Problem Set 3 suggested answers (with minor corrections)

Ch 6. Problems and Complements:

1. (page 132) In each case, the idea is to write these out in general form (without the lag operators). I also think that it makes sense to express it with y_t on the left-hand side and all other terms on the right-hand side. Note that these expressions hold for all times t, so that's what give me the final steps in parts a and b.

a.

$$\left(L^{\tau}\right)y_t = \varepsilon_t$$
$$y_{t-\tau} = \varepsilon_t$$
$$y_t = \varepsilon_{t+\tau}$$

b.

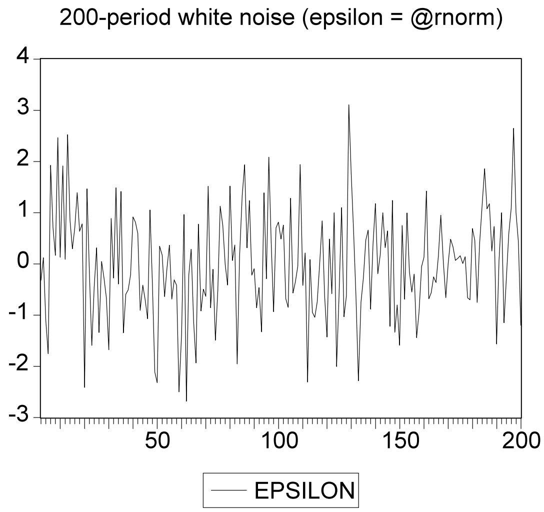
$$y_t = \frac{\left(2+5L+0.8L^2\right)}{\left(L-0.6L^3\right)}\varepsilon_t$$
$$\left(L-0.6L^3\right)y_t = \left(2+5L+0.8L^2\right)\varepsilon_t$$
$$y_{t-1} - 0.6y_{t-3} = 2\varepsilon_t + 5\varepsilon_{t-1} + 0.8\varepsilon_{t-2}$$
$$y_{t-1} = 0.6y_{t-3} + 2\varepsilon_t + 5\varepsilon_{t-1} + 0.8\varepsilon_{t-2}$$
$$y_t = 0.6y_{t-2} + 2\varepsilon_t + 5\varepsilon_t + 0.8\varepsilon_{t-1}$$

c.

$$y_t = 2\left(1+\frac{L^3}{L}\right)\varepsilon_t$$
$$y_t = 2\left(1+L^2\right)\varepsilon_t$$
$$y_t = 2\varepsilon_t + 2\varepsilon_{t-2}$$

8. (p. 134) Simulating time series.

a. Create an undated workfile and then create a white noise variable: **smpl 1 200**. **series epsilon = @rnorm**

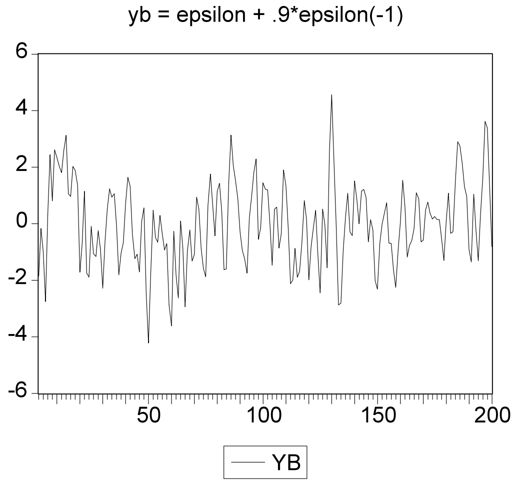


There are no significant autocorrelations in the correlogram, nor are there any significant partial autocorrelations. None of the Q-stats are significant (since none of the p-vales ("Prob") are .05 or smaller. So the series is not serially correlation, as expected for white noise.

Date: 03/03/03 Time: 09:23
Sample: 2 200
Included observations: 199

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|* |	.|* |	1	0.074	0.074	1.1053	0.293
.|* |	.|. |	2	0.070	0.065	2.0994	0.350
.|. |	.|. |	3	0.003	-0.006	2.1016	0.552
.|* |	.|* |	4	0.072	0.068	3.1678	0.530
*|. |	*|. |	5	-0.068	-0.079	4.1168	0.533
.|* |	.|* |	6	0.095	0.098	5.9769	0.426
.|. |	.|. |	7	-0.024	-0.031	6.1006	0.528
.|* |	.|* |	8	0.127	0.118	9.4710	0.304
.|. |	.|. |	9	0.025	0.019	9.6043	0.383
.|* |	.|* |	10	0.101	0.069	11.772	0.301

b. Set the sample: **smpl 2 200** and use the series command: **series yb = epsilon + .9*epsilon(-1)**. This is an MA(1) series:

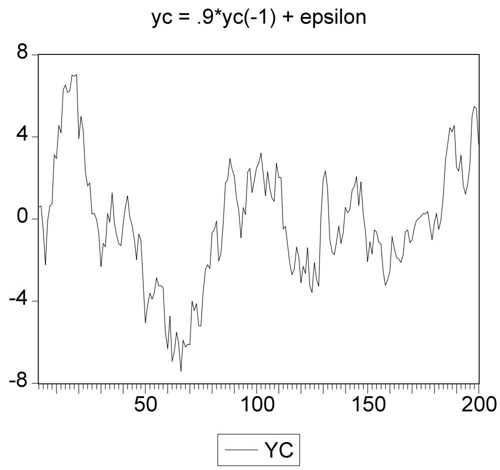


Its correlogram shows a sharp spike at lag 1 with no other significant autocorrelations. The pattern of partial autocorrelations oscillates and gradually damps, as expected for an MA(1) series. The low p-values (Prob) for the q-stats confirm that the series has significant serial correlation.

Date: 03/03/03 Time: 09:36
Sample: 2 200
Included observations: 199

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|**** |	.|**** |	1	0.560	0.560	63.416	0.000
.|* |	**|. |	2	0.107	-0.301	65.752	0.000
.|* |	.|** |	3	0.087	0.284	67.282	0.000
.|. |	**|. |	4	0.045	-0.227	67.693	0.000
.|. |	.|* |	5	0.009	0.186	67.712	0.000
.|. |	*|. |	6	0.043	-0.088	68.100	0.000
.|* |	.|* |	7	0.067	0.125	69.037	0.000
.|* |	.|. |	8	0.109	0.034	71.526	0.000
.|* |	.|. |	9	0.124	0.038	74.753	0.000
.|* |	.|. |	10	0.069	-0.025	75.746	0.000

c. First, you need to put a value of "1" into the first observation of your new series: **smpl 1 1; series yc = 1**. Then reset the sample: **smpl 2 200**, and use the series command: **series yc = .9*yc(-1) + epsilon**. This is an AR(1) series. Note that it is much more "persistent" than the MA series, tending to wander only slowly from where it is at a point in time.



Its correlogram shows a pattern of gradual damping, as expected for an AR process. Only the first partial autocorrelation is significant, again as expected for an AR(1) series.

Date: 03/03/03 Time: 09:40
Sample: 2 200
Included observations: 199

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|*******|	.|*******|	1	0.929	0.929	174.28	0.000
.|*******|	*|. |	2	0.854	-0.060	322.50	0.000
.|****** |	.|. |	3	0.781	-0.032	447.00	0.000
.|****** |	.|. |	4	0.720	0.044	553.20	0.000
.|***** |	.|. |	5	0.660	-0.024	643.05	0.000
.|***** |	.|* |	6	0.620	0.105	722.69	0.000
.|**** |	*|. |	7	0.574	-0.069	791.35	0.000
.|**** |	.|. |	8	0.536	0.032	851.48	0.000
.|**** |	*|. |	9	0.482	-0.122	900.49	0.000
.|*** |	.|. |	10	0.432	-0.010	939.95	0.000

Problems for Ch. 8

4. (p. 163) Note: these take quite a bit of time. If you get hung up and can't finish all the calculations, just explain what steps need to be done to perform the calculations.

a. $y_t = \mu + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}$

This is an MA(2) with a non-zero mean, μ.

The unconditional mean is:

$$\begin{aligned} Ey_t &= E(\mu + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}) \\ &= E(\mu) + E(\varepsilon_t) + \theta_1 E(\varepsilon_{t-1}) + \theta_2 E(\varepsilon_{t-2}) = \mu \end{aligned}$$

The conditional mean is (the information set that is relevant is all past values of epsilon):

$$\begin{aligned} &E(y_t | \varepsilon_{t-1}, \varepsilon_{t-2}, ...) = \\ &\quad = \mu + E(\varepsilon_t | \varepsilon_{t-1}, \varepsilon_{t-2}, ...) + \theta_1 E(\varepsilon_{t-1} | \varepsilon_{t-1}, \varepsilon_{t-2}, ...) + \theta_2 E(\varepsilon_{t-2} | \varepsilon_{t-1}, \varepsilon_{t-2}, ...) \\ &\quad = \mu + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} \end{aligned}$$

The unconditional variance is (note that the variance of the constant μ is zero):

$$\begin{aligned} \operatorname{var}(y_t) &= \operatorname{var}(\mu + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}) \\ &= \operatorname{var}(\mu) + \operatorname{var}(\varepsilon_t) + \theta_1^2 \operatorname{var}(\varepsilon_{t-1}) + \theta_2^2 \operatorname{var}(\varepsilon_{t-2}) \\ &= (1 + \theta_1^2 + \theta_2^2)\sigma^2 \end{aligned}$$

The conditional variance is:

$$\begin{aligned} \operatorname{var}(y_t | \varepsilon_{t-1}, \varepsilon_{t-2}, ...) &= E\left[(y_t - E(y_t | \varepsilon_{t-1}, \varepsilon_{t-2}, ...))^2 | \varepsilon_{t-1}, \varepsilon_{t-2}, ... \right] \\ &= E\left[\{(\mu + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}) - (\mu + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2})\}^2 \right] \\ &= E(\varepsilon_t^2) \qquad = \sigma^2 \end{aligned}$$

Now the fun part. The autocovariance function is given by:

$$\begin{aligned} \gamma(t,\tau) &= E(y_t - \mu)(y_{t-\tau} - \mu) \\ &= E\big((\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2})(\varepsilon_{t-\tau} + \theta_1\varepsilon_{t-\tau-1} + \theta_2\varepsilon_{t-\tau-2})\big) \\ &= E(\varepsilon_t\varepsilon_{t-\tau}) + E(\varepsilon_t\theta_1\varepsilon_{t-\tau-1}) + E(\varepsilon_t\theta_2\varepsilon_{t-\tau-2}) \\ &\quad + E(\theta_1\varepsilon_{t-1}\varepsilon_{t-\tau}) + E(\theta_1\varepsilon_{t-1}\theta_1\varepsilon_{t-\tau-1}) + E(\theta_1\varepsilon_{t-1}\theta_2\varepsilon_{t-\tau-2}) \\ &\quad + E(\theta_2\varepsilon_{t-2}\varepsilon_{t-\tau}) + E(\theta_2\varepsilon_{t-2}\theta_1\varepsilon_{t-\tau-1}) + E(\theta_2\varepsilon_{t-2}\theta_2\varepsilon_{t-\tau-2}) \end{aligned}$$

(kill me now…)

$$
\begin{aligned}
\gamma(1) = E&(y_t-\mu)(y_{t-1}-\mu)\\
&= E\big((\varepsilon_t+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2})(\varepsilon_{t-1}+\theta_1\varepsilon_{t-2}+\theta_2\varepsilon_{t-3})\big)\\
&= E(\varepsilon_t\varepsilon_{t-1})+E(\varepsilon_t\theta_1\varepsilon_{t-2})+E(\varepsilon_t\theta_2\varepsilon_{t-3})\\
&+\underline{E(\theta_1\varepsilon_{t-1}\varepsilon_{t-1})}+E(\theta_1\varepsilon_{t-1}\theta_1\varepsilon_{t-2})+E(\theta_1\varepsilon_{t-1}\theta_2\varepsilon_{t-3})\\
&+E(\theta_2\varepsilon_{t-2}\varepsilon_{t-1})+\underline{E(\theta_2\varepsilon_{t-2}\theta_1\varepsilon_{t-2})}+E(\theta_2\varepsilon_{t-2}\theta_2\varepsilon_{t-3})
\end{aligned}
$$

Note that only the terms involving two epsilons with the same lag length will be nonzero, since epsilon is not serially correlated:

$$
\begin{aligned}
\gamma(1) &= E(\theta_1\varepsilon_{t-1}\varepsilon_{t-1})+E(\theta_2\varepsilon_{t-2}\theta_1\varepsilon_{t-2})\\
&= (\theta_1+\theta_1\theta_2)\sigma^2
\end{aligned}
$$

The autocorrelation function would be this divided by the variance, so

$$
\rho(1)=\frac{\gamma(1)}{\gamma(0)}=\frac{(\theta_1+\theta_1\theta_2)\sigma^2}{(1+\theta_1^2+\theta_2^2)\sigma^2}=\frac{(\theta_1+\theta_1\theta_2)}{(1+\theta_1^2+\theta_2^2)}
$$

$$
\begin{aligned}
&when\ \tau=2,\\
\gamma(2) = E&(y_t-\mu)(y_{t-2}-\mu)\\
&= E\big((\varepsilon_t+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2})(\varepsilon_{t-2}+\theta_1\varepsilon_{t-3}+\theta_2\varepsilon_{t-4})\big)\\
&= E(\varepsilon_t\varepsilon_{t-2})+E(\varepsilon_t\theta_1\varepsilon_{t-3})+E(\varepsilon_t\theta_2\varepsilon_{t-4})\\
&+E(\theta_1\varepsilon_{t-1}\varepsilon_{t-2})+E(\theta_1\varepsilon_{t-1}\theta_1\varepsilon_{t-3})+E(\theta_1\varepsilon_{t-1}\theta_2\varepsilon_{t-4})\\
&+\underline{E(\theta_2\varepsilon_{t-2}\varepsilon_{t-2})}+E(\theta_2\varepsilon_{t-2}\theta_1\varepsilon_{t-3})+E(\theta_2\varepsilon_{t-2}\theta_2\varepsilon_{t-4})
\end{aligned}
$$

Note that there is only one term involving two epsilons with the same lag length (in the final row above):

$$
\begin{aligned}
\gamma(2) &= E(\theta_2\varepsilon_{t-2}\varepsilon_{t-2})\\
&= \theta_2\sigma^2
\end{aligned}
$$

The autocorrelation function would be this divided by the variance, so

$$
\rho(2)=\frac{\gamma(2)}{\gamma(0)}=\frac{\theta_2\sigma^2}{(1+\theta_1^2+\theta_2^2)\sigma^2}=\frac{\theta_2}{(1+\theta_1^2+\theta_2^2)}
$$

For displacements greater than τ the covariance and correlations would be zero. This explains why the autocorrelation function for an MA(2) has two significant autocorrelations.

[You weren't asked to do this, but if you wanted to compute the partial autocorrelation function you would have to "invert" the MA process into an AR form and then look at the pattern of coefficients.

$$y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

$$\varepsilon_t = y_t - \mu - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$
$$\varepsilon_{t-1} = y_{t-1} - \mu - \theta_1 \varepsilon_{t-2} - \theta_2 \varepsilon_{t-3}$$
$$\varepsilon_{t-2} = y_{t-2} - \mu - \theta_1 \varepsilon_{t-3} - \theta_2 \varepsilon_{t-4}$$

Substitute these into the expression for y_t etc. as in the book to get an expression for y_t as a function of its own lags. The coefficients on these lags will be the partial autocorrelation function. (Note that we can get AR form also by inverting the MA equation in lag operators but not sure how that could easily be used to obtain the coefficients.)]

6. (p. 165)

Based on the text of question (they say 200 "business days") I set this up as 5-day-a-week daily data, and I arbitrarily put it in 2000. I modified the day of the week dummies I created earlier in the term for this kind of data. There are no apparent trend or day-of-the-week effects in this data. (Show this with a regression on TIME and day-of-the-week dummies.) You can fit a quadratic trend, but it doesn't make a whole lot of sense, since it wanders up and then back down. So for the rest, I just include a constant and ARMA terms. (Note that I have set the sample period to begin in the fifth period so that each model can be estimated over the same sample. If you set your sample to the entire data range you will get slightly different numbers.)

Akaike

		MA Order				
		0	1	2	3	4
AR order	0	5.753776	4.560413	3.881434	3.582574	3.363830
	1	3.308993	**3.026679**	3.036772	3.033685	3.039398
	2	3.077914	3.036625	3.025131	3.041388	3.312603
	3	3.070906	3.039158	3.044412	3.015575	3.058988
	4	3.050568	3.040962	3.049766	2.992947	**2.974099**

Schwartz

	MA Order					
	AR MA	0	1	2	3	4
AR order	0	5.803951	4.627313	3.965060	3.682925	3.447455
	1	3.342443	**3.076854**	3.103672	3.117311	3.139749
	2	3.128089	3.103525	3.108756	3.141739	3.429678
	3	3.137806	3.122784	3.144763	3.132651	3.192789
	4	3.134193	3.141312	3.166842	3.126747	3.124624

Although the Akaike criteria suggests an ARMA(4,4)(!), the SIC selects an ARMA(1,1) model. Note also that the ARMA(4,4) model has a set of inverse AR and MA roots that are of approximately equal magnitudes (this is clearer if you compare the ARMA(3,3) with the ARMA(1,1)). If the roots are in fact equal, then they can be canceled out, bringing us back to the ARMA(1,1) specification.

The ARMA(1,1) model looks pretty good. Re-estimating over the full sample:

Dependent Variable: TRANSFERS
Method: Least Squares
Date: 03/01/10 Time: 13:12
Sample (adjusted): 1/04/2000 10/06/2000
Included observations: 199 after adjustments
Convergence achieved after 6 iterations
MA Backcast: 1/03/2000

	Coefficient	Std. Error	t-Statistic	Prob.
C	13.31644	1.736319	7.669353	0.0000
AR(1)	0.928088	0.027228	34.08530	0.0000
MA(1)	0.605901	0.056862	10.65560	0.0000

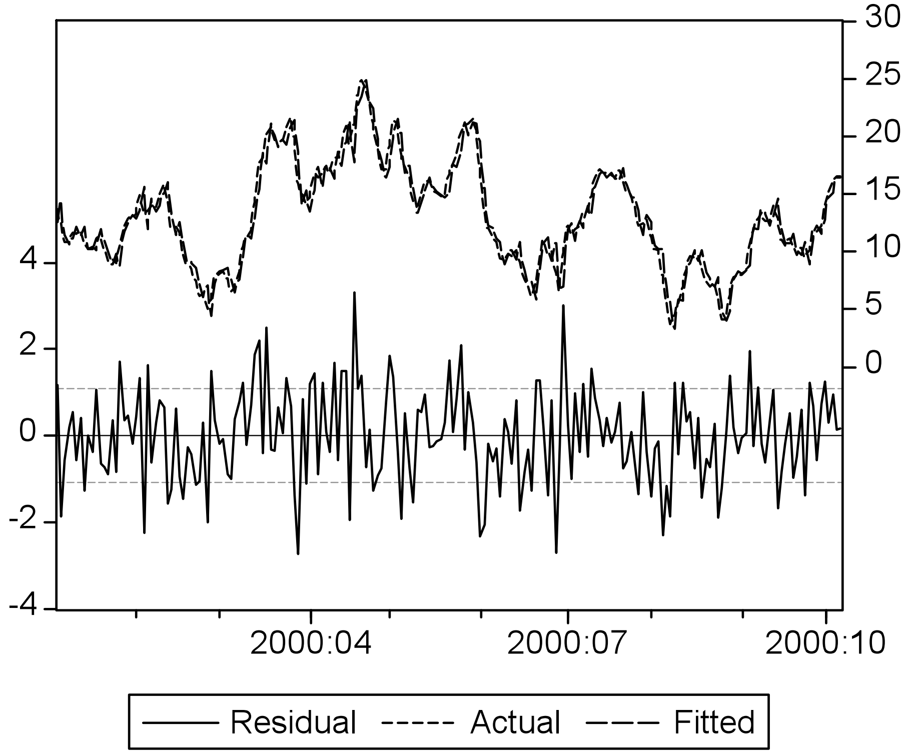
R-squared	0.943728	Mean dependent var	12.98359
Adjusted R-squared	0.943153	S.D. dependent var	4.590735
S.E. of regression	1.094547	Akaike info criterion	3.033518
Sum squared resid	234.8143	Schwarz criterion	3.083166
Log likelihood	-298.8350	Hannan-Quinn criter.	3.053612
F-statistic	1643.531	Durbin-Watson stat	2.012016
Prob(F-statistic)	0.000000		

Inverted AR Roots	.93
Inverted MA Roots	-.61

The model fits well and there is no evidence of serial correlation in the residuals. Also, normality tests (not shown here) do not show any significant deviation from normal residuals.

Date: 03/01/10   Time: 13:14
Sample: 1/04/2000 10/06/2000
Included observations: 199
Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|. |	.|. |	1	-0.009	-0.009	0.0164	
.|. |	.|. |	2	0.069	0.069	0.9731	
.|* |	.|* |	3	0.084	0.086	2.4274	0.119
.|. |	.|. |	4	-0.029	-0.033	2.6048	0.272
.|. |	.|. |	5	-0.049	-0.062	3.0955	0.377
.|* |	.|* |	6	0.099	0.097	5.1367	0.274
.|. |	.|. |	7	0.014	0.030	5.1770	0.395
.|. |	.|. |	8	0.004	-0.003	5.1800	0.521
.|. |	.|. |	9	-0.039	-0.064	5.4921	0.600
.|. |	.|. |	10	-0.023	-0.024	5.6034	0.692
.|. |	.|. |	11	-0.036	-0.017	5.8836	0.752
*|. |	*|. |	12	-0.084	-0.082	7.4083	0.686
.|. |	.|. |	13	-0.025	-0.028	7.5384	0.754
.|. |	.|. |	14	-0.002	0.007	7.5395	0.820


30
25
20
15
10
5
0
4
2
0
-2
-4
2000:04
2000:07
2000:10
Residual
Actual
Fitted