Name: ________________________________

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These two parts are independent.

(a) A discrete-time LTI system has the impulse response \( h[n] = \frac{2}{3} u[n - 1] \).

(i) (3) Is this system stable? Causal? Justify your answer.

(ii) (6) Find the output \( y[n] \) when the input is

\[
x[n] = \begin{cases} 
1 & \text{for } n = 1, 2 \\
0 & \text{otherwise}
\end{cases}
\]

(b) (6) The periodic discrete-time signal \( x[n] \) has period 4.

\[
x[n] = \begin{cases} 
1 & \text{for } n = 1 \\
-1 & \text{for } n = 3 \\
0 & \text{for } n = 0, 2
\end{cases}
\]

Find the Fourier series coefficients and sketch their magnitudes.
Consider the signal $x(t)$ with Fourier transform

$$X(j\omega) = \begin{cases} 
1 & \text{for } |\omega| \leq 20\pi \\
0 & \text{otherwise}
\end{cases}$$

For the following signals, find a mathematical expression for the Fourier transform and plot its magnitude.

(a) $(6)\ y(t) = 2x(t)\cos^2(15\pi t)$
(b) $(6)\ z(t) = x(t) - x(t - 100)$
(c) $(6)\ q(t) = x(-t)$
Let $x(t) = \text{sinc}(t/2)$ where $\text{sinc}(\theta) = \frac{\sin(\pi \theta)}{\pi \theta}$.

(a) (9) Determine and sketch the Fourier transform of $x(t) * x^2(t)$.

(b) (6) Using properties of Fourier transforms, evaluate

$$\int_{-\infty}^{\infty} x^4(t)dt.$$ 

(c) (6) Determine and sketch the Fourier transform of

$$x(t) \sum_{k=-\infty}^{\infty} \delta(t - k/3).$$
(4) Consider a low pass signal with bandwidth 6000Hz.

(a) (3) Determine the minimum sampling frequency such that it can be reconstructed perfectly.

(b) (3) Determine the minimum sampling frequency if a guardband of 2000Hz is required. The guardband is the extra spacing between the spectra of $X(j\omega)$ in the spectra of the sampled signal $X_s(j\omega)$ to further reduce the chances of overlapping spectra.

(c) (6) Determine the minimum sampling frequency for perfect reconstruction using a filter with the following frequency response.

$$H(j\omega) = \begin{cases} K & \text{for } |f| \leq 7000 \\ K - K \frac{|f| - 7000}{3000} & \text{for } 7000 \leq |f| \leq 10000 \\ 0 & \text{otherwise} \end{cases}$$

Graphically, show your reasoning for parts (b) and (c).
Consider an ideal bandpass channel with a frequency response

\[ H(j\omega) = \begin{cases} 
1 & \text{for } 2\pi 100,000 \leq |\omega| \leq 2\pi 115,000 \\
0 & \text{otherwise}
\end{cases} \]

This channel has a passband between 100kHz and 115kHz. Consider two low-pass bandlimited signals \( x_1(t) \) and \( x_2(t) \). Suppose \( |X_1(j\omega)| = 0 \) for \( |\omega| > 2\pi 1000 \) and \( |X_2(j\omega)| = 0 \) for \( |\omega| > 2\pi 10,000 \). Both signals are to be transmitted simultaneously over the channel.

(a) (6) (i) Sketch the spectra for \( X_1(j\omega) \) and \( X_2(j\omega) \) (draw the first as any triangular shape and the second as anything but different). (ii) Propose a modulation scheme that achieves the requirements and show graphically that it is physically realizable.

(b) (9) Sketch a complete realization of the scheme (transmitter and receiver).
(6) (9) Prove the properties listed below which were taken from the table of Fourier transform properties.
(a) If \( x(t) \) is real and even, then \( X(j\omega) \) is real and even.
(b) If \( x(t) \) is real and odd, then \( X(j\omega) \) is imaginary and odd.