

Problem Set 6

Ch 12. Exercises, Problems and Complements:

3. (pp. 280-281)

(a) Betsy's B forecast, because it has a much smaller error variance.

(b) Substituting in to the equation on page 265,

$$w^* = \frac{\hat{\sigma}_{BB}^2 - \hat{\sigma}_{AB}^2}{\hat{\sigma}_{BB}^2 + \hat{\sigma}_{AA}^2 - 2\hat{\sigma}_{AB}^2} = \frac{92.16 - 0.2}{92.16 + 153.76 - 2(.2)} = \frac{91.96}{245.52} = .375$$

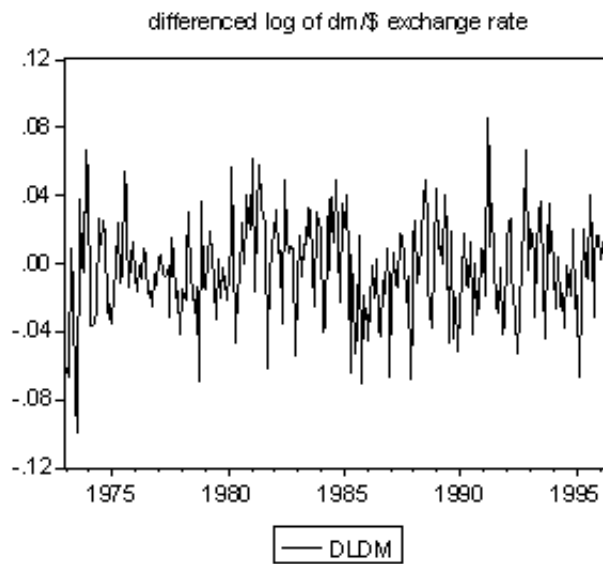
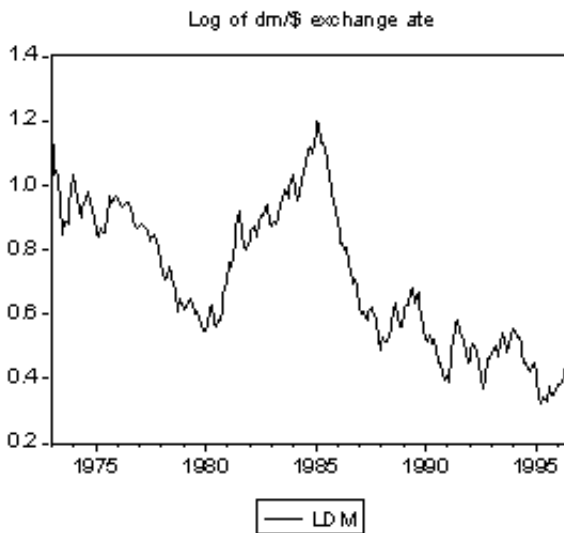
Note that it gives a heavier weight to the more accurate forecast.

(c) As long as the two forecasts are not perfectly correlated (they are not in this case), then improvement is guaranteed in principle, although in practice it might not because of error estimating the variances and covariance. Even in practice, however, combination often produces improvements.

Ch 13. Exercises, Problems and Complements:

1. (p. 320) part (a) only. [Hint: you will need the data in the Diebold data set for Ch. 13, PC13_01_EXCH.DAT.]

Model and forecast the deutsche mark/dollar exchange rate in parallel with the analysis in the text, and discuss the results in detail.



As for the yen/dollar rate, the log level of the dm/\$ rate has a strong AR character and may have a unit root, while the change in the log looks close to white noise. You can graph the correlograms and partial

autocorrelation functions to confirm this.

Including a constant and time trend, we try various ARMA models of the log level:

AIC:

		MA order			
		0	1	2	3
AR order	0	-0.8836256	-2.0952064	-2.8496775	-3.3597980
	1	-4.2571718	-4.3599992	-4.3538387	-4.3530899
	2	-4.3562960	-4.3532225	-4.3462100	-4.3469267
	3	-4.3496197	-4.3518540	-4.3450676	-4.3894823

SIC:

		MA order			
		0	1	2	3
AR order	0	-0.8563113	-2.0542349	-2.7950489	-3.2915121
	1	-4.2162003	-4.3053706	-4.2855528	-4.2711469
	2	-4.3016673	-4.2849367	-4.2642670	-4.2513265
	3	-4.2813339	-4.2699110	-4.2494675	-4.2802250

The more parsimonious model by SIC is the ARMA(1,1). The regression is:

Dependent Variable: LDM

Method: Least Squares

Date: 05/06/03 Time: 21:13

Sample: 1973:04 1994:12

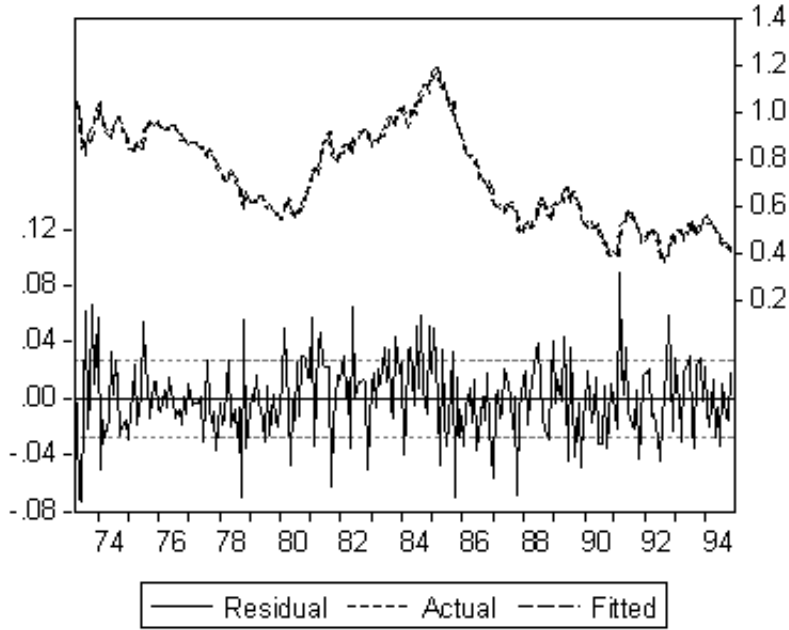
Included observations: 261

Convergence achieved after 6 iterations

Backcast: 1973:03

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.942316	0.200825	4.692226	0.0000
TIME	-0.001707	0.001085	-1.573903	0.1167
AR(1)	0.972604	0.014650	66.39013	0.0000
MA(1)	0.334648	0.059600	5.614902	0.0000
R-squared	0.982591	Mean dependent var		0.732844
Adjusted R-squared	0.982388	S.D. dependent var		0.204547
S.E. of regression	0.027146	Akaike info criterion		-4.359999
Sum squared resid	0.189379	Schwarz criterion		-4.305371
Log likelihood	572.9799	F-statistic		4835.179
Durbin-Watson stat	1.973930	Prob(F-statistic)		0.000000
Inverted AR Roots	.97			
Inverted MA Roots	-.33			

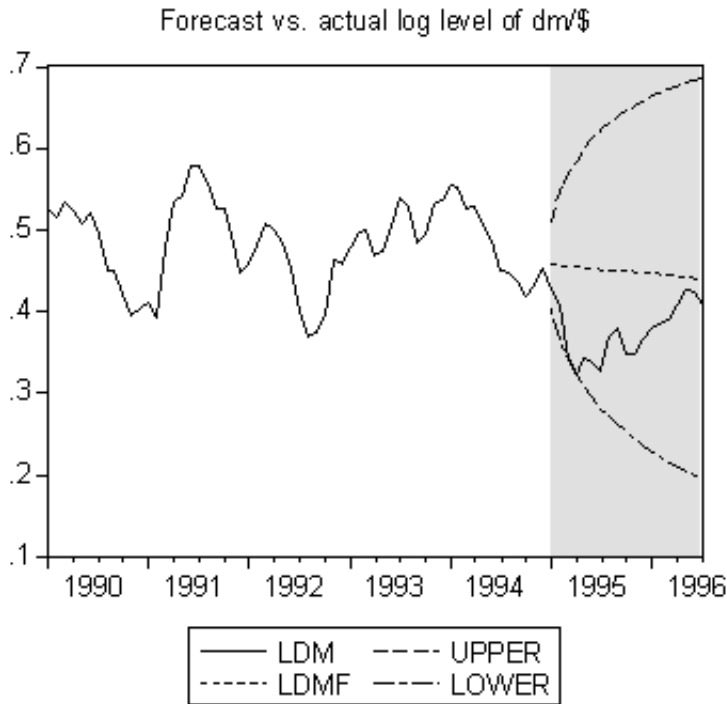
Actual, predicted and residuals from ARMA(1,1) of log level dm/\$



Date: 05/06/03 Time: 21:14
 Sample: 1973:04 1994:12
 Included observations: 261
 Q-statistic probabilities
 adjusted for 2 ARMA
 term(s)

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
. .	. .	1	0.012	0.012	0.0389	
. .	. .	2	0.058	0.058	0.9437	
. .	. .	3	0.049	0.048	1.5815	0.209
. .	. .	4	-0.042	-0.047	2.0521	0.358
. .	. .	5	-0.027	-0.032	2.2468	0.523
. .	. .	6	-0.032	-0.029	2.5303	0.639
. .	. .	7	0.037	0.046	2.8995	0.715
. .	. .	8	0.053	0.058	3.6687	0.721
. .	. .	9	0.053	0.049	4.4405	0.728
. *	. *	10	0.125	0.111	8.6849	0.370
. .	. .	11	-0.012	-0.024	8.7243	0.463
. .	. .	12	0.007	-0.007	8.7362	0.557

The model fits well in sample, and its residuals are not distinguishable from white noise. But note the very large inverted AR(1) root. Here is the out-of-sample forecast through 1996:7:



Again, it is similar to what we saw with the yen/dollar: doesn't track the short-run volatility but actuals are still within the 95% confidence interval.

Now, let's test for a unit root in ldm:

ADF Test Statistic	-2.160978	1% Critical Value*	-3.9966
		5% Critical Value	-3.4284
		10% Critical Value	-3.1373

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LDM)
 Method: Least Squares
 Date: 05/06/03 Time: 21:27
 Sample: 1973:05 1994:12
 Included observations: 260

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LDM(-1)	-0.024011	0.011111	-2.160978	0.0316
D(LDM(-1))	0.336440	0.062080	5.419441	0.0000
D(LDM(-2))	-0.048040	0.064949	-0.739663	0.4602
D(LDM(-3))	0.073432	0.061693	1.190270	0.2351
C	0.021434	0.011271	1.901675	0.0583
@TREND(1973:05)	-4.03E-05	2.99E-05	-1.348268	0.1788
R-squared	0.119184	Mean dependent var		-0.002271
Adjusted R-squared	0.101845	S.D. dependent var		0.028695
S.E. of regression	0.027195	Akaike info criterion		-4.348794
Sum squared resid	0.187844	Schwarz criterion		-4.266625
Log likelihood	571.3433	F-statistic		6.873812
Durbin-Watson stat	1.988294	Prob(F-statistic)		0.000005

As for the yen/dollar, we cannot reject that the dm/\$ rate has a unit root. We will proceed under the assumption that there is a unit root and estimate an ARMA model in differenced logs. Here are the AIC and SIC values:

AIC:

		MA order			
		0	1	2	3
AR order	0	-0.3372439	-4.3581642	-4.3511733	-4.3500634
	1	-4.3561629	-4.3515890	-4.3495500	-4.3423835
	2	-4.3501051	-4.3475792	-4.3402385	-4.3811847
	3	-4.3459192	-4.3402230	-4.3866674	-4.3818860

SIC:

		MA order			
		0	1	2	3
AR order	0	-0.3235490	-4.3307743	-4.3100885	-4.2952837
	1	-4.3287730	-4.3105042	-4.2947703	-4.2739089
	2	-4.3090204	-4.2927995	-4.2717639	-4.2990151
	3	-4.2911395	-4.2717484	-4.3044978	-4.2860215

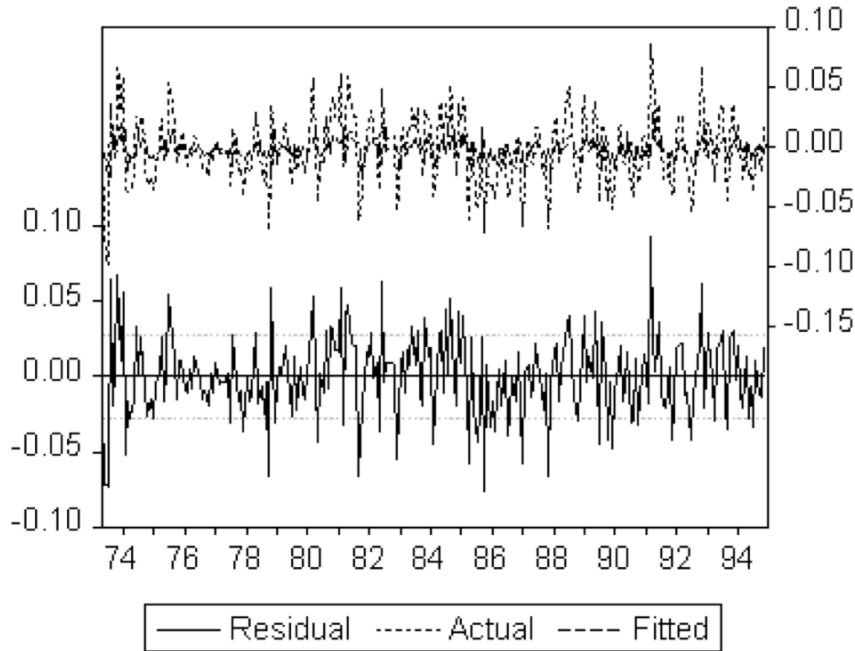
The more parsimonious model by SIC is the MA(1). The regression is:

Dependent Variable: DLOG(DM)
 Method: Least Squares
 Date: 05/06/03 Time: 21:42
 Sample: 1973:05 1994:12
 Included observations: 260
 Convergence achieved after 4 iterations
 Backcast: 1973:04

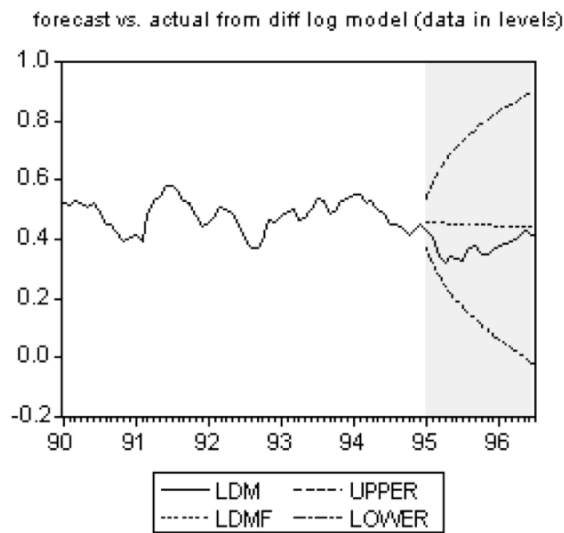
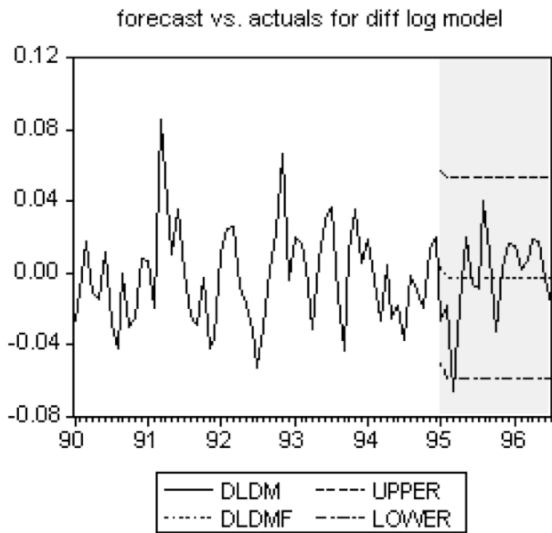
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.002251	0.002240	-1.005063	0.3158
MA(1)	0.325391	0.058882	5.526138	0.0000
R-squared	0.100132	Mean dependent var		-0.002271
Adjusted R-squared	0.096644	S.D. dependent var		0.028695
S.E. of regression	0.027273	Akaike info criterion		-4.358164
Sum squared resid	0.191907	Schwarz criterion		-4.330774
Log likelihood	568.5613	F-statistic		28.70879
Durbin-Watson stat	1.980549	Prob(F-statistic)		0.000000
Inverted MA Roots	-0.33			

As expected, the differenced log model does not fit as well as a levels model would. The negative constant is capturing the downward trend over time.

Actual, fitted and residuals for differenced log of dm/\$



Residual correlogram and normality tests (not shown) do not show any departures from white noise errors. Here are the forecasts first of the modeled differenced log (growth rate) series and then of the implied levels:



Notice that the confidence intervals are wider than for the levels model above, but the forecast looks very similar.

5. (p. 321) parts (a)-(c) only. *Automatic forecasting software...*

(a) *What do you think are the benefits of such software?* Automatic forecasting software may permit you to get reasonable forecasts for a large number of different series at low cost. It may also insulate you

from the forecast results, so that your (possibly incorrect) beliefs do not color the model identification process.

(b) *What do you think are the costs?* The results are only as good as the “expert system” itself, and there is no room for “extra-model” judgment that may improve the forecasts. There is actually quite a lot of evidence that the experienced opinions of forecasters add value to purely mechanical forecasts.

(c) *When do you think it would be most useful?* I guess I answered this in (a): when large numbers of series have to forecast with limited resources.

9. (p. 323) *Cointegration occurs when a linear combination of two I(1) series is stationary, I(0).*

(a) *Consider the bivariate system:*

$$\begin{aligned}x_t &= x_{t-1} + v_t, & v_t &\sim WN(0, \sigma^2) \\y_t &= x_t + \varepsilon_t, & \varepsilon_t &\sim WN(0, \sigma^2)\end{aligned}$$

To show that the series are I(1), we have to show that their first differences are stationary. This is trivial for x_t :

$$\begin{aligned}x_t &= x_{t-1} + v_t, & v_t &\sim WN(0, \sigma^2) \\x_t - x_{t-1} &= v_t\end{aligned}$$

For y_t , substitute for x_t :

$$\begin{aligned}y_t &= (x_{t-1} + v_t) + \varepsilon_t, & \varepsilon_t, v_t &\sim WN(0, \sigma^2) \\ \Delta y_t &= y_t - y_{t-1} = [x_{t-1} + v_t + \varepsilon_t] - [x_{t-2} + v_{t-1} + \varepsilon_{t-1}] \\ \Delta y_t &= (x_{t-1} - x_{t-2}) + (v_t - v_{t-1}) + (\varepsilon_t - \varepsilon_{t-1}) \\ \Delta y_t &= \Delta x_t + \Delta v_t + \Delta \varepsilon_t\end{aligned}$$

The sum of white noise processes is white noise, so the difference of y_t is stationary, so y_t is I(1).

To show that they are cointegrated, just take the difference of y_t and x_t and note that this is:

$$y_t - x_t = \varepsilon_t, \text{ which of course is stationary.}$$

Since there exists a linear combination of the two I(1) processes that is stationary, we say that they are *cointegrated*, and the *cointegrating relationship* is $y_t - x_t = 0$.

(b) *Engle and Yoo(1987) show that optimal long-run forecasts of cointegrated variables obey the cointegrating relationship exactly. Verify their result for the system at hand.*

We saw in class that the optimal h-step ahead forecast of a random walk at time T is $\hat{y} = y_{T+h,T} = y_T$ (just today's value). (Because for a general AR(1) process $y_t = \phi y_{t-1} + \varepsilon_t$. Forecast is $y_{T+h,T} = \phi^h y_T$. And for a

random walk, $\phi=1$.) So the optimal forecasts in our case are:

$$\hat{x}_t = x_t,$$

$$\hat{y}_t = \hat{x}_t = x_t$$

Clearly the difference between the optimal forecasts of x and y is zero, so they satisfy the cointegrating relationship, $y_t - x_t = 0$.