AN ANALYTICAL FRAMEWORK FOR PREDICTING THE PERFORMANCE OF AUTONOMOUS UNDERWATER VEHICLE POSITIONING

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Abstract—In this paper we present a set of simple models to predict the performance of underwater positioning solutions for autonomous underwater vehicles (AUVs). Improvements in underwater navigation continue to contribute to the expansion of our capability for exploration, scientific discovery and environmental monitoring. Despite the diversity instruments and algorithms employed to improve performance, the fundamental trade-offs involved in estimating position can be understood with a few general stochastic observation models and information-centric analysis.

We present models for the most common oceanographic navigation observations: absolute reference solutions such as long-baseline (LBL) acoustic positioning, relative positioning solutions such as Doppler velocity log (DVL) dead-reckoning and accurate heading such as a fiber optic gyro (FOG) reference. We combine these simple models using the Cramér Rao lower bound to predict both the precision and accuracy of common AUV positioning solutions. To demonstrate the utility of such predictions, we model the uncertainty of current solutions and proposed future implementations. Many current solutions use a combination of LBL positioning and DVL odometry. We show how this approach can predict the performance of such a configuration.

Finally, we substantiate the use of this framework by comparing model-based performance predictions to experimental evidence. The field experiment we present is a controlled evaluation of DVL dead-reckoning, using the Jason remotely operated vehicle (ROV). We observe a total error growth that can be approximated as 0.04% of distance traveled.

I. INTRODUCTION

Autonomous Underwater Vehicles (AUVs) are powerful tools for exploring, investigating and managing our ocean resources. As the capabilities of these platforms continue to expand and AUVs continue to mature as operational assets, navigation remains a fundamental technological component. Determining position, in real-time or via post-processing, is a foundational capability; improved navigational techniques make possible the application of AUV technology to a wider variety of scientific, military and industry needs.

The two basic building blocks of an underwater navigation solution are absolute and relative positioning. There are many methods and a variety of sensors for each type of positioning (see [1]–[3] for review articles), but the fundamental trade-off is captured by the dichotomy of absolute versus relative positioning.

The goal of the research presented here is to predict the performance of state-of-the-art AUV navigation solutions. Predicting the resulting navigation uncertainty requires a fundamental understanding of how the system accumulates information in order to estimate position.

A. Approach

We pose the underwater positioning as an estimation problem, allowing us to predict the performance of a variety of AUV navigation solutions. To do this we use simple stochastic models of both absolute and relative position references. These models quantify the salient contributors to system-level positioning performance: uncertainty in range observations, geometry of fixed reference locations, uncertainty in velocity observations, uncertainty in heading observations, update rate of observations and vehicle speed. The stochastic models combine each of these parameters, and, by using the Cramér Rao lower bound (CRLB), we can predict how each component of the system affects the overall navigation performance. To illustrate the efficacy of these simple models we use data from field operations to qualify the agreement between the model and real-world performance.
II. BACKGROUND AND CLOSELY RELATED WORK

A. Underwater Positioning and Navigation

Absolute position is estimated using an external reference at a known location. At the surface the global positioning system (GPS) [4] can provide such a reference using orbiting satellites. Because seawater is opaque to electromagnetic radiation, underwater applications use acoustic positioning solutions such as long baseline (LBL), short baseline (SBL) or ultra-short baseline (USBL) to provide an absolute position reference [5]. For AUV applications the most common of these is LBL positioning where transponders moored to the seafloor provide a fixed reference. In either case, using GPS during a surface interval or acoustic positioning while submerged, the result is a position estimate in three dimensions with respect to an absolute Earth-centered, Earth-fixed reference (ECEF) frame with bounded uncertainty.

The performance of LBL positioning can vary greatly depending on the geometric configuration of the fixed transponders and the uncertainty in the estimation of range between the AUV and these fixed locations. Hunt et al. provided one of the early descriptions of LBL positioning in 1974. They estimated the accuracy of this early deep-water implementation to be between 10 and 20 m, depending on signal-to-noise ratio of the transmitted pulse [6]. Using similar continuous wave pulse transmissions, we estimated the range standard deviation to be 3.97 m during a using Autonomous Benthic Explorer (ABE) AUV mission using Benthos transponders [7]. The REMUS AUV comes standard with LBL capabilities with a reported repeatability of approximately 1.5 m [8]. Using higher frequencies and more complex signal processing can yield higher performance positioning. We report have achieved sub-centimeter resolution using high-frequency spread-spectrum signals [9], [10].

Not only are these reported values for performance highly variable, but the reader will also note that it is challenging to compare the performance because the reporting of uncertainty is not standard. While some authors refer to precision, some attempt to quantify the overall accuracy of the solution. Even the metrics are variable as some report summary statistics such as standard deviation, CEP or 95% confidence while some simply report a single value for measured error.

As a complement to absolute positioning, dead-reckoning provides an estimate of relative position, typically provided by either a inertial navigation system (INS) or, more commonly, a DVL integrated with an accurate heading reference. Dead-reckoning provides an estimate of the distance traveled from an arbitrary reference point. A key advantage of this mode of navigation is that it is self-contained without the need for an external reference, making it ideal for rapid deployments. The disadvantage is that the uncertainty in dead-reckoning is not bounded, the quality of the position estimate degrades with time and distance traveled.

Just as with acoustic positioning performance, dead-reckoning performance can vary greatly depending on implementation. For example, Brokloff reported position errors between 0.33 and 0.51% distance traveled [11] and Bluefin Robotics reports 0.5% of distance traveled (three-sigma) for real-time dead-reckoning and 0.3% distance traveled after post-processing [12]. Using a more sophisticated INS, McEwen et al. report a position error growth of less than 0.05% distance traveled for using a DVL/INS combination developed by Kearfott and RD Instruments [13].

Neither absolute or relative positioning is typically used in a standalone fashion, nor is there a standard, one-size-fits-all solution. By combining the right dead-reckoning solution with periodic updates from an absolute position reference, the advantages of both methods are combined to deliver high-update rate position updates with bounded absolute precision [14].

B. Modeling Positioning Uncertainty

This research builds on previous efforts in the modeling of navigation uncertainty. The accuracy of multilateration solutions as been investigated in the context of underwater [15], terrestrial (GPS) [4], [16] and space [17] navigation. Specifically for long-distance AUV navigation, Eustice et al., developed a uncertainty model to predict the performance of one-way-travel-time acoustic navigation that utilizes both absolute and relative navigation references [18].

To integrate the stochastic models we make use of the Cramér Rao lower bound, a standard tool of estimation theory [19], [20].

III. STOCHASTIC MODEL FOR PERFORMANCE PREDICTION

The framework we propose uses two simple component models for quantifying the uncertainty in absolute (range-based) and relative (dead-reckoning) underwater positioning solutions. We then use the CRLB to combine these two observations models allowing us to predict the performance of today’s integrated navigation solutions.

1Teledyne Benthos, Inc., TR-6001 acoustic transponders.

2The Seaborne Navigation System/Doppler Velocity Log (SEA DeViL) from Kearfott Corporation.
A. Model for Range-Based Absolute Positioning

The fundamental concept of estimating position through time-of-arrival measurements is the basis for many underwater, terrestrial and space navigation solutions. For the purposes of predicting the performance of AUV positioning, we use a spherical positioning model common for both LBL and GPS applications. This concept is illustrated in two-dimensions in Figure 1 and Figure 2.

Spherical positioning is based on observing individual range values \( z_r_i \) between known fixed beacon locations \( (x_{b_i}) \) and an unknown mobile host position \( (x_h) \) where the individual range measurements is indexed by \( i \).

\[
z_{r_i} = \|x_h - x_{b_i}\| + w_{r_i}
\]

(1)

The vector \( x \) is used generically to denote a three-dimensional position, i.e., \( x = (x, y, z) \). We consider the additive noise in each measurement \( w_{r_i} \) as an independent, zero-mean, Gaussian variable with variance \( \sigma^2_{r_i} \).

\[
w_{r_i} \sim \mathcal{N}(0, \sigma_{r_i}^2)
\]

(2)

We do not claim that this normal probability distribution captures the full complexity of range observations, but instead that this Gaussian error model represents the salient characteristics of the measurements while enabling a prediction of overall system performance.

B. Model for Velocity-Based Relative Positioning

The second stochastic observation model we employ concerns the measurement of vehicle velocity and heading to estimate course over ground. As discussed above, this is typically accomplished using the combination of a DVL with a heading reference. The DVL provides independent measurements of velocity \( (v_k) \) in each of three dimensions (indexed by \( k \)).

\[
z_{v_k} = v_k + w_{v_k}
\]

(3)

We characterize the uncertainty as mutually independent additive, zero-mean, Gaussian white noise.

\[
w_{v_k} \sim \mathcal{N}(0, \sigma_{v_k}^2)
\]

(4)

Transforming these sensor frame measurements into a local coordinate frame requires knowledge about sensor and vehicle attitude. Heading is the most important measurement for this coordinate rotation [21]. Again we use a simple additive Gaussian noise model to represent the heading \( (\psi) \) measurement.

\[
z_{\psi} = \psi + w_{\psi}
\]

(5)

\[
w_{\psi} \sim \mathcal{N}(0, \sigma_{\psi}^2)
\]

(6)

It is possible to carry forward the complete three dimensional \( (k = 1, 2, 3) \) formulation [18], for the clarity we present the two-dimensional case. For the remainder of this development we use the symbol \( x \) to signify the along track dimension and \( y \) to signify the across track dimension. This is a non-limiting simplification.
for two reasons: most underwater vehicles are passively stable in pitch and roll, furthermore pitch and roll are typically measured much more accurately than heading. We also consider the uncertainty along track to be independent of the uncertainty across track. This consideration captures the dominant dynamics of error growth and allows us to simplify our two-dimensional model, preserving intuition. This approach leads to a two-dimensional observation model of odometry where each discrete measurement of incremental distance \((z_{o_j}, \text{where } j \text{ is the index for velocity measurements in three directions})\) is subject to additive, zero-mean, Gaussian noise.

\[
z_{o_j} = (x_{h_j} - x_{h_{j-1}}) + w_o
\]  

The additive noise is characterized by a two-dimensional covariance matrix \((\Sigma_o)\) in the along track and across track directions.

\[
w_o \sim \mathcal{N}(0, \Sigma_o)
\]

\[
\Sigma_{o_i} = \begin{bmatrix} t\sigma_v^2 & 0 \\ 0 & d^2\sigma_\psi^2 \end{bmatrix}
\]  

The diagonal matrix in Equation (9) is a consequence of the independent along track and across track uncertainty growth. The along track term \((\sigma_x = \sqrt{t}\sigma_v)\) captures growth of position uncertainty as a function of velocity uncertainty, based on random walk uncertainty growth. The across track term \((\sigma_y = d\sigma_\psi)\) captures the linear uncertainty growth with distance traveled.

Figure 3 is an illustration of this simple odometry model. The aspect ratio of error ellipses increases with time, illustrating the linear growth of the along track uncertainty (growing with distance traveled) and the growth of the along track position with the square root of time.

C. The Cramér Rao Lower Bound for Predicting Performance

The third and final step in developing our prediction framework is to use the CRLB to quantify the informational contribution of absolute and relative positioning observations to our ability to estimate the true position. When it exists, the CRLB gives the lower bound on the variance of any valid unbiased estimator \(\hat{x}(\cdot)\) of the parameters \(x\). The Fisher information, \(I_z(x)\), is the information about the parameters, \(x\), contained in the observations, \(z\).

\[
I_z(x) = E \left[ \left( \frac{\partial}{\partial x} \ln p_z(z; x) \right)^2 \right] \tag{10}
\]

Where \(p_z(z; x)\) is the known probability density of the observations \(z\) which are dependent upon the parameters \(x\) and \(E[\cdot]\) is the expectation operator. The CRLB, \(A_x(x)\), is the inverse of the Fisher information.

\[
A_x(x) = [I_z(x)]^{-1}
\]

The CRLB is the minimum uncertainty achievable by an unknown optimal estimator. An estimator that approaches this bound is efficient, but the bound does not guarantee that an efficient estimator exists or that one can be found.

To apply this tool we must express the complete observation model for integrated absolute and relative positioning measurements. We present this in two dimensions, recalling that \(x\) is the along track dimension and \(y\) is the across track dimension. The result is a combined measurement model including \(n\) measurements of the absolute \(x\) and \(y\) positions along with \(n-1\) measurements of the relative distance traveled in each direction.

\[
\mathbf{Z}_c = \begin{bmatrix}  \tilde{x}_1 \\ \vdots \\ \tilde{x}_n \\ \tilde{y}_1 \\ \vdots \\ \tilde{y}_n \\ \delta \tilde{x}_2 \\ \vdots \\ \delta \tilde{x}_n \\ \delta \tilde{y}_2 \\ \vdots \\ \delta \tilde{y}_n \end{bmatrix} = [\mathbf{A}] \begin{bmatrix}  x_1 \\ \vdots \\ x_n \\ y_1 \\ \vdots \\ y_n \end{bmatrix} + \Sigma_c
\]

\[
\mathbf{A} = \begin{bmatrix}  \mathbf{I} & \mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}
\]
where

\[ A = \begin{bmatrix} I_{n \times n} & 0_{n \times n} \\ 0_{n \times n} & I_{n \times n} \\ \Delta_{n-1 \times n} & 0_{n \times 1} \\ 0_{n \times 1} & \Delta_{n-1 \times n} \end{bmatrix} \]  

\[ \Delta_{n-1 \times n} = \begin{bmatrix} 0 \\ \vdots \\ I_{n-1} \\ 0 \end{bmatrix} - \begin{bmatrix} I_{n-1} \\ \vdots \\ 0 \end{bmatrix} \]  

\[ \Sigma_c = \begin{bmatrix} \sigma_p^2 I_{2n} \\ 0 \\ \sigma_v^2 (dt) I_{n-1} \\ 0 \\ 0 \end{bmatrix} \]  

(13)

(14)

(15)

where \( I_n \) is an \( n \times n \) identity matrix, \( dt \) is the elapsed time between absolute position updates and \( dx \) is the corresponding distance traveled along track between successive absolute position updates.

The measurement model in Equation (12) relates the measurements of absolute and relative position to the parameters we wish to estimate, the actual position. Because the integrated model is linear and the error is modeled as having a Gaussian distribution, the CRLB is simply

\[ \Lambda_{\hat{x}}(x) = [A^T \Sigma_c^{-1} A] \]  

(16)

The resulting \( n \times n \) matrix bounds the position covariance matrix.

\[ \Lambda_{\hat{x}}(x) \leq \mathbb{E} \left[ [\hat{x}_h - \bar{x}_h][\hat{x}_h - \bar{x}_h]^T \right] \]  

(17)

where \( \hat{x}_h \) is the unknown, true \( n \times 2 \) position matrix of the vehicle and \( \hat{x}_h \) is the corresponding estimated 2D position matrix at each of the \( n \) positions.

D. Evaluating Integrated LBL/DVL Solutions

Using the framework developed in the previous section we develop a generalizable prediction of navigation performance for integrated solutions that employ both absolute and relative positioning techniques. Figure 4 illustrates this prediction for a large range of system parameters. To succinctly capture the summative positioning performance we use the notion of horizontal dilution of precision (HDOP). This scalar figure of merit summarizes the two-dimensional position uncertainty.

\[ \sigma_{HDOP} = \frac{\sqrt{\sigma_x^2 + \sigma_y^2}}{\sigma_p} \]  

(18)

We normalize the uncertainty in the \( x \) and \( y \) directions with the overall absolute positioning uncertainty (\( \sigma_p \)). To evaluate the HDOP for a particular integrated navigation solution requires quantifying the following parameters:

- Position uncertainty (\( \sigma_p \)) which is the standard deviation of the overall absolute position reference combining both contributions from range uncertainty and the geometric effects of the multilateration geometry.
- Along track uncertainty (\( \sigma_{along} \)) which is a function of the velocity uncertainty (\( \sigma_v \)) and the acoustic update rate (\( dt \)) which specifies the elapsed time between absolute position updates.
- Across track uncertainty (\( \sigma_{across} \)) which is a function of the heading uncertainty (\( \sigma_h \)) and the distance traveled between acoustic updates (\( dx \)). The distance traveled is simply the product of the vehicle velocity and the acoustic update rate.

To interpret the results in Figure 4 we can examine the three regions labeled A, B and C. On the left, in region A, the along track uncertainty is much smaller than the absolute position uncertainty. This scenario indicates that the velocity error is small and that the update rate is relatively fast, preventing large drift in the relative positioning between updates. The result, indicated by a low HDOP, is that the overall positioning uncertainty is 10% of the standalone absolute positioning. Because of the accuracy of the relative measurements between individual absolute updates, the information from each absolute reference cycle accumulates, lowering the overall uncertainty. In fact, this asymptote achieves the lower bound for the overall uncertainty

\[ \lim_{(\sigma_{along}/\sigma_p)\to 0} \sigma_{HDOP} = \frac{1}{\sqrt{n}} \]  

(19)
where \( n \) is the number of absolute position updates. In this case \( n = 100 \).

Another interesting aspect of this limiting case is that the across track uncertainty does not affect the resulting performance. This is an important result because for solutions that employ only relative positioning (dead-reckoning), the heading reference is a key determinate of performance. In contrast, the prediction illustrated in Figure 4 shows that when both absolute and relative positioning are combined and the velocity measurements have low uncertainty, the quality of the heading reference has little impact on the overall performance.

Region C, in the far right of Figure 4, illustrates the performance prediction for the opposite extreme when the relative positioning is poor compared to the absolute reference. In this case the HDOP asymptotically approaches 1.0, indicating that the standalone absolute position uncertainty is equivalent to the combined uncertainty. In other words, between each absolute position update the dead-reckoning drift is so large that the information from the last update is lost. Therefore, overall accuracy is no better than the standalone accuracy of the absolute reference.

1) Illustrative Examples: To further illustrate the results in Figure 4 we use the framework to analyze two possible scenarios for AUV navigation, quantified by the parameter values listed in Table I.

The first example (Ex 1) is based on a typical high-performance AUV deployment using instrumentation that will be familiar to many practitioners. Absolute position references are delivered through an array of Benthos LBL transponders (\( \sigma_p \approx 3.0 \text{ m} \)) which are interrogated once every 10 s to allow for acoustic propagation times. Relative positioning is accomplished using a 1,200 kHz RDI DVL (\( \sigma_v = 3 \text{ mm/s} \)) in combination with an Octans true north heading reference (\( \sigma_\psi = 0.1 \text{ degrees} \)). In order to calculate the across track uncertainty we also specify a typical vehicle speed of 1.0 m/s. The resulting system-level positioning performance for this scenario is illustrated by the annotation in Figure 4. In this case the relative positioning is sufficiently accurate achieve significant gains in the overall performance due to the complementary nature of these two positioning modalities. The HDOP value of 0.1 indicates that the overall performance is an order of magnitude more accurate than the standalone absolute reference.

In the second example (Ex 2) we examine the effect of using a high-performance absolute positioning reference. In this case the absolute reference accuracy is 1.0 cm, a lower limit of the performance of available solutions. This scenario also employs a heading solution with much more uncertainty (1.0 degrees). The results of this scenario are also illustrated with an annotation in Figure 4. Here we see that the HDOP is 0.75 indicating that the positioning is only 25% more accurate than the standalone absolute reference.

### Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ex 1</th>
<th>Ex 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position Uncertainty (( \sigma_p ))</td>
<td>3.0 m</td>
<td>1.0 cm</td>
</tr>
<tr>
<td>Velocity Uncertainty (( \sigma_v ))</td>
<td>0.003 m/s</td>
<td>0.003 m/s</td>
</tr>
<tr>
<td>Heading Uncertainty (( \sigma_\psi ))</td>
<td>0.1 degrees</td>
<td>1.0 degrees</td>
</tr>
<tr>
<td>Vehicle Speed</td>
<td>1.0 m/s</td>
<td>1.0 m/s</td>
</tr>
<tr>
<td>Acoustic Update Rate</td>
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<td>10 s</td>
</tr>
<tr>
<td>Along Track (( \sigma_{\text{along}} ))</td>
<td>0.009 m</td>
<td>0.009 m</td>
</tr>
<tr>
<td>Across Track (( \sigma_{\text{along}} ))</td>
<td>0.017 m</td>
<td>0.17 m</td>
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<tr>
<td>( \sigma_{\text{along}} / \sigma_p )</td>
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<td>0.9</td>
</tr>
<tr>
<td>( \sigma_{\text{across}} / \sigma_{\text{along}} )</td>
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<td>19</td>
</tr>
<tr>
<td>HDOP</td>
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</tr>
<tr>
<td>System Accuracy</td>
<td>30 cm</td>
<td>0.75 cm</td>
</tr>
</tbody>
</table>

IV. EXPERIMENTAL RESULTS

To evaluate our dead-reckoning model we performed an at-sea trial using the ROV Jason, part of the National Deep Submergence Facility operated by the Woods Hole Oceanographic Institution.

A. Experimental Setup

This experiment took place during an engineering lowering of the ROV. The lowering took place on June 21, 2007 as part of a scientific expedition to the Gulf of Mexico at roughly 2,200m total water depth. The navigation system aboard the ROV has been described by other researchers [14], [22]. For this trial the ROV was outfitted with an RDI 1,200 kHz Navigator DVL\(^3\) to measure velocity relative to the bottom and an Octans FOG compass and attitude sensor\(^4\).

The experiment provided a visual ground-truth for evaluating the precision (repeatability) of the relative position estimated via dead-reckoning. The ground-truth was provided by deploying a navigation benchmark as shown in Figure 2. Using the closed-loop control capability of the Jason ROV, we were able to position the vehicle directly above the benchmark at both the beginning and end of the experiment.

\(^3\)Teledyne RD Instruments

\(^4\)IXSEA, Inc.
Fig. 5. The dead-reckoning error was quantified using a stationary benchmark fixed on the seafloor. This down-looking image illustrates how the ROV was positioned directly over the benchmark at the start and the end of the experiment. Two parallel, calibrated laser markers were present in the image to aid in repeatable positioning relative to the bottom.

Fig. 6. Two-dimensional plot of the dead-reckoning track during the test. At both the "Start" and "End" points the ROV was positioned directly over the bottom-mounted benchmark (Figure 2). The difference between the recorded "Start" and "End" conditions serve as a measure of the DR drift.

B. Experiment Results

Figure 6 and Figure 7 show the two-dimensional position estimates reported from the dead-reckoning solution. At both the “Start” and “End” locations the vehicle was positioned directly over the navigation benchmark. The ROV was navigated roughly 90 m due south and returned to the original start position on a reciprocal heading during an elapsed time of approximately 23 minutes.

The experiment described above allows us to compare the predicted performance of the relative positioning with the actual measured error. Table II quantifies this comparison. The predicted standard deviation is derived from Equation (9) using published values for the sensor uncertainties. The measured error is smaller than predicted, 36% and 20% of the expected standard deviation in the along and across track direction. This measured two dimensional error corresponds to a 18% cumulative probability given the predicted values for the standard deviation. To compare these results to previous reports, we also calculate the total uncertainty as a fraction of the distance traveled. The measured value of 0.04% distance traveled is smaller than, but comparable to, those reported in the literature.

V. Conclusion

This article presents two results: a modeling framework for predicting the accuracy of combined absolute/relative positioning and an experimental evaluation of state-of-the-art relative positioning (dead-reckoning). The theoretical results show how the performance of both the absolute reference (eg., LBL acoustic positioning) and the relative navigation (eg., DVL dead-reckoning) affect the overall system-level accuracy. Figure 4 pro-
vides a succinct summary that captures the salient trade-offs in position uncertainty, velocity uncertainty, heading uncertainty, update rate and vehicle speed. Each of these parameters is an important determinate of overall positioning accuracy, and this framework provides a tool for making decisions on how best to configure a particular solution.

The experimental evaluation of the dead-reckoning navigation provides an example of one way of directly measuring positioning uncertainty. The positioning error was less that predicted, indicating that the combination of our simple model for error growth along with the manufacturers specifications is a conservative estimate of performance.

The goal of this work has been to provide a generalizable tool to guide future decisions. The stochastic models we choose are simple enough to provide system-level performance accounting, but detailed enough to capture the important dynamics. This ability to predict the performance of such integrated navigation solutions should provide guidance for both vehicle and instrument improvement. By targeting development efforts on the most important aspects of the solution, future efforts will have the most impact on the overall system performance.

VI. ACKNOWLEDGMENTS

We wish to thank the entire Deep Submergence Lab at the Woods Hole Oceanographic Institution for making room in the operation for this navigation evaluation during the RB-07-04 expedition. Similarly, Dr. Chuck Fisher, the Chief Scientist during the expedition, was patient enough to allow for a full engineering lowering.

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