Outline

- trees
- binary search trees
- tree traversal
- binary search tree algorithms: add, remove, traverse
- binary node class

trees

- imagine having to store, in an organized fashion, all the descendants of a given person
- as in this family tree:

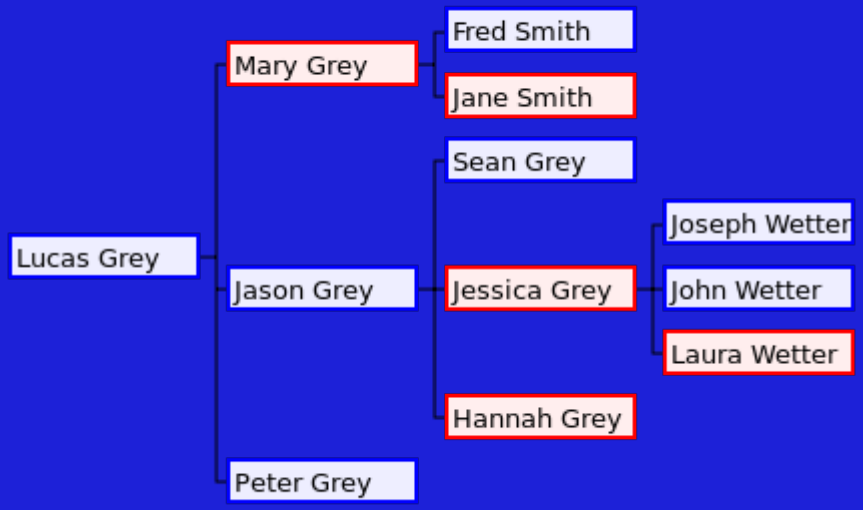


image by Derrick Coetzee, in entry “family tree” in Wikipedia

Trees in Computer Science

- trees can hold any kind of data, even numbers:

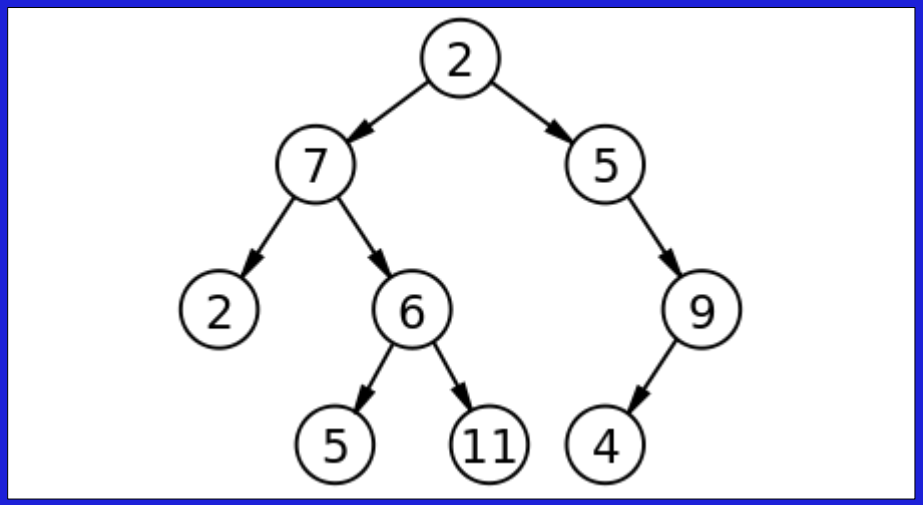


image by Derrick Coetzee, in entry “Tree (data structure)” in Wikipedia

- a node in a tree is similar to a node in a linked list, except:
  - a linked list node has a reference to zero or one link nodes, whereas
  - a tree node has a reference to zero or more other tree nodes
- later we will consider how trees are implemented

Tree Properties

- a tree has one **root** node, from which all other nodes can be reached by following links
- each node in a tree, except the root node, has exactly one **parent**
- each node in a tree can have zero or more **children**
- the other children of a node's parent are the node's **siblings**
- nodes are in a hierarchical relationship:
  - node X is an ancestor of another node Y, or
  - node X is a descendant of another node Y, or
  - node X is on a different branch than node Y
- nodes without children are **leaf nodes**
- nodes with children and a parent are **interior nodes**

Tree Properties Exercise

- in-class exercise (everyone together): identify the

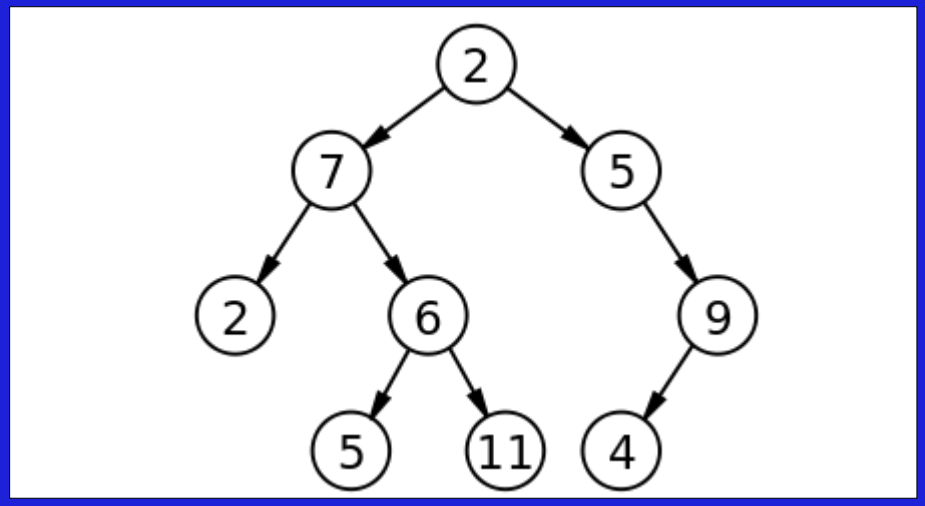


image by Derrick Coetzee, in entry “Tree (data structure)” in Wikipedia

- root node
- interior nodes
- leaf nodes
- parent of node 6
- children of node 6
- ancestors of node 9
- descendants of node 7

More Tree Definitions

- a **subtree** is a node with all its descendants
- the **level** of a node is the number of its ancestors (including itself), so e.g. the root is always at level 1:
  - the children of the root are at level 2
  - their children are at level 3
  - the children of a node at level n are at level n+1
- sometimes the word **depth** is used instead of level
  - sometimes the level or depth does not include the node itself, in which case the root is at level 0
- the **height** of a tree is the maximum depth of any node in the tree
  - this may also be called the depth of the tree
- in a **binary tree** each node has at most two children
- likewise, in a **ternary tree** each node has at most three children

types of binary trees

- in a **balanced binary tree** the height of each subtree of every node differs by at most one

  (this is a recursive definition)

  there are also other definitions of balanced binary trees
- in an **expression tree**, each internal node contains an operator, and each leaf node is an operand (value)
  - subtrees are evaluated first, yielding a value. Once all its children have been evaluated, we can evaluate a node
- in a **Huffman tree**, each internal node has two children. Each leaf node represents something to encode, such as a letter. The binary string to encode the letter is found by going down the tree from the root to the leaf. Each time we go left contributes a 0 bit to the code, and each time we go right contributes a 1 bit to the code.
- in a **binary search tree** each node has a value greater than every node in its left subtree, and less than every node in its right subtree
- in a **full binary tree**, each node has either 0 or two children (non-leaf nodes are full)
- a **perfect binary tree** of height n has $2^n - 1$ nodes, of which 2^{n-1} are leaves
- a **complete binary tree** is a perfect binary tree up to the last level. All the leaves at the lowest level are as far to the left as possible

expression trees

- an expression tree is a tree where:
  - every node with children is an operator
  - every leaf node is an operand
- each operator operates on the values of its children

binary search trees

- in-class exercise: how do we know this is a binary search tree?

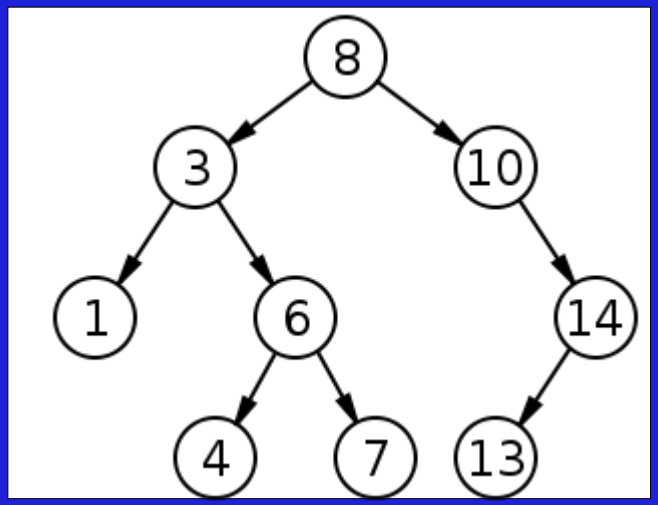


image by Derrick Coetzee and Booyabazooka, in entry “Binary search tree" in Wikipedia

binary search in binary search trees

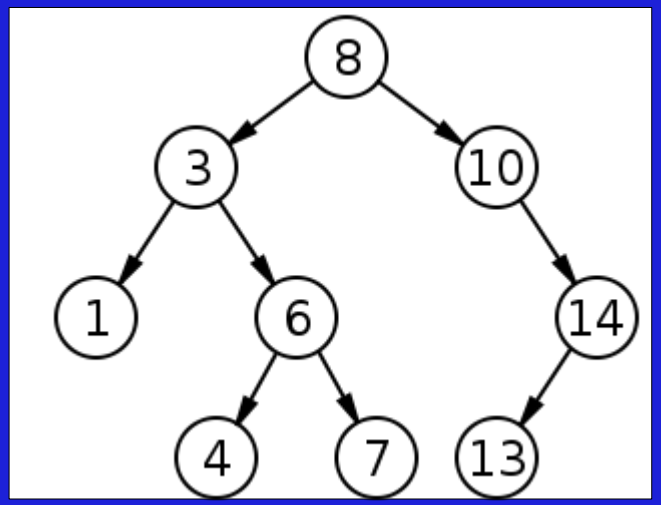


image by Derrick Coetzee and Booyabazooka, in entry “Binary search tree" in Wikipedia

- binary search (for value x) is now much easier:
  - if the value in the root is x, we are done
  - otherwise, if x < the value in the root, search in the left subtree
  - otherwise, x > the value in the root, so search in the right subtree
  - if the subtree we need to search is empty, x is not in the tree
- note this is a recursive definition
- in-class exercise: what is the runtime of this algorithm?

binary tree traversals

- if we want to visit each node in a binary tree, we have a choice of how to do it:
  - preorder traversal: visit the root node, then recursively visit the left subtree, then the right subtree
  - inorder traversal: recursively visit the left subtree, then visit the root node, then recursively visit the right subtree
  - postorder traversal: recursively visit the left subtree, then the right subtree, then the root node
- what does “visit a node” mean?
  - could mean printing the value
  - could mean saving the value in a data structure

binary tree traversal exercises

- do pre-order, inorder, and post-order traversals of these trees

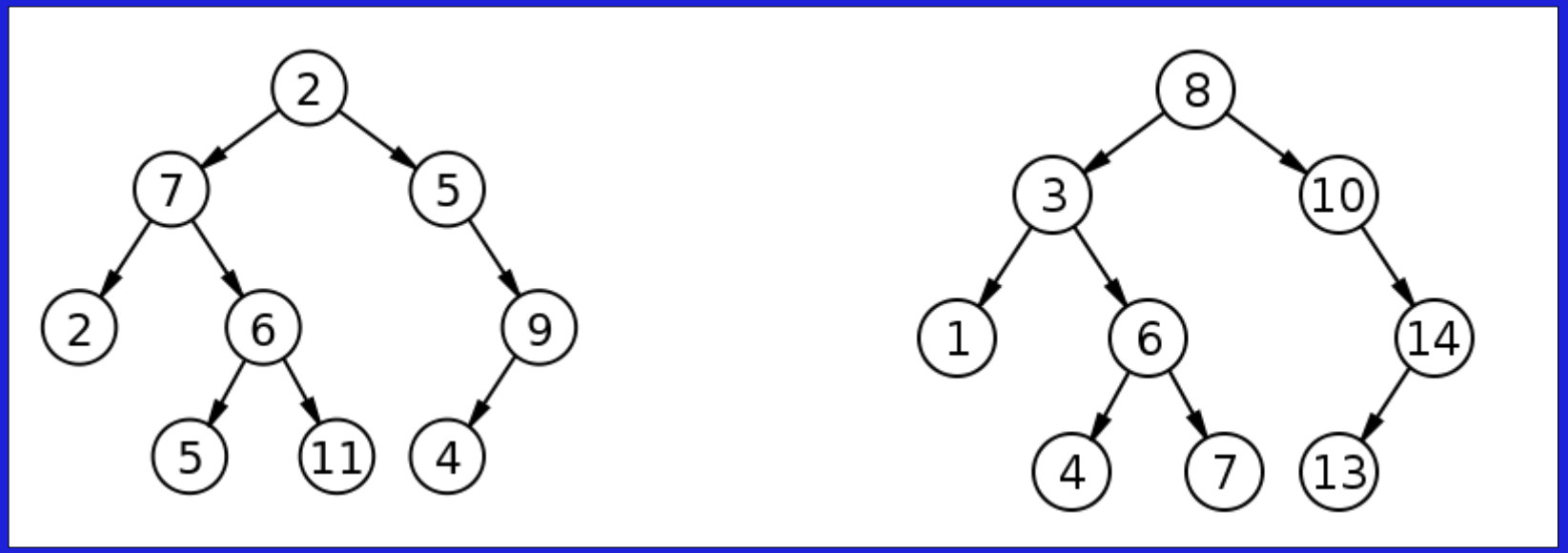
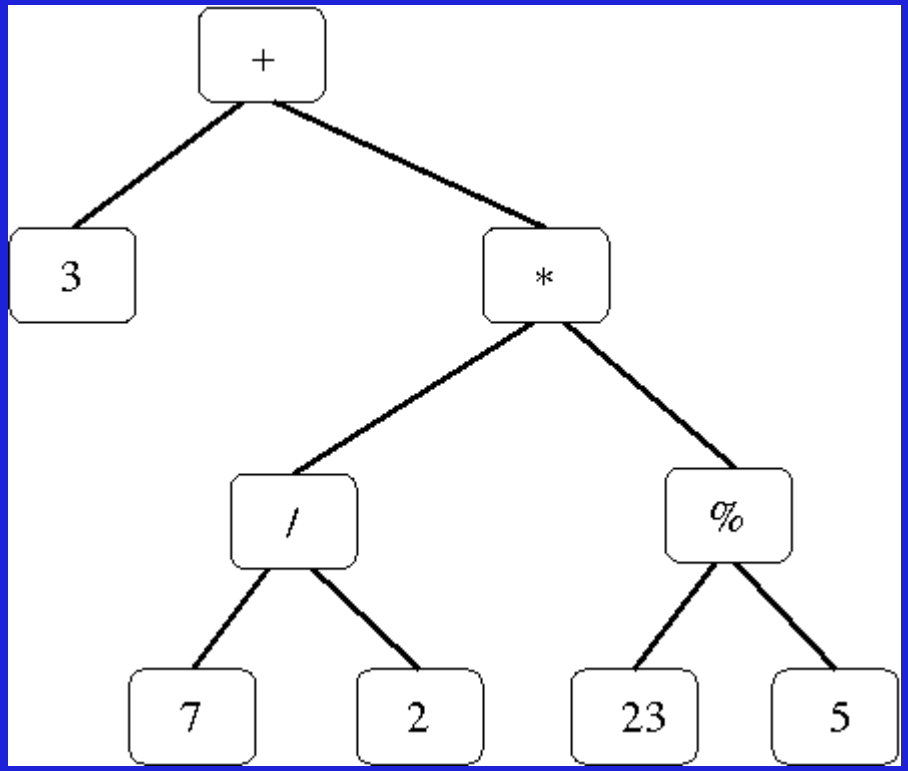


image by Derrick Coetzee, in entry " Tree (data structure)" in Wikipedia

image by Derrick Coetzee and Booyabazooka, in entry " Binary search tree" in Wikipedia

expression tree exercise

- do a pre-order, in-order, and post-order traversal of this tree



binary search tree properties

- adding, finding (get), or removing a node are all O(depth), which, **if the tree is balanced**, is O(log n), with n the number of nodes
  - in a sorted array, binary search is always O(log n), inserting is O(n)
- binary search trees are an efficient way to search data and to sort data, as long as the trees remain balanced
- an unbalanced binary tree might look like a linked list
- in-class exercise: what is the runtime of the `get` method when the tree is not balanced?
- data can be identified with a unique key
- in-class exercise: add everyone's name to a search tree (in groups of five-ten people)
- in-class exercise: is the resulting tree balanced?

binary search tree `add` operation

- if key is less than the key of the current node, recursively add to the left subtree
- if key is greater than the key of the current node, recursively add to the right subtree
- otherwise, add at this node:
  - if there is no current node, create one and return it
  - if there is a current node, replace its contents with the new contents
- this assumes:
  - the method returns a new root for the new (sub)tree with the desired value inserted
  - the caller of this method knows what to do with the new root
- in-class exercise: create a new binary tree using as keys the following letters, in the order given: "hello world". These keys each carry the value 1, 2, 3, 4, 5, 6, 7 8, 9, 10, 11

binary search tree `remove` operation

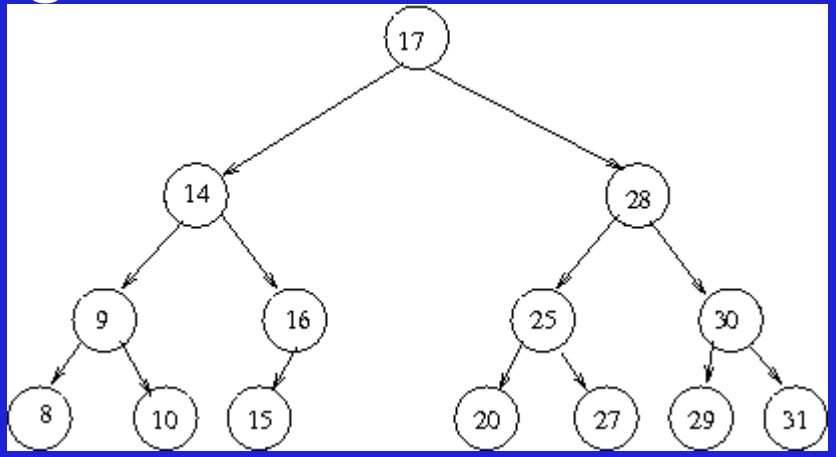
- find node with the given key
- if the node is a leaf node, delete it, return `null`
- if the node only has a left subtree, return the left subtree
- if the node only has a right subtree, return the right subtree
- if the node has both subtrees, must replace it with another node that fits in the slot

removing a node that has both subtrees

- the rightmost node in the left subtree can be put in place of the current node without altering the sorted property
- likewise, the leftmost node in the right subtree can be put in place of the current node
- that node might have a left (right) subtree, which can be used in its old position
- the rightmost node in the left subtree is the inorder predecessor of the current node
- the leftmost node in the right subtree is the inorder successor of the current node
- so either can be used

node removal exercises

- in-class exercise (everyone together): delete node 17 from this tree
- in-class exercise (individually): delete node 14 from the original tree
- in-class exercise (individually): delete node 31 from the original tree



BinaryNode class

- a singly-linked list node has (at most) one reference to another node
- a binary tree node has (at most) two references to other nodes
- for example, see BinaryNode.java
- both types of nodes also need a reference to the locally stored value (possibly including a key)
- data fields are item, left, right
- methods include constructors, accessor methods, mutator methods, toString
- test program builds small tree, tests the methods

BinaryNode exercises

- in-class exercise: write a recursive static method to print in post-order a tree, given its root
- in-class exercise: build the following tree using the methods from class BinaryNode