ICS 661 Midterm Solution

1. Given the following relative bigram probabilities, show the formula for the Perplexity of “Footballs can ice up.”

<table>
<thead>
<tr>
<th></th>
<th>can</th>
<th>footballs</th>
<th>ice</th>
<th>up</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;s&gt;</td>
<td>0.08</td>
<td>0.07</td>
<td>0.05</td>
<td>0.007</td>
</tr>
<tr>
<td>can</td>
<td>0.001</td>
<td>0.01</td>
<td>0.02</td>
<td>0.002</td>
</tr>
<tr>
<td>footballs</td>
<td>0.03</td>
<td>0.00001</td>
<td>0.0002</td>
<td>0.003</td>
</tr>
<tr>
<td>ice</td>
<td>0.04</td>
<td>0.005</td>
<td>0.00002</td>
<td>0.06</td>
</tr>
<tr>
<td>up</td>
<td>0.0007</td>
<td>0.008</td>
<td>0.009</td>
<td>0.00003</td>
</tr>
</tbody>
</table>

In the above table, \( P(\text{up} \mid \text{ice}) = 0.06 \)

Unfortunately, the \(<s>\) column on the far right got left out in transferring from Excel to Word, so \( P(<s> \mid \text{up}) \) could not be included in the exam or solution.

\[
P(\text{“Footballs can ice up”})
\]

\[
= \frac{1}{\sqrt[5]{P(\text{footballs}|<s>) \cdot P(\text{can}|\text{footballs}) \cdot P(\text{ice}|\text{can}) \cdot P(\text{up}|\text{ice}) \cdot P(<s>|\text{up})}}
\]

\[
= \frac{1}{\sqrt[5]{0.07 \cdot 0.03 \cdot 0.02 \cdot 0.06 \cdot P(<s>|\text{up})}}
\]

\[
= 13.2 \text{ if } P(<s>|\text{up}) = 1
\]
2. Given the following HMM and the relevant observation likelihoods, show the Viterbi algorithm in action (the trellis) for “bats should fly.”

<table>
<thead>
<tr>
<th></th>
<th>bats</th>
<th>should</th>
<th>fly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aux</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>N</td>
<td>0.02</td>
<td>0</td>
<td>0.06</td>
</tr>
<tr>
<td>V</td>
<td>0.009</td>
<td>0</td>
<td>0.02</td>
</tr>
</tbody>
</table>

![Trellis diagram showing the Viterbi algorithm applied to the sequence “bats should fly.”]
3. Given the following grammar and lexicon, show the top-down parse chart for “I saw her with glasses.” Be sure to show all constituents and all partial matches. Be sure to label all constituents with a unique number and also specify their sub-constituents.

Grammar:

S → NP VP
NP → Pro
NP → N
NP → NP PP
VP → V NP
VP → VP PP
PP → Prep NP

Lexicon:

Pro → I | her
N → glasses | saw
Prep → with

[0,0]:
S → NP VP
[1,1]:
VP → V NP
[2,2]:
NP → Pro
NP → N
NP → NP PP
[3,3]:
PP → Prep NP
[4,4]:
NP → Pro
NP → N
NP → NP PP

S1(NP1,VP1)
S2(NP1,VP2)
S3(NP1,VP3)
VP3(V1,NP5)
VP2(VP1,PP1)
NP5(NP2,PP1)

NP1(Pro1)
Pro1(I)

NP3(Pro2)
V1(saw)
Pro2(her)

NP4(N1)
Prep1(with)
N2(glasses)

0 1 saw 4 her 3 with 4 glasses

S → NP • VP
NP → NP • PP
VP → V1 • NP
VP → VP1 • PP
NP → NP • PP
VP → VP2 • PP
VP → VP3 • PP
4. Given the Probabilistic Context-Free Grammar below, show the probabilistic CKY parse table for “I saw her with glasses.” Be sure to label all constituents with a unique number and also specify their sub-constituents.

**Grammar:**
- \( S \rightarrow NP \ VP \ [1] \)
- \( NP \rightarrow NP \ PP \ [0.01] \)
- \( VP \rightarrow V \ NP \ [0.9] \)
- \( VP \rightarrow VP \ PP \ [0.1] \)
- \( PP \rightarrow Prep \ NP \ [1] \)

**Lexicon:**
- \( NP \rightarrow I \ [0.3] \)
- \( NP \rightarrow her \ [0.4] \)
- \( NP \rightarrow glasses \ [0.1] \)
- \( NP \rightarrow saw \ [0.2] \)
- \( V \rightarrow saw \ [1] \)
- \( Prep \rightarrow with \ [1] \)

<table>
<thead>
<tr>
<th></th>
<th>saw</th>
<th>her</th>
<th>with</th>
<th>glasses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td>S1(NP1,VP1) ( .3 \times .36 \times 1 =.108 )</td>
<td>S2(NP1,VP2) ( .3 \times .036 \times 1 =.0108 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NP1 .3</td>
<td>VP1(V1,NP3) ( 1 \times .4 \times .9 =.36 )</td>
<td>VP3(VP1,PP1) ( .036 \times 1 \times 1 =.036 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NP2 .2</td>
<td>V1 1</td>
<td>VP2(V1,NP5) ( 1 \times .004 \times .9 =.0036 )</td>
<td>VP5(NP3,PP1) ( .4 \times 1 \times .001 =.0004 )</td>
<td></td>
</tr>
<tr>
<td>NP3 .4</td>
<td>PP1(Prep1,NP4) ( 1 \times 1 \times 1 =.1 )</td>
<td>NP4 .1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>