Wireless Sensor Placement For Reliable and Efficient Data Collection

Edoardo S. Biagioni
Department of Information and Computer Sciences, University of Hawaii
esb@hawaii.edu

Galen Sasaki
Department of Electrical Engineering University of Hawaii
sasaki@spectra.eng.hawaii.edu

Abstract

Sensors can be paired with radio units and deployed to form a wireless ad-hoc sensor network. Actual deployments must consider the coverage that can be achieved with a given number of sensors: this coverage varies with the range of the radios and the maximum allowable distance between any point in the area and the nearest sensor. Deployments must also preserve connectivity in spite of possible failure or energy depletion in a subset of the units. This paper presents and analyzes a variety of regular deployment topologies, including circular and star deployments as well as deployments in square, triangular, and hexagonal grids.

1. Introduction

We consider an application of wireless ad hoc network technology for data collection in biological and environmental studies. The wireless network relays information from environmental sensors to one or more collection points. We assume that the network is composed of *sensor unit*s, each of which has sensors (to measure data) and wireless communication equipment (to transmit and relay data). These units may be placed in a geographic area where data is of interest. We will often refer to these units as *nodes*.

The PODs project at the University of Hawaii [1] is a specific example of such an application. The project has undertaken the design and development of units that can both sense the environment -- temperature, sunlight, rainfall, humidity, and high-resolution images -- and communicate the results, via ad-hoc wireless communications, back to one or more base stations. These units are designed so that they can be deployed to provide information about the environment in which endangered plant species grow, as well as about neighboring areas where the plants are not found. Studying these environments will make it possible to determine what conditions are and are not suitable for these plants to thrive in.

Though we focus on data collection for biological and environmental studies, our work can be applied to other applications, including helping robots interact with their environment and helping humans guide robots through difficult terrain.

There are many constraints in deploying such a wireless sensor network. For example, the units must be low power. If the units are battery-powered, energy depletion is a major cause of node failures. Units deployed outdoors must be secured against weather and human and animal visitors. The network must have a way of delivering results back to the "real world", often by way of Internet- or satellite-connected base stations. Deploying units in rugged terrain can itself be a major challenge. In this paper, we focus on optimal strategies for placing sensor units.

Individual sensor units must be placed close enough to each other that wireless communication is possible, and must be arranged so they form a network to relay data back to data collection points. In addition, nodes can be prone to failure due to events such as loss of power, operating system bugs, and equipment glitches. It is important that the network provide reliable communication that can survive node outages.

A second constraint is that units must be placed so as to observe events of interest. For environmental sensing, for example, it may be important to measure variations in temperature and the gradient of temperature variation across the area of interest. Such gradients may be known before deployment, but for many variables of interest they may not be known until after deployment. For the PODS project, data has to be available relatively quickly, so that scientists can decide when studying a phenomenon is of sufficient interest to justify expending scarce resources. There are many other constraints in the placement of environmental sensors [7, 8]. Finally, financial or other considerations usually limit the number of units that can be deployed to study a given area.

Figure 1 shows examples of node placements. In Figure 1(a), nodes are placed in a 1-dimensional region, e.g., along a mountain ridge or a river. In Figure 1(b), nodes are placed in a 2-dimensional region. They are placed to sample data in the entire region and, in this sense, they "cover" the region. Note that large regions will require a large number of nodes, which may be cost prohibitive. Figure 1(c) shows a cost efficient placement. Nodes are placed to cover a sub-region of particular interest. Outside the sub-region, data is still of interest, but not as much as in the sub-region. Some of the nodes are used to provide communication to one or more units (typically base stations) that have to be placed at a particular location [2]. The nodes are arranged as "lines" to sparsely sample data over long distances. Note that the configuration of Figure 1(c) is a combination of the configurations of Figures 1(a) and 1(b).

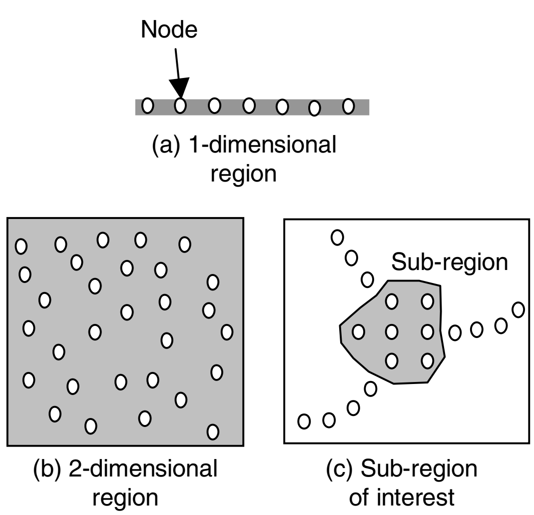

Figure 1. Examples of node placements.

To simplify our study of the node placement problem, we make the following assumptions.

- Data is to be sampled over a region denoted by *A*. The region could be a 2-dimensional or 1-dimensional. If it is 2-dimensional, then *A* is a unit square. Euclidean distances are used over the regions. The distance between two nodes *x* and *y* is denoted by $dist(x, y)$. In the case of 2-dimensional regions, we assume that *A* is large. This leads to simpler formulas, and perhaps better insight.

- The number of nodes is denoted by *N*, and the nodes are numbered 1, 2, ..., *N*. The location of node *i* in *A* are denoted by x_i. *N* is assumed to be large. This leads to simpler formulas.

- There is a parameter *r*, which determines which pairs of nodes can directly communicate. In particular, if a pair of nodes *x* and *y* have $dist(x, y) \le r$ then they can directly communicate, otherwise, they cannot. We refer to *r* as the *transmission radius*. In many practical cases, this threshold model is not entirely accurate. However, it is used in research and is supported by many simulators, including ns-2 [5], which assumes that antennas are omnidirectional and no obstacles are present. In practice, many other factors [4] will determine wireless connectivity, and research has been done on dynamically configuring node topology [6].

- Nodes should form a network that is resilient (i.e., remain connected) to node failures. In particular, it should be resilient to all *single* node failure. For a 1-dimensional region, we will also assume that there is a base station at one end of the region.

- There is a parameter δ which determines how sensors are placed to sufficiently cover region *A* for data sampling. The nodes should be placed so as to be sufficiently "close" to all points in *A*. To be more precise, consider the following definitions. Note that for a point $y \in A$, $\min_i dist(x_i, y)$ is the distance of *y* from its closest node. Then all points in *A* have distance at most

$\max_{y \in A} \min_i dist(x_i, y)$ from some node. The measure $\max_{y \in A} \min_i dist(x_i, y)$ will be referred to as the *maximum distance* from a placement of nodes. We will require that nodes be placed so that the resulting maximum distance at most δ. We refer to δ as the *required sampling distance*.

The placement problem that we consider is as follows. We are given region A, transmission radius r, and required sampling distance δ. The problem is to find a placement of nodes such that (*i*) it forms a network that is resilient to all single node failures, (*ii*) it has maximum distance at most δ, and (*iii*) it has a minimum number of nodes. Note that (*i*) is a communication constraint, (*ii*) is a data collection constraint, and (*iii*) is a cost issue.

The paper is organized as follows. In Section 2, we consider placement in a 2-dimensional region. We focus on three specific, regular placements. They are not necessarily optimal but their symmetry lead to simple formulas. In addition, their simplicity could lead to easier deployment than optimal placements. We compare these arrangements in two ways. The first measure of comparison is the number N of nodes to cover the area A. It will be shown that the best placements depend on the value of r with respect to δ. The second measure of comparison is the survivability of the network beyond single failures.

In Section 3, we consider 1-dimensional regions. We provide two placements and compare them with the two measures of Section 2.

We consider node placement for 2-dimensional regions in Section 4 but for special data we refer to as *linear data*. The following is an example. Suppose we want to observe environmental variations from east to west. The underlying assumption is that the data does not significantly change from south to north. Therefore, when a node collects sensor data then that data is representative of all locations that are directly north or south of it. We refer to such type of data as *linear* data since it varies along a linear direction but does not vary perpendicular to that direction.

In Section 5 we have our final remarks.

2. Node placement in 2-dimensional regions

We consider the three node placements shown in Figure 2 over a square region A. Figure 2(*a*) shows a node placement that corresponds to a "tiling" of squares. The nodes are located at the corners of the squares. The other node placements correspond to "tilings" of equilateral triangles and hexagons as shown in Figures 2(*a*) and 2(*b*). Again, the nodes are placed at the corners of the tiles.

We denote the length of a side of a tile by a. Note that a node must have transmission range at least a in order to transmit to another node. Thus, $a \geq r$. Also note that if $a \geq r$ then all these node placements are resilient to single node failures. To see this, consider two facts. First, nodes on a tile are connected to form a cycle. If any single node fails then they still remain connected. Second, adjacent tiles are connected by a side, which has two nodes. If a node fails then nodes in adjacent tiles remain connected. These two facts imply that the nodes in the network remain connected after any single node failure.

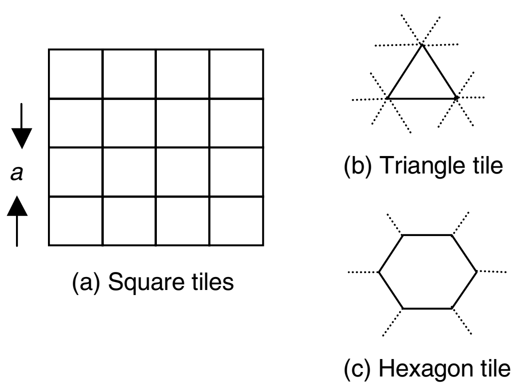

Figure 2. (*a*) Nodes arranged in a square "tiling". (*b*) Equilateral triangle tile, and (*c*) hexagonal tile.

Table 1 has approximate formulas of parameter values for the three tile shapes. The first two parameters are the maximum distance d and the length of a side of a tile a. They are functions of N and the area of A, which we denote by $|A|$. The calculations for the formulas for d and a are given in Appendix A.

Since we have a formula for d as a function of N, we can write a formula for N as a function of d.

We denote this by $N_s(d)$ in the table. We use a subscript "s" because parameter *d* is a measure of how well the nodes *sample* data. Similarly, since we have a formula for *a* as a function of *N*, we can write a formula for *N* as a function of *a*. We denote this by $N_t(a)$ in the table. The subscript "t" is used because parameter *a* is the minimum *transmission* range for a node to communicate with another node.)

Table 1. Approximate formulas of parameters for the three tiles.

	Triangle	*Square*	*Hexagon*
d	$\approx 0.62\sqrt{\frac{\lvert A\rvert}{N}}$	$\approx 0.71\sqrt{\frac{\lvert A\rvert}{N}}$	$\approx 0.88\sqrt{\frac{\lvert A\rvert}{N}}$
a	$\approx 1.07\sqrt{\frac{\lvert A\rvert}{N}}$	$\approx \sqrt{\frac{\lvert A\rvert}{N}}$	$\approx 0.88\sqrt{\frac{\lvert A\rvert}{N}}$
N_s	$\approx 0.38\frac{\lvert A\rvert}{d^2}$	$\approx 0.50\frac{\lvert A\rvert}{d^2}$	$\approx 0.77\frac{\lvert A\rvert}{d^2}$
N_t	$\approx 1.15\frac{\lvert A\rvert}{a^2}$	$\approx \frac{\lvert A\rvert}{a^2}$	$\approx 0.77\frac{\lvert A\rvert}{a^2}$
*Deg** *(r=a)*	6	4	3

The values of *d* and *a* are restricted. In particular, $a \geq r$, to insure that nodes can communicate with one another, and $\delta \geq d$, to insure that the placement of nodes does not exceed the required sampling distance δ. Thus, the minimum number of nodes *N* should be

$$N_{\min} \equiv \max\{N_s(\delta), N_t(r)\}.$$

Note that if *r* is much smaller than δ, i.e., transmission range is relatively short, then $N_{\min} = N_t(r)$ and the hexagonal tile leads to smallest *N* and the triangular tile leads to the largest *N*. If *r* is much larger than δ then $N_{\min} = N_s(r)$ and the opposite is true.

The final row in Table 1 has a measure of the survivability of the network for the three tiles beyond single node failures. It corresponds to the *degree* of a node, which is the number of nodes within its transmission range *r*. Note that we can disconnect a node from the network if all nodes within its transmission range fail. The higher the degree, the more difficult it is to disconnect the node. Therefore, the degree of a node is a measure of how resilient the network is to individual node failures.

To measure the degrees of nodes we assume that *a* = *r*, which is the minimum *a* can be and still have some nodes within transmission range. Note that nodes have different degrees. However, since *N* is assumed to be large, the vast majority of nodes are in the interior of *A* rather than its borders. *Deg** is the degree of an interior node. Thus, it is the degree of a "typical" node. For the triangle, square, and hexagon arrangements, *Deg** is, respectively, 6, 4, and 3 neighbors, and this is shown in the table. This means that the triangle tiles lead to the most survivable network and the hexagonal tiles lead to the least survivable network.

Note that the best and worst choice of tiles is either the triangle or hexagon. However, the square has good performance for any of the parameters.

Now we consider the case when *r* may be too small for any of the three arrangements, i.e., *r* < *a*. An option in this case is to place nodes along the sides of the tiles such as shown in Figure 3 for the square tile. Nodes are placed at each corner of a tile, and along the sides with spacing at most *r*. We call such a placement a *sparse grid*. Note that the placement still leads to a network that can survive single node failures because each tile is a cycle of nodes and adjacent tiles share a side with at least two nodes.

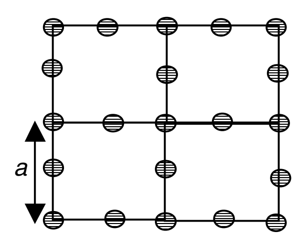

Figure 3. A sparse square grid with *a/r* = 2 and *s* = 3. Nodes are placed at the corners of the squares and along the sides.

For any sparse grid, each tile side has at least $s = \lceil a/r \rceil + 1$ nodes. In Appendix B, we derive formulas for *N* as a function of |*A*|, *r*, and *a* for the sparse triangular, square, and hexagonal grids. These formulas are shown in Table 2. Note that with these formulas, we can numerically compute the appropriate value of *a* given *N*, and in turn the appropriate value of *d* given *N*.

Table 2. Number of nodes for sparse arrangements.

	N
Sparse Triangle	$\frac{4}{\sqrt{3}}\frac{\lvert A\rvert}{a^2}\left(\frac{3}{2}\left(\left\lceil\frac{a}{r}\right\rceil-\frac{1}{2}\right)\right)$
Sparse Square	$\frac{\lvert A\rvert}{a^2}(2\lceil a/r\rceil)$
Sparse Hexagon	$\frac{4}{3\sqrt{3}}\frac{\lvert A\rvert}{a^2}(3\lceil a/r\rceil+1)$

Note that as r becomes smaller, the "sides" of the triangles, squares, and hexagons have a larger fraction of the nodes. Sides are linear arrangements of nodes spaced apart by the transmission range r. The nodes along the sides now have "degree" 2, rather than 3 or higher as in the original grid arrangements. This means the network is less reliable.

If additional nodes are available for a sparse grid then reliability can be improved. One option is to place them to reduce the value of a. Yet another option is to deploy twice as many nodes as necessary along the sides of the tiles. As a result, each "side" node would have connectivity not with two, but with four neighbors. In the next section we analyze the reliability of such a strategy.

3. Node placement in 1-dimensional regions

We consider the case when A is a line, and we denote its length by L. We assume that there is a base station node that is fixed at right end of A as shown in Figure 4. The placement problem is to place the other $N-1$ nodes.

Note that a linear arrangement of equally spaced nodes will minimize the maximum distance as shown in Figure 4. Let a denote the spacing between nodes. Note that the last node on the left is a distance $a/2$ away from the end of A. Thus, the spacing is $a = 2\frac{L}{2N-1} = \frac{L}{N-1/2}$. The formula for the maximum distance is $d = \frac{a}{2} = \frac{L}{2N-1}$. Note that $r \geq a$ is required for a connected network.

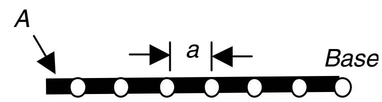

Figure 4. Nodes spaced equally along *A*, which is shown as a boldfaced line.

However, if $r = a$ then this arrangement is vulnerable to single node failures. Two arrangements that survive single node failures are shown in Figure 5, and are referred to as the *paired* and *in-line* arrangements. They require a minimum transmission range to survive any single node failures. We denote this transmission range by $r1$. Next, we will discuss the two arrangements and for simplicity, we assume that the base station never fails and that the number of nodes N is odd.

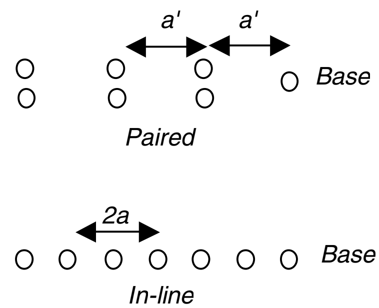

Figure 5. Two protected node arrangements.

For the paired arrangement, nodes are paired, and pairs are co-located (or nearly co-located) except for the base node. The space between adjacent pairs is $a' = \frac{2L}{N+1}$, and the last pair on the left is a distance $a'/2$ away from the end of A. The value of $r1$ is a', and the maximum distance d is $\frac{a'}{2} = \frac{L}{N+1}$.

The in-line arrangement is the same as the one in Figure 4. To insure that the network survives a single node failure, we have $r1 = 2a = \frac{2L}{N-1/2}$. This insures that if a node has a neighbor who fails then it can reach the next one down the line.

Recall that the maximum distance is $d = \frac{L}{2N-1}$. The formulas for *r1* and *d* are given in Table 3 for the paired and in-line arrangements.

Note that since we have *r1* as a function of *N*, we can write *N* as a function of *r1* which we denote by $N_t(r1)$. (Just as in Section 2, the subscript "t" corresponds to *transmission* range.) Similarly, since we have *d* as a function of *N*, we can write *N* as a function of *d* which we denote by $N_s(d)$. (Just as in Section 2, the subscript "s" corresponds to *sampling* data.) Table 3 has the formulas for $N_t(r1)$ and $N_s(d)$. Recall that *r1* and *d* are constrained so that $r1 \geq r$, to insure that nodes can communicate with one another, and $\delta \geq d$, to insure that the placement of nodes is within the required sampling distance δ. Thus, the minimum number of nodes *N* should be

$$N_{\min} \equiv \max\{N_s(\delta), N_t(r)\}.$$

Table 3. Formulas for parameter values for paired and in-line arrangements.

	Paired	*In-Line*
d	$\frac{L}{N+1}$	$\frac{L}{2N-1}$
$r1$	$\frac{2L}{N+1}$	$\frac{2L}{N-1/2}$
$N_s(d)$	$\left\lceil \frac{L}{d} - 1 \right\rceil$	$\left\lceil \frac{L}{2d} + \frac{1}{2} \right\rceil$
$N_t(r1)$	$\left\lceil \frac{2L}{r1} - 1 \right\rceil$	$\left\lceil \frac{2L}{r1} + \frac{1}{2} \right\rceil$

Let us compare the minimum number of nodes for the paired and in-line arrangements. If the transmission range *r* is small compared to δ then $N_{\min} \equiv N_t(r)$. Then the number of nodes is about the same for paired and in-line, differing by at most 2. If the transmission range *r* is large compared to δ then $N_{\min} \equiv N_s(\delta)$. Then the number of nodes for in-line is half of that for paired. It seems that in-line is a good choice for all values of *r* and δ.

However, if $r \leq \delta$ and multiple node failures can be expected, then paired is more reliable. The following example illustrates this. Consider both the in-line and paired arrangements, and in both cases suppose $r = r1$ to minimize the number of nodes. Note that they have about the same number of nodes. Let us suppose there is a single node failure. Both paired and in-line arrangements will survive this. Let us further suppose that the failed node is not one of the last two nodes at either end of *A*. This is reasonable because *N* is large, and so the failed node is likely to be in the interior somewhere. Suppose there is a second failure and each node is equally likely to be it. In the case of the paired arrangement, the network becomes disconnected if the second failure is in the same pair as the first failure. Thus, the probability of a disconnection is $\frac{1}{N-2}$. In the case of in-line, the network becomes disconnected if the second failure is adjacent to the first. Since the first failed node has two adjacent nodes, the probability of disconnection is $\frac{2}{N-2}$. Thus, for this example, the in-line arrangement has twice the risk of becoming disconnected after two failures than the paired.

From the table, we can conclude that in-line can lead to smaller numbers of nodes. However, if *r* is small compared to δ then the paired arrangement should be chosen because it is more reliable for the same number of nodes.

4. Node placements for linear data

We consider arrangement of nodes in a 2-dimensional region *A*, which is very large. We are interested in placing nodes to collect data that can tell us something about environmental changes in a particular direction. For example, we may be interested in temperature variations from east to west, or humidity variations from southeast to northwest.

We assume that the sensor data is of a particular type, which we refer to as *linear* data. This leads to a smaller number of nodes than the numbers suggested in Section 2. *Linear data* has a reference direction vector *D*. For example, if we want to measure temperature variation from east to west then *D* would be a vector from east to west. For linear data, it can be assumed that along any line perpendicular to *D*, data values will be the same. For example, if *D* is east to west then locations on any line going south to north have the same data values.

We assume that the data of interest goes along some vector *D* through the center *c* of *A* as shown in Figure 6. The data of interest is within a geographical range *R* about the center. Hence, the region to collect data is a sub-segment of *D* centered at *c*. We denote this segment region as *D** as shown in the figure.

The figure also shows nodes that are not necessarily placed on *D*. However, since the data is linear, the nodes can provide accurate measurement of certain points on *D*. For example, in Figure 6, consider the node located at x_k. Note the dashed line that is perpendicular to *D* and intersects x_k and *D*. Note there is a point *p* that is at the intersection of the dashed line and *D*. Locations x_k and *p* have the same sensor data values because they both lie on the dashed line that is perpendicular to *D*. Point *p* is called the *projection* of x_k onto *D*. Note that it is the closest point on *D* to x_k. For $k = 1, 2, \ldots, N$, we denote the projection of x_k onto *D* by p_k. Thus, the *N* nodes provide accurate measurements along *D* at the projection points $p_1, p_2, \ldots, p_N$.

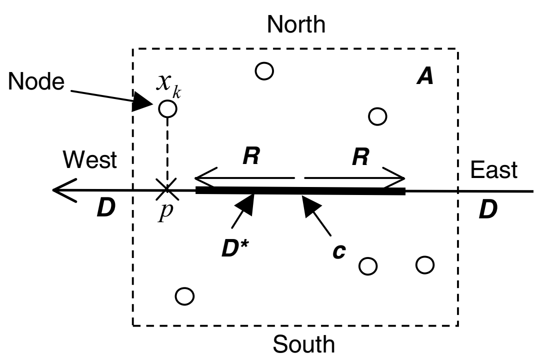

Figure 6. Direction *D* goes from east to west. Point *p* (shown as "x") is the projection of x_k onto *D*.

We can use the nodes (that are arranged in a 2-dimension region) to measure data on *D** (which is a 1-dimensional region). To determine how accurate this measurement is, we use the following measure

$$\max_{y \in D^*}\left[\min_k dist(y, p_k)\right]$$

Note that the "min" is the distance between a point *y* (in *D**) and the projection points. Thus, the measure is the maximum distance between the points in *D** and the projection points. This is similar to the maximum distance that we defined in Section 1. We call it the *maximum distance for* (*D,R*).

Now placing nodes for linear data with a particular direction *D* and geographical range *R* is the same as placing nodes in a 1-dimensional region as discussed in Section 3. Such arrangements would place nodes along *D*. However, we are interested in collecting all types of data, and each may have a different direction. Thus, the placements described in Section 3 are inadequate. For example, suppose there are two types of data to collect, say *B1* and *B2*, and they have direction vectors that are orthogonal. If nodes are arranged along the direction vector for *B1* then they cannot collect much data for *B2*, i.e., *B2* is in a "blind spot".

The node placement problem we consider is as follows. We are given a geographical radius *R*, transmission range *r*, and required sampling distance δ. The problem is to find a placement of nodes with the minimum number of nodes such that for all possible direction vectors *D* (through center *c*), the maximum distance for (*D,R*) is at most δ. In addition, the network must survive (i.e., remain connected) after one node failure. In this way, our node placement is "future proof" for future data that may have different and at present unknown directions.

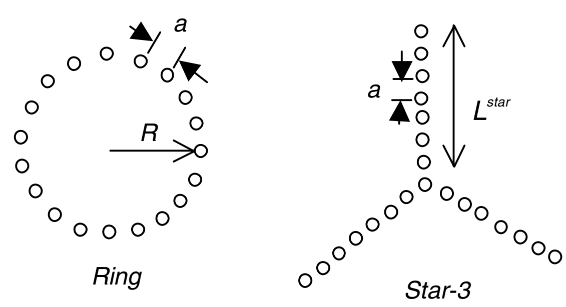

Figure 7. The Ring and Star-*m* arrangements, where *m* = 3.

We consider two types of node arrangements, shown in Figure 7, and discuss how well they solve the problem. All nodes in the arrangements are equally (or nearly equally) spaced. The "Ring" is a ring arrangement with radius *R*. The "Star-*m*" arrangement is a star with *m* equally spaced spokes of the same length $L^{star} = \dfrac{R}{\cos(\pi/m)}$, $m \geq 3$. The length L^{star} is sufficient for the geographical radius *R* as shown in Appendix C.

The star topology places many nodes near the center of the area, which may be advantageous for some deployments.

Table 4 compares a number of attributes of the arrangements assuming *N* is large. The first attribute is *a*, which is the spacing between adjacent nodes given *N* nodes. For the ring, $a \approx \dfrac{2\pi R}{N}$, and for Star-*m*, $a \approx \dfrac{mL^{star}}{N}$.

Table 4. Comparison of Ring and Star-*m*. The formulas are approximate, but the approximation sign ("≈") has been left out for simplicity.

	Ring	*Star-3*	*Star-4*	*Star-5*
a	$6.28\frac{R}{N}$	$6\frac{R}{N}$	$5.64\frac{R}{N}$	$6.18\frac{R}{N}$
d	$3.14\frac{R}{N}$	$3\frac{R}{N}$	$2.82\frac{R}{N}$	$3.09\frac{R}{N}$
r1	$6.28\frac{R}{N}$	$12\frac{R}{N}$	$11.28\frac{R}{N}$	$12.36\frac{R}{N}$
$N_s(d)$	$3.14\frac{R}{d}$	$3\frac{R}{d}$	$2.82\frac{R}{d}$	$3.09\frac{R}{d}$
$N_t(r1)$	$6.28\frac{R}{r1}$	$12\frac{R}{r1}$	$11.28\frac{R}{r1}$	$12.36\frac{R}{r1}$

The attribute *d* = *a/2* is an upper bound on the maximum distance for (*D,R*) over all possible *D*. To see this, consider an arbitrary direction *D*. First, consider the Ring arrangement, and the projections of its nodes on *D*. Since the Ring has radius *R*, the projections cover the segment region *D**. Since the space between projections is at most *a*, the maximum distance for (*D,R*) is *a/2*. Next, consider the Star-*m* arrangement, and the projections of its nodes on *D*. The length L^{star} has been chosen so that the projections completely cover the segment region *D** (Appendix C explains why L^{star} is sufficiently long). Since the spacing between projections is at most *a*, the maximum distance for (*D,R*) is *a*/2. Thus, for the Ring and Star-*m*, *a*/2 is the maximum distance for (*D,R*). Since *D* is arbitrary, *a*/2 is the maximum distance for (*D,R*) over all possible *D*. Therefore, we have *d* = *a*/2.

The value *r1* is the required transmission range so that the network will survive a single node failure. For the Ring, *r1* = *a* because the ring topology provides redundant connectivity. On the other hand, Star-*m* requires *r1* = 2*a* because the star topology does not provide the redundancy. This is similar to the *r1* for the in-line arrangement in Section 3. Note that we do not considered a paired arrangement because paired has an advantage over in-line only when there are multiple failure.

Since we have *d* and *r1* as functions of *N*, we can write *N* as a function of *d* and then as a function of *r1*. These functions are denoted by $N_s(d)$ and $N_t(r1)$, respectively. Recall that *r1* and *d* are constrained so that $r1 \geq r$, to insure that the network is survivable, and $\delta \geq d$, to insure that the placement of nodes is within the required sampling distance δ. Thus, the minimum number of nodes *N* should be

$$N_{\min} \equiv \max\{N_s(\delta), N_t(r)\}$$

Note that if transmission range *r* is small compared to required sampling distance δ then $N_{\min} = N_s(\delta)$ and Star-4 minimizes the number of nodes. On the other hand, if the transmission range *r* is large compared to the required sampling distance δ then $N_{\min} = N_t(r)$ and Ring minimizes the number of nodes.

5. Final remarks

We have studied a number of deployment strategies for nodes that are used both to sense the environment and communicate with other units. Cost considerations require using the smallest possible number of nodes. In addition, nodes must be placed to sample the environment sufficiently, and provide a network that survives node failures. We found that the required sampling distance δ and transmission radius r are key parameters.

We considered regular placements for 2-dimensional and 1-dimensional regions. We also considered Ring and Star-*m* deployments for a special type of data that varies in one direction *D* and is constant in the direction orthogonal to *D*. The placements were compared and we observed that the choice of placements depended on δ and r.

In practice each deployment is unique and must be customized to its specific location, and may be a hybrid of the techniques presented. Nonetheless, these results should help design effective and efficient deployments of wireless sensor networks.

Acknowledgements

We are indebted to Kim Bridges, University of Hawaii, for providing the fundamental motivation for this work as well as continuing support and encouragement.

Appendix A. Triangular, square, and hexagonal arrangements

We will estimate parameters for the triangular, the square, and then finally the hexagonal tile arrangements as functions of the number of nodes N and the area of A, which we denote by $|A|$. The parameters of interest are the maximum distance d, and the length of a side of a tile a. Recall that A is assumed to be a square, and N is assumed to be large.

Triangle: The area of a triangle is $\frac{\sqrt{3}}{4}a^2$ [3] and $d = \frac{\sqrt{3}}{3}a$ which implies that the area of the triangle is $\frac{3}{4}\sqrt{3}d^2$. In addition, each node will be a corner of six triangles. Since a triangle has 3 corners, each triangle requires approximately half a node. There are N nodes, so there are approximately $2N$ triangles. Since each triangle has an area of $\frac{3}{4}\sqrt{3}d^2$, the triangles cover an area of $2N\frac{3}{4}\sqrt{3}d^2$, which must be equal to $|A|$. Thus, $d \approx 0.62\sqrt{\frac{|A|}{N}}$. We can combine this with $d = \frac{\sqrt{3}}{3}a$ to get $a \approx 1.07\sqrt{\frac{|A|}{N}}$.

Square: The area of a square tile is a^2 and $d = \frac{\sqrt{2}}{2}a$ which implies that the area of a square is $2d^2$. In addition, each node will be a corner of four squares. Since a square has four corners, each square requires a node. There are N nodes, so there are approximately N squares. Each square has an area of $2d^2$. Therefore, the squares cover an area of $2Nd^2$, which must be equal to $|A|$. Thus, $d \approx 0.71\sqrt{\frac{|A|}{N}}$ which is larger than for the triangle tiles. Note that $a \approx \sqrt{\frac{|A|}{N}}$ because the squares cover an area of Na^2, which must be equal to $|A|$.

Hexagon: A hexagon is composed of six triangles as shown in Figure 8. Each triangle has sides of length a. Thus, the area of a hexagon with side of length a is $6\frac{\sqrt{3}}{4}a^2 = \frac{3}{2}\sqrt{3}a^2$. Figure 8 shows that each node is a corner to approximately 3 hexagons, and each hexagon has 6 corners. Therefore, there are approximately 2 nodes per hexagon. Therefore, the total area covered by N nodes is approximately $\frac{3}{4}\sqrt{3}Na^2$. In addition, for a hexagon, $d = a$. Therefore, the total area covered by N nodes is approximately $\frac{3}{4}\sqrt{3}Nd^2$. This area is equal to $|A|$, so $|A| = \frac{3}{4}\sqrt{3}Nd^2$. This implies $d \approx 0.88\sqrt{\frac{|A|}{N}}$. We can combine this with $d = a$ to get $a \approx 0.88\sqrt{\frac{|A|}{N}}$.

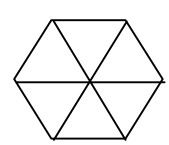

Figure 8. A hexagon divided into 6 equilateral triangles.

Appendix B. Sparse grids

We will derive the formulas of N as a function of $|A|$, r, and a for the sparse grids. These grids are the triangular, square and hexagonal sparse grids.

For a triangular sparse grid, the two nodes at the end of a tile side are shared among six triangles, and the remaining $s-2$ nodes are shared between two triangles each. The total number of nodes per tile is then $3\left(\frac{s-2}{2}+\frac{1}{3}\right) = \frac{3}{2}s$. The area of a tile is still $\frac{\sqrt{3}}{4}a^2$, so for T tiles to cover an area $|A|$ we have $T = \frac{4}{\sqrt{3}}\frac{|A|}{a^2}$. The total number of nodes is then $N = \frac{4}{\sqrt{3}}\frac{|A|}{a^2}\left(\frac{3}{2}\left(\left\lceil\frac{a}{r}\right\rceil+1\right)\right)$, where we have used $s=\lceil a/r\rceil+1$.

A similar computation for square grids gives $s-2$ as the total number of nodes per square tile. The total number of nodes is $N = \frac{|A|}{a^2}(2\lceil a/r\rceil)$. For hexagonal grids, each tile requires $3s-2$ nodes, and the total number of nodes to fill an area is $N = \frac{4}{3\sqrt{3}}\frac{|A|}{a^2}(3\lceil a/r\rceil+1)$.

For all these grids, the factor a is found both in the numerator and, squared, in the denominator, so as a increases, the number of nodes decreases, as would be expected.

Appendix C. Calculation of L^{star}

In this appendix, we consider the Star-m arrangement of Section 4. Recall that each "spoke" of the star has length $L^{star} = \frac{R}{\cos(\pi/k)}$. We will show that this is long enough to insure that the projections of the nodes "cover" an arbitrary direction vector D. In particular, the projections must "cover" segment D^*, which is the part of D that is within R of center c (see Figure 6).

Now notice that D^* is halved at c. To simplify our discussion we will only consider one half of the segment. We will determine the value of L^{star} so that the projections cover the half-segment. Note that the half-segment and an arbitrary spoke form an angle between them because they meet at center c. Consider the spoke σ that forms the smallest angle θ. This angle can be at most π/m because the angle between consecutive spokes is $2\pi/m$. Then the projections made by the nodes of spoke σ run along the half-segment starting from the center c for a distance $L^{star}\cos(\pi/m)$. Since $L^{star} = \frac{R}{\cos(\pi/m)}$ and the half-segment has length R, the projections "cover" the half-segment.

References

[1] E. Biagioni and K. Bridges, "The Application of Remote Sensor Technology to Assist the Recovery of Rare and Endangered Species", *International Journal of High Performance Computing Applications*, vol. 16, no. 3, Fall 2002.

[2] E. Biagioni, B. Chee, and K. Bridges, "A Remote Ecological Micro-Sensor Network", Grant proposal (overview), funded by DARPA in June 2000. http://www.botany.hawaii.edu/pods/overview.htm

[3]http://mathworld.wolfram.com/EquilateralTriangle.html

[4] T. S. Rappaport, "Wireless Communications: Principles and Practice", Prentice Hall, New Jersey, 1996.

[5] The CMU Monarch Project, "Wireless and Mobility Extensions to ns", Snapshot release 1.1.1, August 5, 1999. http://www.monarch.cs.cmu.edu/

[6] R. Ramanathan and R. Rosales-Hain, "Topology Control of Multihop Wireless Networks using Transmit Power Adjustment", Proceedings of the IEEE INFOCOM, pages 404-413, Tel Aviv, Israel, March 2000. http://www.ir.bbn.com/~ramanath/

[7] *The State Climatologist* Publication of the American Association of State Climatologists: Heights and Exposure Standards for Sensors on Automated Weather Stations, vol. 9, no. 4, October 1985.

[8] Guide to Meteorological Instruments and Methods of Observation. World Meteorological Organization no. 8, 5th edition, Geneva Switzerland, 1983.