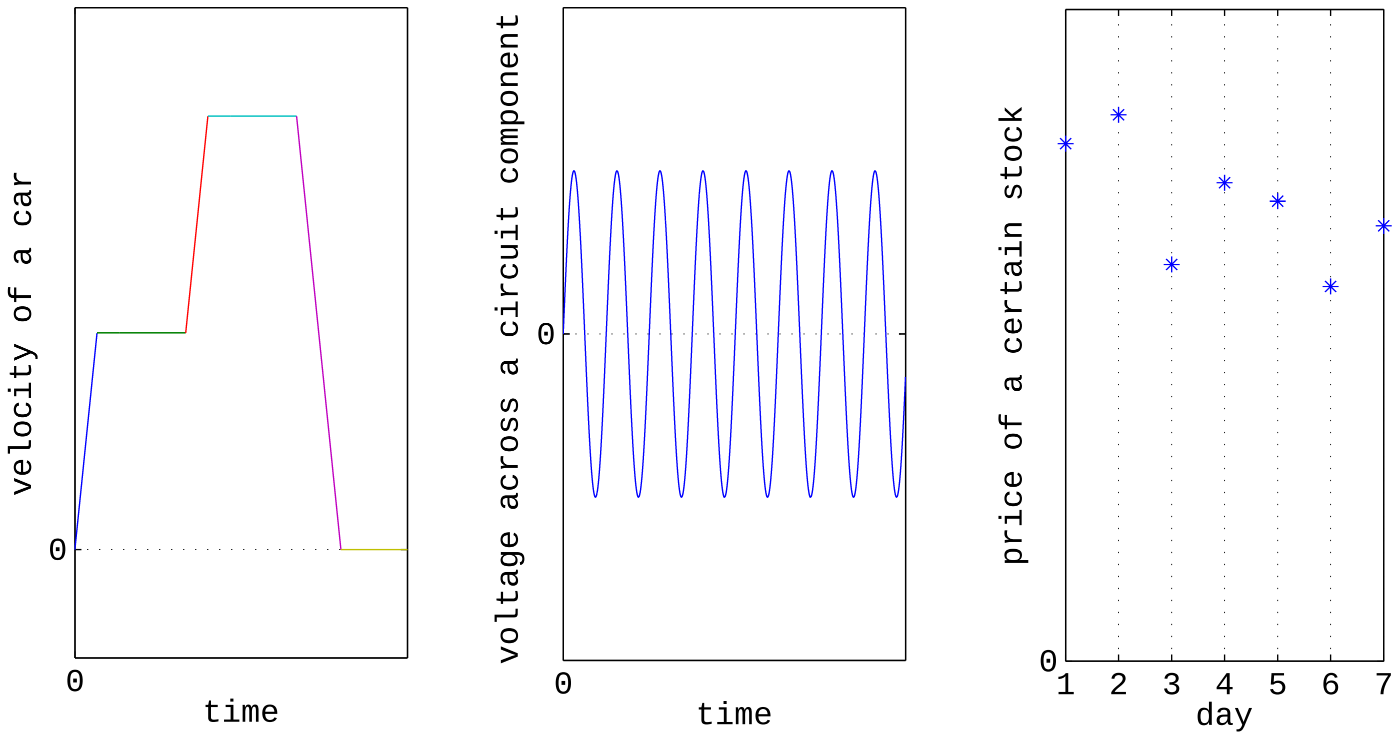
CLASS 1

(**Sections 1.0-1.2**)

What is a signal?

In this class, a signal is some function of time and it represents how some physical quantity changes over some window of time.

Examples:



- In general, a signal can be a function of some variable other than time, for example, location.
- Moreover, a signal can be a function of more than two variables, for example, brightness of a computer screen as a function of the x-y coordinates.
- However, we will focus on signals that are functions of only one variable and that is time.

Continuous-time signal:

- Defined at every point in time such as the velocity of a car or voltage across a circuit component in the previous figure.
- We use the notation $x(t)$.

Discrete-time signal:

- Defined only a set of discrete points in time such as the price of a certain stock in the previous figure.
- We use the notation $x[n]$.

Comments:

- In applications, some signals may naturally arise as continuous time signals such as the voltage across a circuit component measured by an analog device.
- In other applications, some signals may naturally arise as discrete time signals such as the daily change in the price of a certain stock.
- An important class of discrete-time signals arise from sampling a continuous-time signal, that is, by recording the signal value at certain discrete time instances and throwing away the rest of the signal. This is important because in many modern applications signals are processed by digital computers that require discrete-time data. For example, in many modern automatic control systems, measurements are continuous-time signals, but they are first converted to discrete-time and then fed to a digital microprocessor that computes the necessary control signal which is also discrete-time.

Signal energy and power:

Energy and average power of a continuous-time signal $x(t)$ over a finite interval $t_1 \le t \le t_2$:

$$\text{Energy} : E_{[t_1,t_2]} = \int_{t_1}^{t_2} |x(t)|^2 dt, \qquad \text{Average Power} : P_{[t_1,t_2]} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} |x(t)|^2 dt.$$

Energy and average power of a continuous-time signal $x(t)$ over the infinite interval $-\infty < t < \infty$:

$$\text{Energy} : E_{(-\infty,\infty)} = \lim_{T\to\infty}\int_{-T}^{T} |x(t)|^2 dt, \qquad \text{Average Power} : P_{(-\infty,\infty)} = \frac{1}{2T}\lim_{T\to\infty}\int_{-T}^{T} |x(t)|^2 dt.$$

Energy and average power of a discrete-time signal $x[n]$ over a finite interval $n_1 \le n \le n_2$:

$$\text{Energy} : E_{[n_1,n_2]} = \sum_{n=n_1}^{n_2} |x[n]|^2, \qquad \text{Average Power} : P_{[n_1,n_2]} = \frac{1}{n_2 - n_1}\sum_{n=n_1}^{n_2} |x[n]|^2.$$

Energy and average power of a discrete-time signal $x[n]$ over the infinite interval $-\infty < n < \infty$:

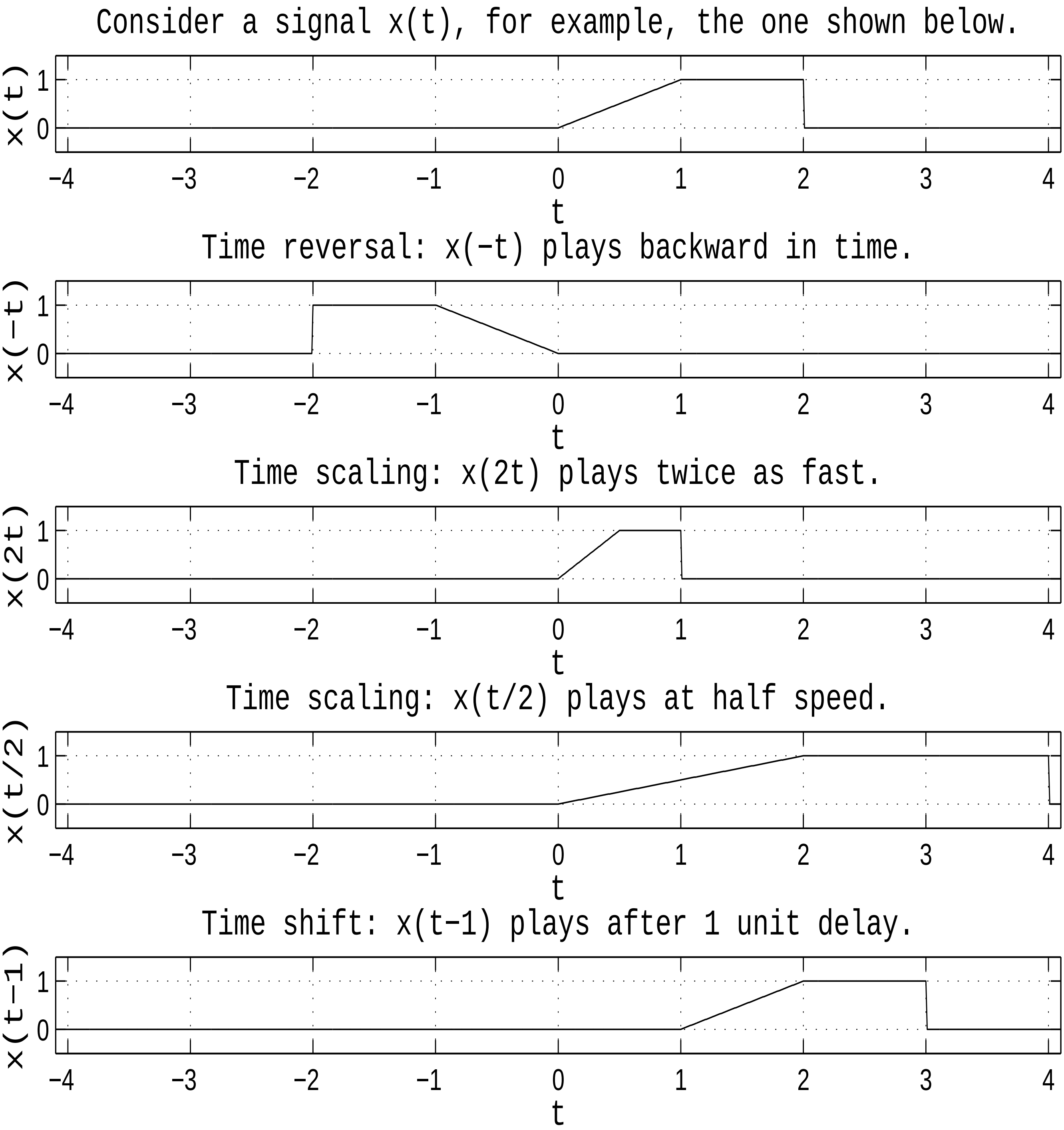
$$\text{Energy} : E_{(-\infty,\infty)} = \lim_{N\to\infty}\sum_{n=-N}^{N} |x[n]|^2, \qquad \text{Average Power} : P_{(-\infty,\infty)} = \frac{1}{2N+1}\lim_{N\to\infty}\sum_{n=-N}^{N} |x[n]|^2.$$

- Energy or power of a signal may not always be related to physical energy. Nevertheless, this terminology is still useful.

Three important class of signals defined over $(-\infty,\infty)$:

1) Those with finite energy, i.e., $E_{(-\infty,\infty)} < \infty$. These signals must have zero average power, i.e., $P_{(-\infty,\infty)} = 0$. For example, $x(t) = e^{-|t|}$ or $x[n] = e^{-|n|}$.
2) Those with finite power, i.e., $P_{(-\infty,\infty)} < \infty$. If $P_{(-\infty,\infty)} > 0$, then these signals must have infinite energy, i.e., $E_{(-\infty,\infty)} = \infty$. For example, $x(t) = \sin(t)$ or $x[n] = \sin(n)$.
3) Those with infinite power and infinite energy. For example, $x(t) = t$ or $x[n] = n$.

Transformations of time:

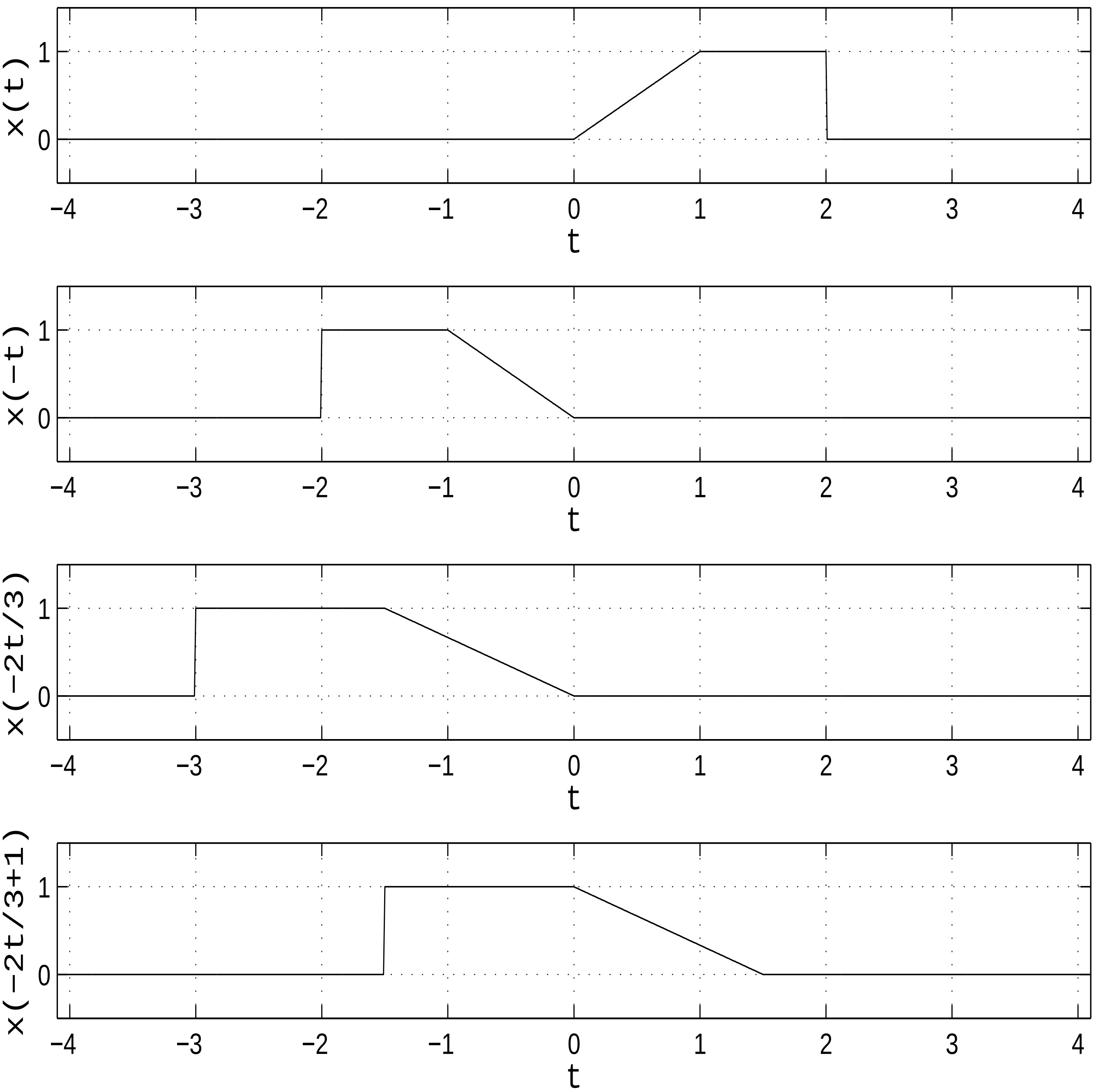


- Transformation of time for discrete-time signals are analogous.

Example: Obtain $x(-2t/3+1)$.

- One solution:

$$\begin{aligned} \text{Time reversal} &: \quad y(t) := x(-t) \\ \text{Time scaling by } 2/3 &: \quad z(t) := y(2t/3) = x(-2t/3) \\ \text{Time delay by } 3/2 &: \quad w(t) := z(t-3/2) = y(2t/3-1) = x(-2t/3+1) \end{aligned}$$

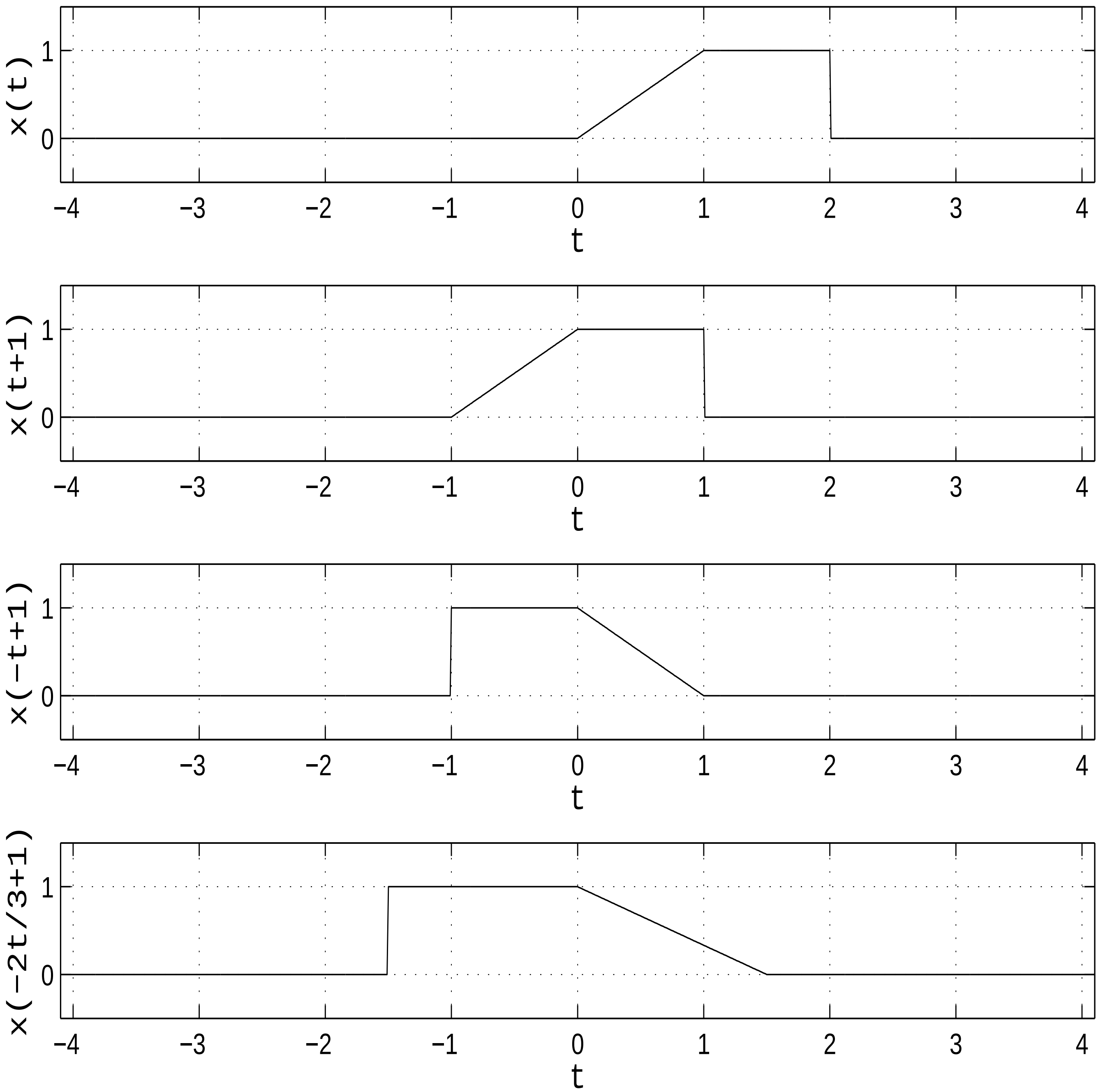


- Another solution:

$$\text{Time advancement by } 1 \quad : \quad y(t) := x(t+1)$$

$$\text{Time reversal} \quad : \quad z(t) := y(-t) = x(-t+1)$$

$$\text{Time scaling by } 2/3 \quad : \quad w(t) := z(2t/3) = y(-2t/3) = x(-2t/3+1)$$



Periodic signals:

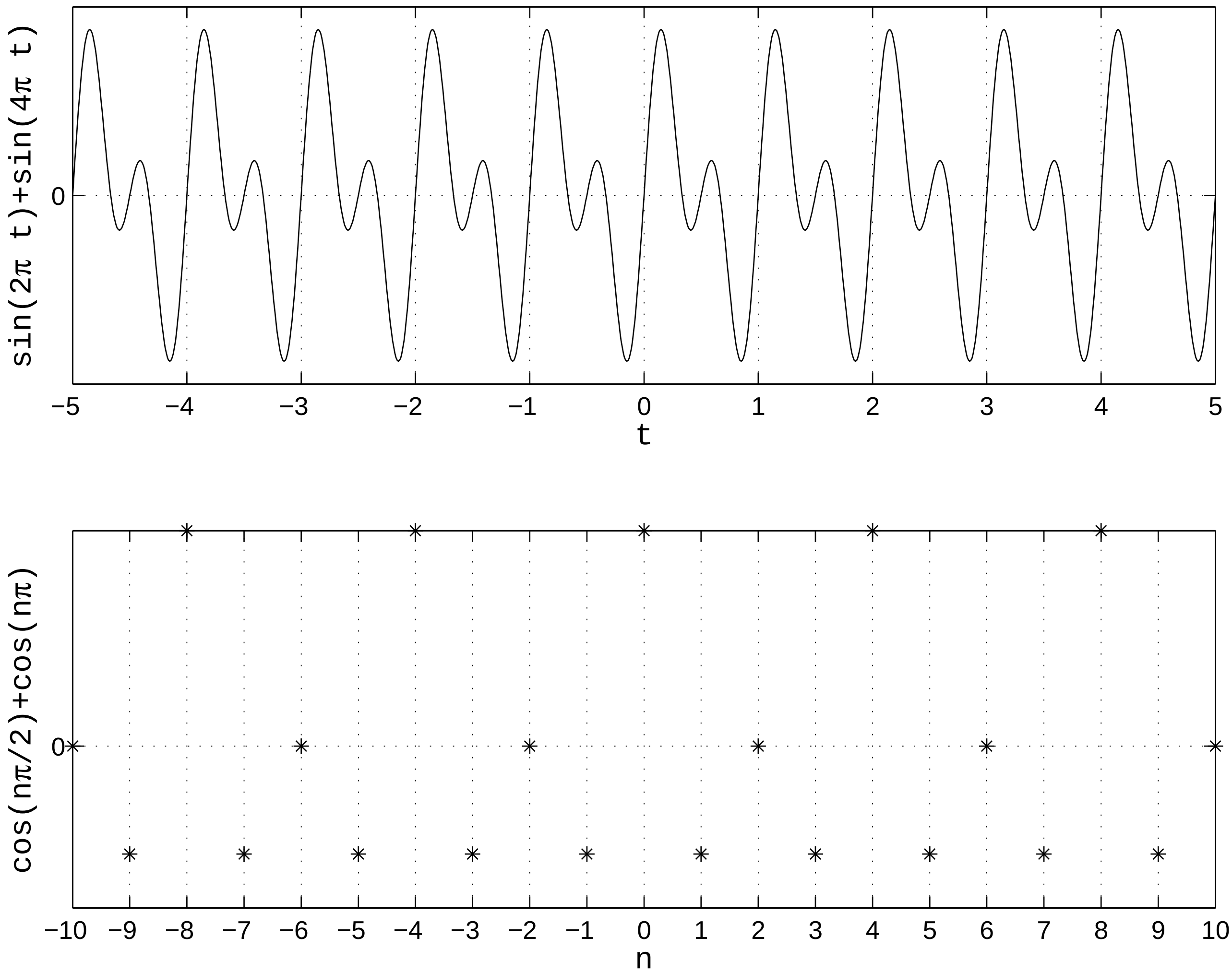
$$x(t) \text{ is periodic with period } T > 0 \quad \text{if} \quad x(t+T) = x(t) \text{ for all } t.$$

- The smallest T such that $x(t)$ has this property is called the fundamental period of $x(t)$.

$$x[n] \text{ is periodic with period } N > 0 \quad \text{if} \quad x[n+N] = x[n] \text{ for all } n.$$

- The smallest N such that $x[n]$ has this property is called the fundamental period of $x[n]$.

Examples:



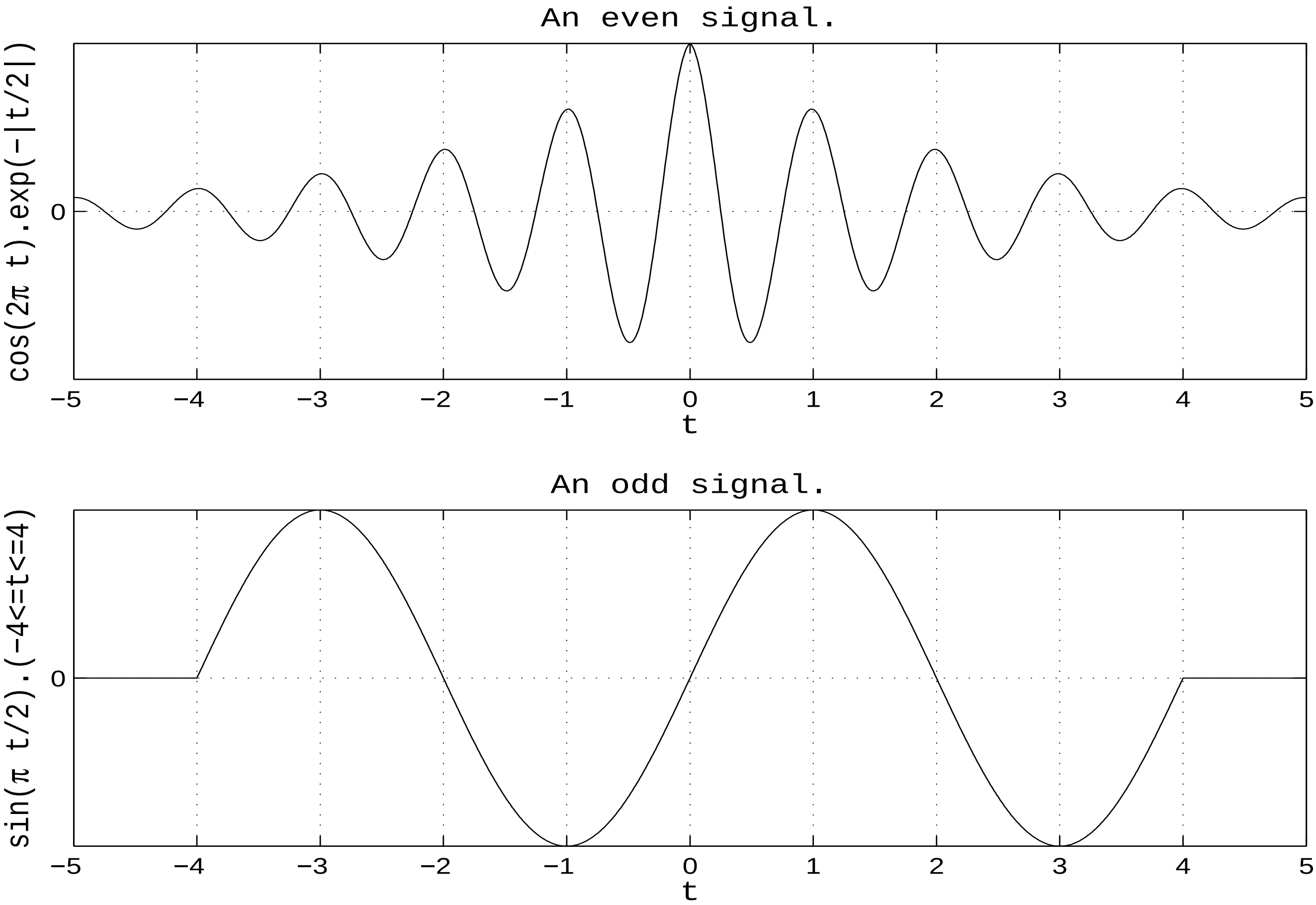
- Continuous-time signal above is periodic with fundamental period $T = 1$.
- Discrete-time signal above is periodic with fundamental period $N = 4$.

Even and odd signals:

$$x(t) \text{ is even} \quad \text{if} \quad x(t) = x(-t) \text{ for all } t.$$

$$x(t) \text{ is odd} \quad \text{if} \quad x(t) = -x(-t) \text{ for all } t.$$

Examples:



- For an odd signal $x(t)$, we always have $x(0) = -x(0) = 0$.
- Every real valued signal $x(t)$ can be written as the sum of an even signal and odd signal:

$$x(t) = x_e(t) + x_o(t)$$

where

$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad \text{and} \qquad x_o(t) = \frac{x(t) - x(-t)}{2}.$$

- $x_e(t)$ and $x_o(t)$ are referred to as even and odd parts of $x(t)$, respectively.
- Evenness and oddness and the related concepts are analogously introduced for discrete-time signals.