

Anatomy of a Constant Elasticity of Substitution Type Production/Utility Function in Three Dimensions

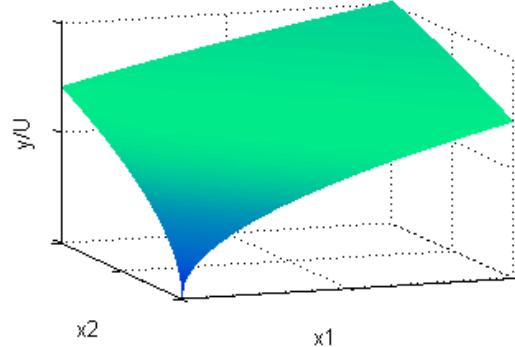
(A Visual Guide for Econ Majors)

Peter Fuleky
Department of Economics, University of Washington

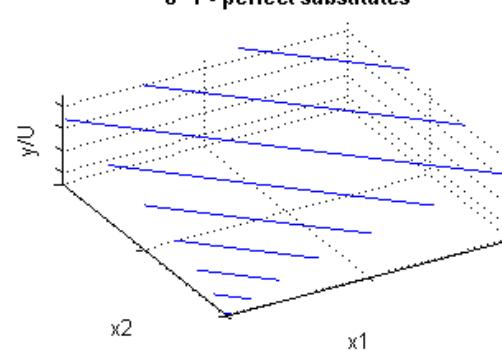
October 2006

Decreasing returns to scale: r=0.5 (Strongly concave y/U)

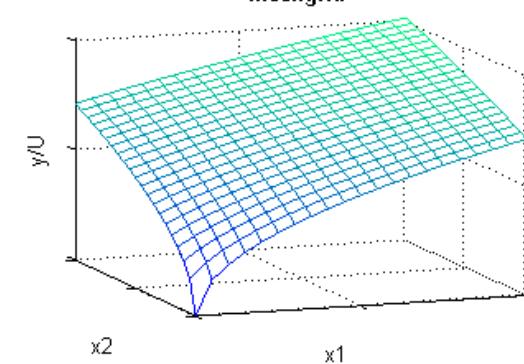
production/utility function = $A [\lambda x_1^\sigma + (1-\lambda) x_2^\sigma]^{1/\sigma}$
 $A=1, \lambda=0.5, \sigma=0.99, r=0.5$



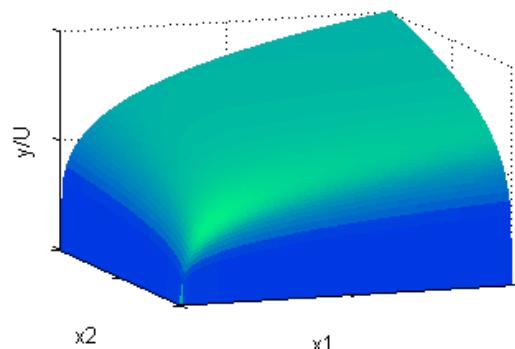
isoquants/indifference curves
 $\sigma=1$ - perfect substitutes



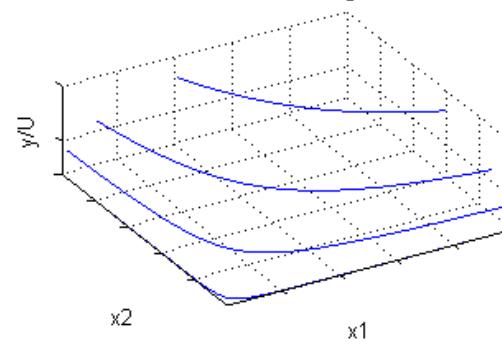
meshgrid



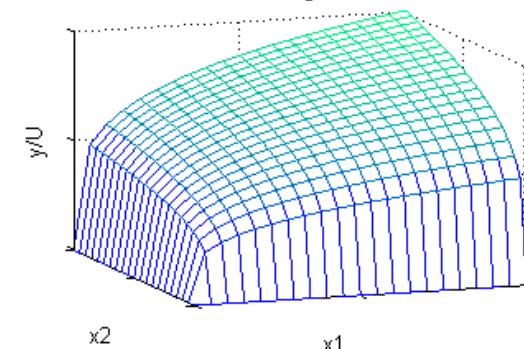
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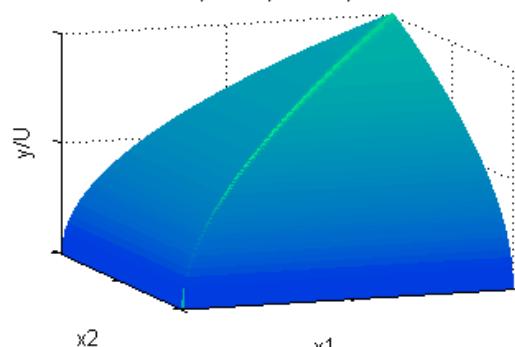
isoquants/indifference curves
 $\sigma=0$ - Cobb-Douglas



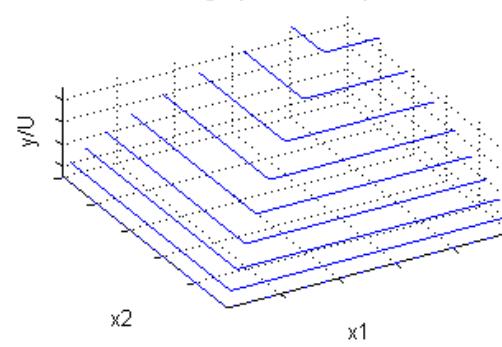
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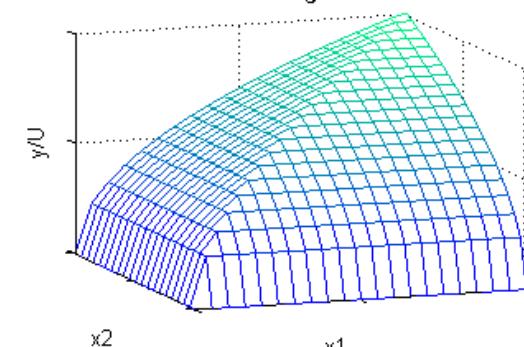
production/utility function = $A [\lambda x_1^\sigma + (1-\lambda) x_2^\sigma]^{1/\sigma}$
 $A=1, \lambda=0.5, \sigma=-100, r=0.5$



isoquants/indifference curves
 $\sigma=-\infty$ - perfect complements

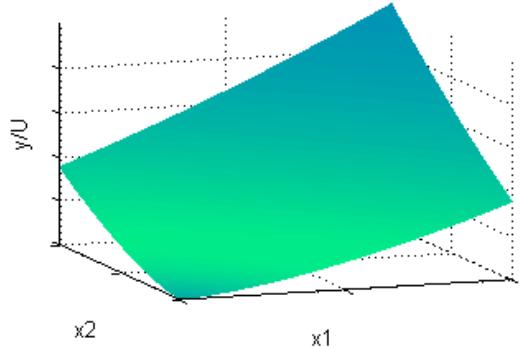


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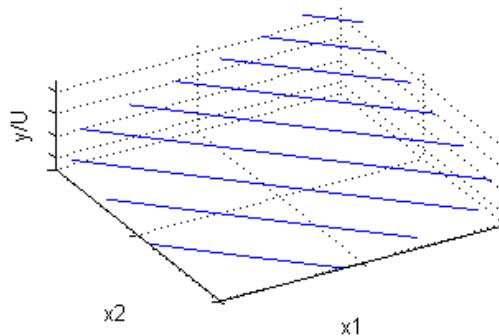


Increasing returns to scale: r=1.5 (Quasiconcave y/U)

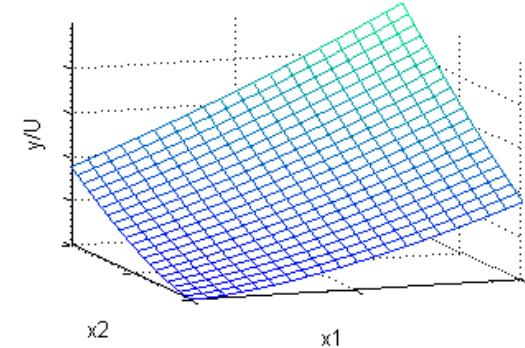
production/utility function = $A [\lambda x_1^\sigma + (1-\lambda) x_2^\sigma]^{r/\sigma}$
 $A=1, \lambda=0.5, \sigma=0.99, r=1.5$



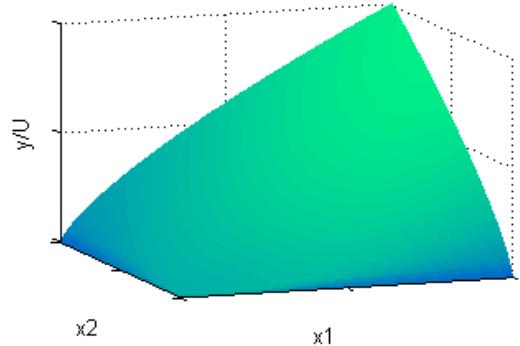
isoquants/indifference curves
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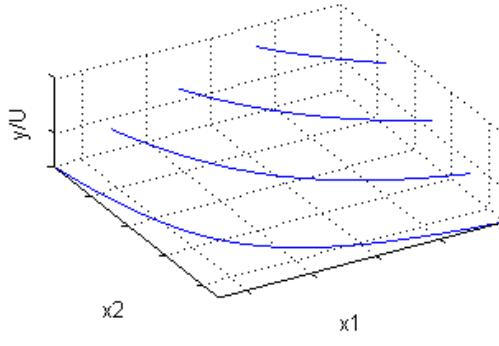
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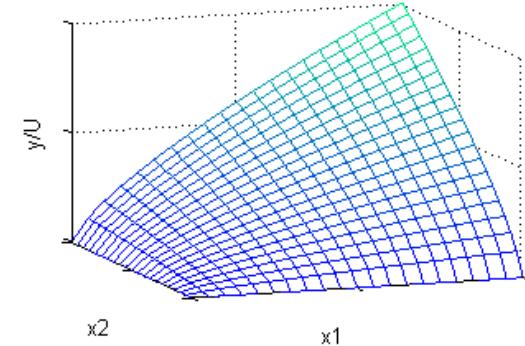
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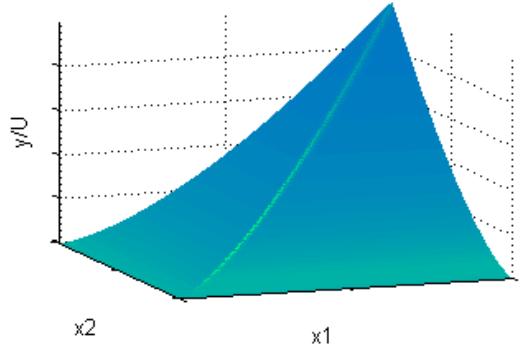
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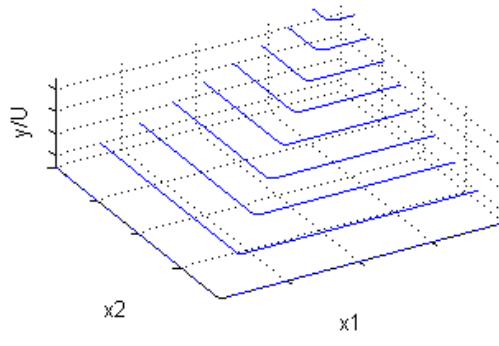
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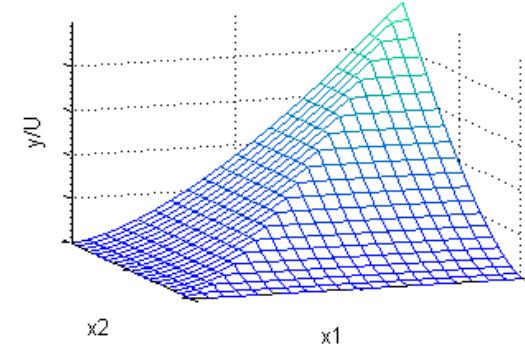
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isoquants/indifference curves
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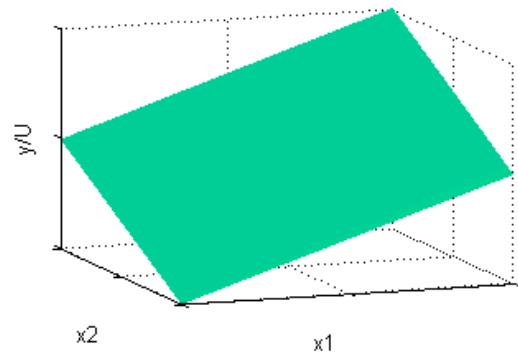


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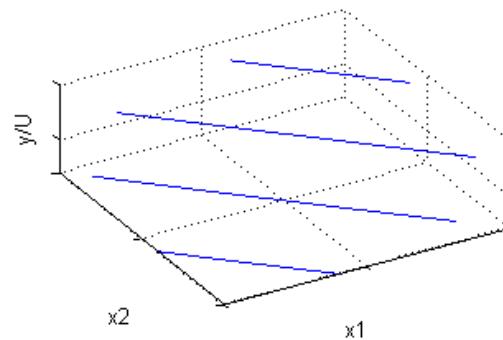


Constant returns to scale: r=1 (Weakly concave y/U)

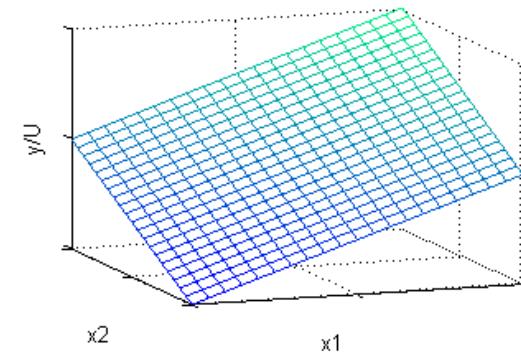
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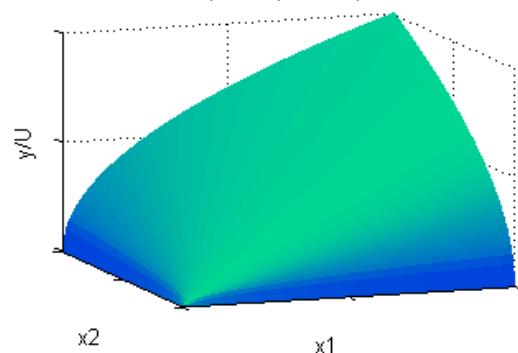
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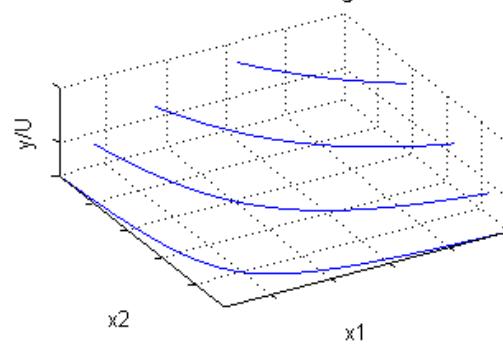
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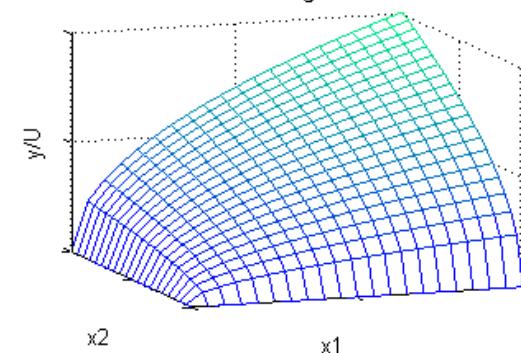
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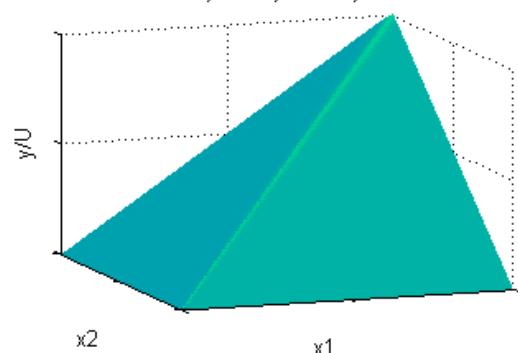
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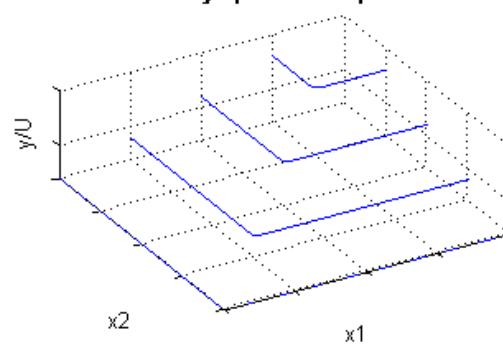
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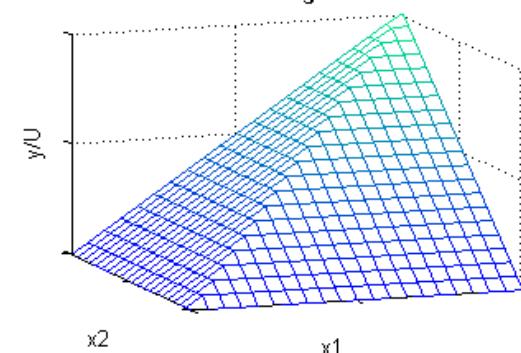
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 $\sigma=-\infty$ - perfect complements

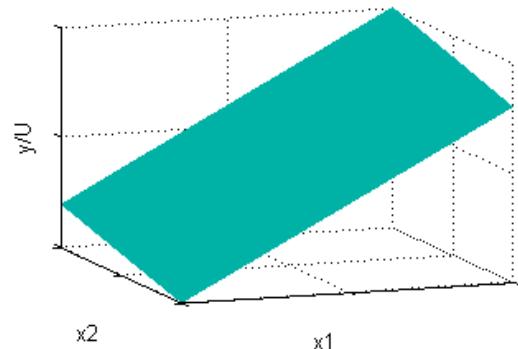


meshgrid

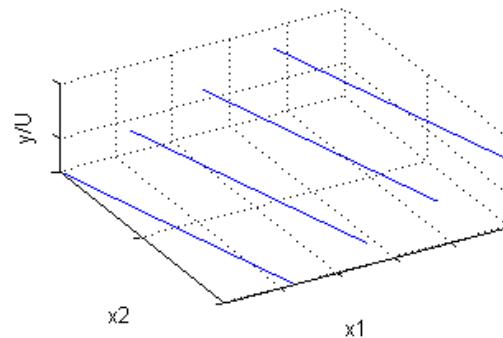


Constant returns to scale with asymmetric weighting: r=1, lambda=0.8

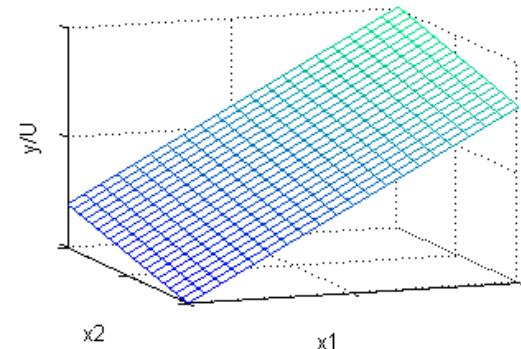
production/utility function = $A [\lambda x_1^\sigma + (1-\lambda) x_2^\sigma]^{r/\sigma}$
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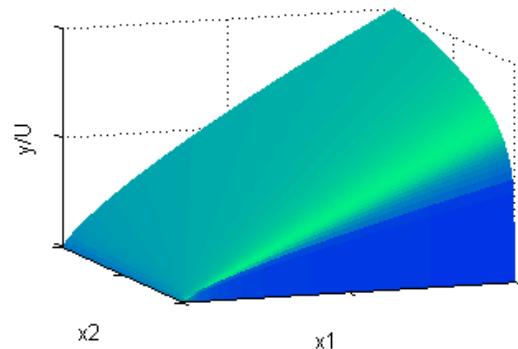
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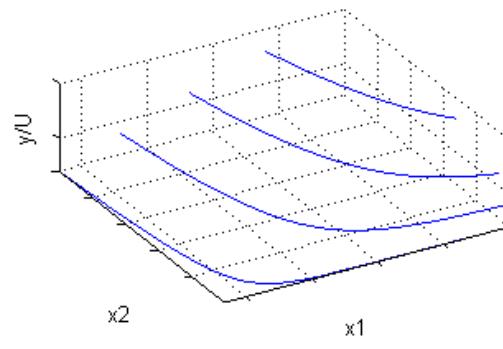
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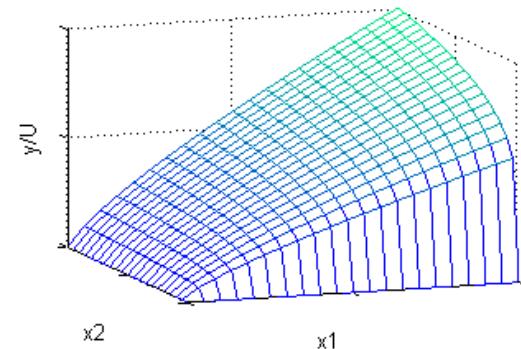
production/utility function = $A [\lambda x_1^\sigma + (1-\lambda) x_2^\sigma]^{r/\sigma}$
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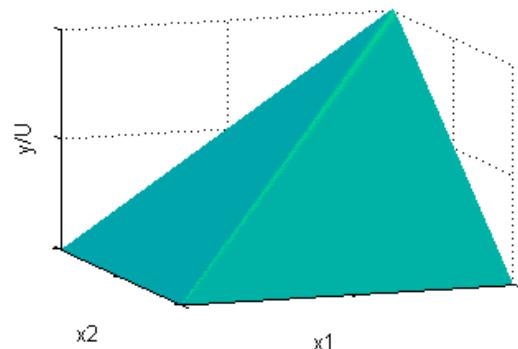
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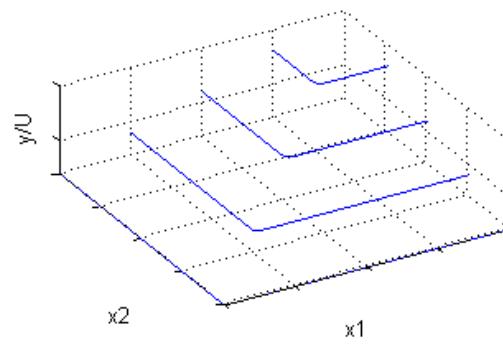
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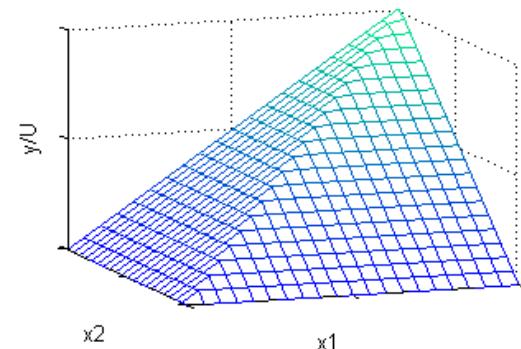
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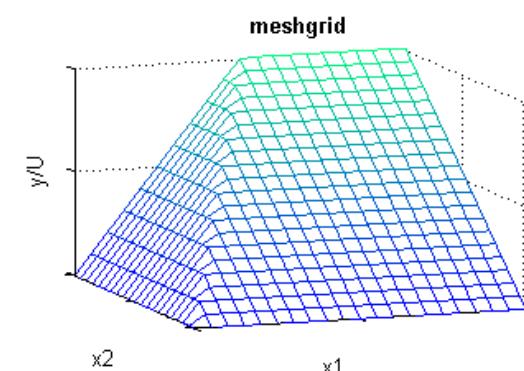
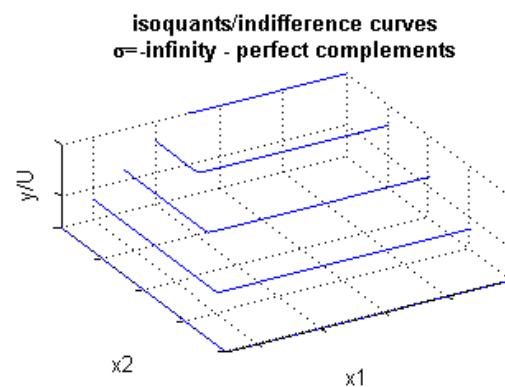
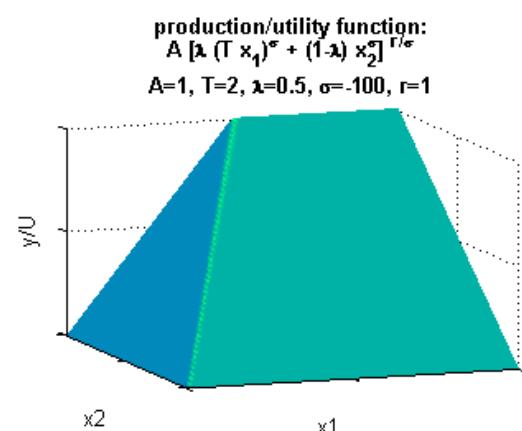
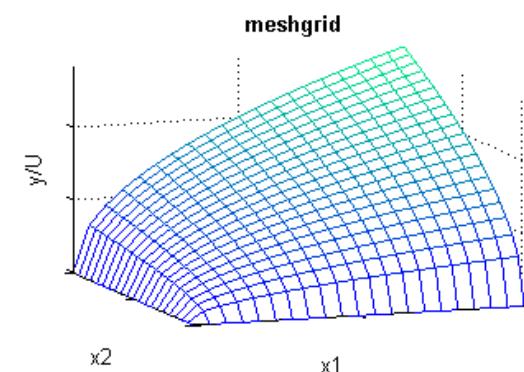
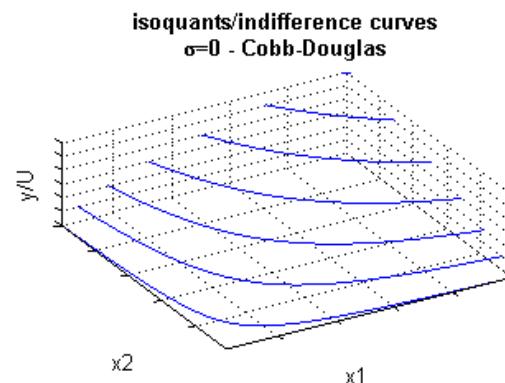
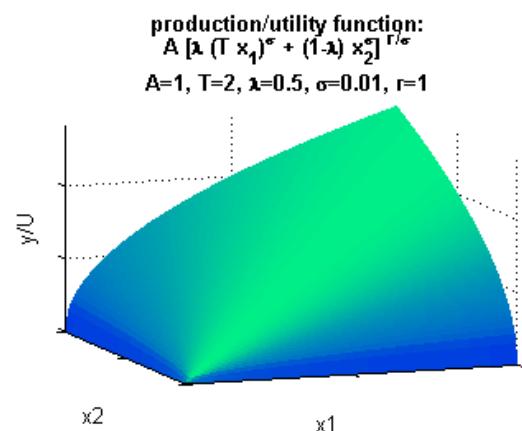
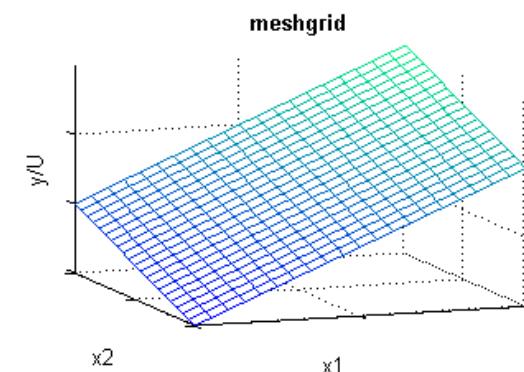
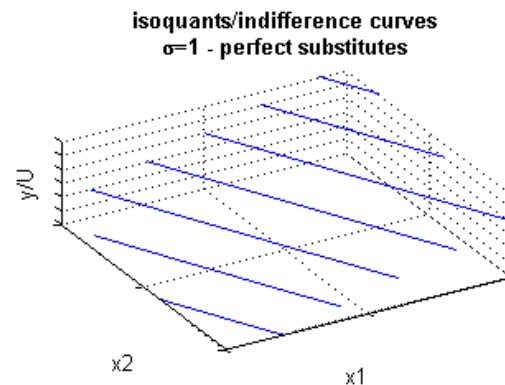
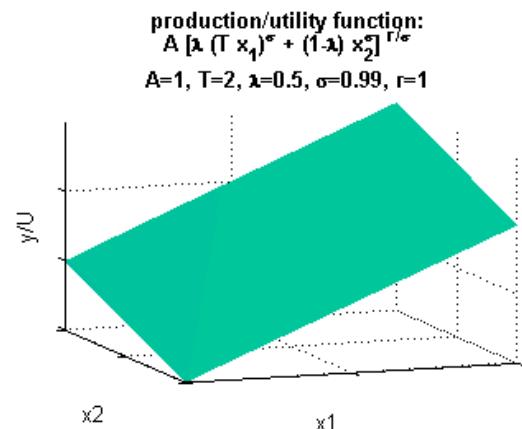
isoquants/indifference curves
 $\sigma=-\infty$ - perfect complements



meshgrid



Constant returns to scale with asymmetric productivity/utility: r=1, T=2



Literature and further reading:

The Structure of Economics, 3rd ed., Eugene Silberberg
MATLAB Documentation, MathWorks

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