Hypothesis Grids: Improving Long Baseline Navigation for Autonomous Underwater Vehicles

Brian Bingham, Member, IEEE, and Warren Seering

Abstract—Navigation continues to fundamentally limit our ability to understand the underwater world. Long baseline navigation uses range measurements to localize a remote vehicle using acoustic time-of-flight estimates. For autonomous surveys requiring high precision navigation, current solutions do not satisfy the performance or robustness requirements. Hypothesis grids represent the survey environment capturing the spatial dependence of acoustic range measurement, providing a framework for improving navigation precision and increasing the robustness with respect to non-Gaussian range observations. Prior association probabilities quantify the measurement quality as a belief that subsequent observations will correspond to the direct-path, a multipath, or an outlier as a function of the estimated location. Such a characterization is directly applicable to Bayesian navigation techniques. The algorithm for creating the representation has three main components: Mixed-density sensor model using Gaussian and uniform probability distributions, measurement classification and multipath model identification using expectation-maximization (EM), and grid-based spatial representation. We illustrate the creation of a set of hypothesis grids, the feasibility of the approach, and the utility of the representation using survey data from the autonomous benthic explorer (ABE).

Index Terms—Author, please supply your own keywords or send a blank e-mail to keywords@ieee.org to receive a list of suggested keywords.

I. INTRODUCTION

NAVIGATION, the combination of localization and guidance, remains a fundamental challenge to field robotics. For many robotic missions on land and in the air the global positioning system (GPS) [1] has revolutionized this problem, but not all applications can rely on a GPS position estimate. Sensor networks operate in places where GPS is not practical reasons of power and size [2], and robotic planetary exploration requires self contained navigation solutions independent of GPS. Saltwater is impenetrable to most forms of electromagnetic radiation, including GPS signals, challenging underwater remote-sensing and navigation schemes [3].

Navigation for underwater survey is sensor-limited. A localization solution is most often the product of combining complimentary sensing modalities emphasizing the strengths of each instrument. Long baseline (LBL) provides a low-update-rate, fixed-reference, bounded-error estimate of location based on acoustic travel times between transponders in fixed locations on the seafloor [4]. In contrast, a Doppler velocity log (DVL) or inertial navigation system (INS) provides a high-update-rate, internal-reference, unbounded-error position estimate based on integrating velocity or acceleration. Combining these types of measurements can provide a solution with the advantages of each, but requires an external reference, i.e., instrumenting the environment with transponders, a time consuming task not appropriate for all operations [5], [6].

Underwater navigation continues to improve while autonomous underwater vehicles (AUVs) prove their utility as operational assets. The challenge of autonomous underwater navigation, operating without a human in the loop, requires a new level of robustness to sensor operation. This paper presents a step in this direction—an algorithm compensating for the unique characteristics of LBL navigation. The hypothesis grid (Hgrid) approach represents the sensor environment of an underwater robot navigating with fixed acoustic transponders. The approach builds a probabilistic representation of the environment based on empirical evidence, quantifying the quality of subsequent time-of-flight range measurements based on the estimated observer position.

Acoustic range measurement error cannot be fully characterized with a Gaussian distribution. Acoustic multipaths cause consistently erroneous range observations; whales and outboard motors cause storms of false data; transponders refuse to reply; and measurements are often just wrong. Capturing all the possible sources of error in range measurement is not possible; there is always a new type of failure. This paper argues that by modeling range sensors with mixtures of simple distributions and inferring probabilistic characteristics from past evidence autonomous systems realize more performance and gain new navigation capabilities.

Hypothesis grids quantify the quality of an acoustic range measurement using prior association probability, the belief in the particular source of a subsequent range observation: Direct-path, multipath, or outlier. The representation captures the relationship between the probabilistic source and the estimated location of the observer, i.e., the spatial dependence of the range measurement quality. As an environmental representation capturing the prior probability, Hgrids increase the precision of Bayesian inference-based navigation algorithms: Multiple hypothesis tracking (MHT), Bayesian filters, particle filters, or Kalman filtering approaches to simultaneous localization and mapping (SLAM). Fig. 1 illustrates implementation of a hypothesis grid within a precision acoustic ranging environment. The figure shows the two-dimensional (2-D) hypothesis grid based

Manuscript received June 2, 2004; accepted September 15, 2005. This work was supported by the Ocean Exploration office of the National Oceanic and Atmospheric Administration (NOAA-OE). Associate Editor: R. Blidberg.
Brian Bingham is with the Franklin W. Olin College of Engineering, Needham, MA 02492 USA (e-mail: bbing@olin.edu).
Warren Seering is with the Massachusetts Institute of Technology (MIT), Cambridge, MA 02139 USA.
Digital Object Identifier 10.1109/JOE.2006.872220

0364-9059/$20.00 © 2006 IEEE
on empirical observations from the mobile autonomous platform. The vehicle operates within this grid, using the prior probability for estimation and decision. This environmental awareness increases the navigation precision and raises the robustness with respect to non-Gaussian sensor behavior.

### A. Problem Statement

To state the problem clearly and succinctly requires three basic assumptions.

1) There are three exhaustive, mutually exclusive, hypothetical associations for each range measurement: The direct-path (DP), a multipath (MP), and an outlier (OL), i.e., the association hypothesis \( \theta \) has three possible values, \( \theta = \{DP, MP, OL\} \).

2) An environmental representation estimates the prior association probabilities \( P(\theta) \), the probability that a particular measurement is from one of the three sources without knowledge of the measurement.

3) This prior association probability depends on the estimated location of the observer \( \hat{x} \).

With this formalism, the challenge is representing the sensor environment.

- Given range observations (evidence) and corresponding position estimates, approximate the dependence between the prior association probabilities and the estimated location, i.e., approximate the function, \( f(\hat{x}) \), in the relationship \( P(\theta) = f(\hat{x}) \).

### B. Approach

Guiding this approach are two observations from experience. First, the statistics of range errors are not captured by a well-behaved Gaussian distribution. Range measurements contain spurious outliers, multipath returns, and systematic errors. Second, the quality of range data is often dependent on the location within the survey area, e.g., a particularly reliable corner of a survey near a transponder with good line of sight or a problematic region where observations are prone to error. Hypothesis grids are an effort to formalize this operator experience, making it available to autonomous navigation systems. Hypothesis grids summarize past experience through Bayesian inference, succinctly representing the belief that future measurements from a particular location will be from a particular source.

The hypothesis grid method builds this representation using three steps. First, we use a mixed Gaussian and uniform distribution sensor model to parameterize the sensor model. Second, we apply expectation-maximization (EM) to find parameters fitting model to empirical data. Expectation-maximization is a process of iterative identification and classification to simultaneously estimate the parameters of the mixed distribution model and probabilistically associate each observation with a particular source. Third, we represent the sensor environment with a grid representation. Each rectangular region of the grid contains three association probabilities corresponding to the three potential observation sources—DP, MP, and OL. Fig. 2 illustrates this Hgrid representation.

---

**Fig. 1.** Discrete, instrumented environment. The AUV performs a sonar survey using the two transponders shown in the corners of the illustration. The wire-frame grid illustrates a 2-D hypothesis grid containing the prior probabilities for estimation and decision.

**Fig. 2.** Conceptual sketch of a hypothesis grid representation.

### II. Closely Related Work and Background

#### A. Closely Related Work

The Hgrid approach is not a navigation algorithm, but a method for representing the environment. Hgrids provide a framework for utilizing the well-studied techniques from robot exploration [7], [8] and map-based navigation [9]–[11].

Bayesian techniques are the basis for many estimation techniques [12]. Bayes filters, occupancy grids, Kalman filters, etc. are each based on Bayesian inference. Thrun’s summary develops the techniques from the fundamentals and draws connections between the disparate approaches [13]. Hypothesis grids operate in concert with such Bayesian estimation techniques. By explicitly modeling prior probabilities, the algorithm forms a foundation for other navigation techniques and supplies an informed estimate of values that are typically approximated heuristically.

Occupancy grids are similar to hypothesis grids because they share a rectangular representation of a sensor environment [14]–[16]. An occupancy grid divides the environment into small cells. The information from sensor readings (typically laser-line-scanners) leads to an estimate of whether each cell contains an object or is unoccupied. This technique has been applied to underwater applications where a scanning sonar builds a geometric map from a localized platform [17]. The differences are greater than the similarities. Hgrids model the probability of association; occupancy grids capture the physical geometry of the environment. Hgrids consider multiple \( (\geq 3) \) possible hypotheses; occupancy grids use only two hypotheses—free or occluded space. Simple Hgrids, dividing
the space into modest number of cells, are shown to capture the critical sensor behavior; dense occupancy grids, requiring complex computation, are necessary to model even simple environments.

B. Background

Three areas summarize the important background for the development of the hypothesis grid approach: Deffenbaugh’s work using multipath range data for navigation [18], Leonard’s application of multiple hypothesis tracking to underwater navigation [19], and Blimes’ basics of the expectation-maximization algorithm [20]. These three areas—multipath modeling, Bayesian tracking and navigation, and data association—are each distinct research topics that are brought together within the proposed hypothesis grid method.

A general model of the acoustic communication channel is an open research question. At the coarsest level there are two basic multipath models: A probabilistic reflector model prevalent in applications to electromagnetic wave communications [21], [22] and a deterministic ray-tracing model typical in the ocean acoustics community [23], [24]. The latter methods are based on the physics of large scale acoustic propagation and have originated from military sonar research. Ray-tracing techniques demand knowledge of the source-receiver geometry, the geometry of the environment, and the sound-speed structure of the medium. Since these quantities are not often completely known, the challenge is to simultaneously identify the model and use it to classify the measurements.

A fundamental challenge of this paper is the association of range observations with their individual sources. Our approach follows the probabilistic association techniques in radar tracking where measurements are classified based on the multiple sources of a particular return [25], [26]. This challenge is also related to the correspondence problems in robot mapping and machine vision. Correspondence in robot mapping is the problem of matching features across disparate sensor scans, i.e., to find a feature known from a previous measurement in the current observation [27]. Correspondence in machine vision is the difficulty finding the same portions within multiple images.

Background on EM ranges from mathematical fundamentals [20], [28] to applications such as simultaneous localization and mapping [29] and image segmentation [30], [31]. The operation of this batch processing algorithm for simultaneous model identification and observation classification will become clear in the following development and example.

III. Algorithm Development

Hypothesis grids are both a representation and a synthesizing algorithm. The algorithm, illustrated in Fig. 3, consists of three components: A mixed distribution measurement model, an EM algorithm for simultaneous measurement classification and model identification, and a cellular decomposition of the survey area.

Two restrictions are important. First, hypothesis grids complement navigation and do not produce a localization directly. Uncertain localization estimates are available corresponding to each range observation. This does not mean that we assume perfect navigation; instead the framework incorporates uncertain estimates of location for classification and representation. Second, building grids is a batch process requiring a full data set of range measurements. This does not constrain the application to postmission processing; initialization may happen once enough evidence is available, followed by continued accumulation of observations.

A. Measurement Model: Mixing Gaussian and Uniform Distributions

For any particular acoustic range observation, \( z(k) \) at time \( k \), three classifications of measurement are possible—a direct-path (DP) range, a multipath (MP) range, or an outlier (OL). The measurement model captures this structure by expanding an additive noise model where three hypotheses, \( \theta(k) \), exist for each observation

\[
z(k) = \begin{cases} &|\hat{x}(k) - x_0(k)| + \nu_{\text{DP}} \quad \theta(k) = \text{DP} \\ &f_{\text{mp}}(\hat{x}(k); x_0(k), \Phi) + \nu_{\text{MP}} \quad \theta(k) = \text{MP} \\ &\nu_{\text{OL}} \quad \theta(k) = \text{OL} \end{cases}
\]

\[
\nu_{\text{DP}} \sim N(0, \sigma_{\text{DP}}) \quad \nu_{\text{MP}} \sim N(0, \sigma_{\text{MP}}) \quad \nu_{\text{OL}} \sim \text{Uniform},
\]

The direct-path measurement is the Cartesian distance metric between the estimated observer position vector, \( \hat{x}(k) \), and the known transponder location, \( x_0(k) \). The additive zero-mean Gaussian noise has a standard deviation of \( \sigma_{\text{DP}} \). The multipath observation is a function, \( f_{\text{mp}} \), of the observer location, the transponder location, and the multipath model parameter vector \( \Phi \). The outlier observations are modeled as uniformly distributed over a specified range. Fig. 4 illustrates this combination of probability density functions. The prior probability of a particular association, \( P(\theta) \), is the belief, without evidence, that a measurement will be from one of the three sources. For example, a value of \( P(\text{MP}) = 0.25 \) predicts a 25% chance of subsequent measurements corresponding to multipath ranges.

We specify the multipath geometry with a ray-trace model. For small spatial scales we assume a constant speed of sound and consider a single specular reflector to generate an alternate acoustic path—a single multipath. Fig. 5 illustrates this geometry in a vertical plane containing the vehicle and transponder. Because we know the estimated position of the vehicle and the surveyed position of the transponder, the height of the reflecting plane (\( H \)) determines the length of the multipath ray. This simple geometry defines the relationship between the parameter \( H \) and the estimated multipath range (\( z_{\text{mp}} \))

\[
H = \frac{1}{2} \left[ \Delta d + \frac{\Delta r}{\tan(\frac{\Delta r}{z_{\text{mp}}})} \right]
\]

\[
z_{\text{mp}} = \frac{\Delta r}{\sin(\theta)}
\]

\( ^{1} \)Time-of-flight is actually measured and the range is estimated based on this measurement and an uncertain estimate of the speed of sound.
where

$$\theta = \arctan \left( \frac{\Delta r}{2H - \Delta d} \right). \quad (4)$$

The relative location of the vehicle and transponder are known
- $\Delta r$—the estimated horizontal distance between the vehicle and transponder position;
- $\Delta d$—the estimated vertical distance between the vehicle and transponder depth.

This multipath model completes the sensor model (1). The proposed geometry specifies the functional relationship $f_{\exp}$ (1) with a single unknown geometric parameter $\Phi = H$, the height of the reflecting plane. To simplify this discussion, the geometry presented here includes a single multipath reflector. We can use the same model for multiple reflectors, each corresponding to a particular classification of the range data. In the case-study application which follows we use a separate reflector model for each of the four transponder transponders.
and the

distributions. For the OL association, the probability of a par-
tions the estimated mean and variance values characterize the
The direct-path and outlier parameters of the model are static;
standard deviation of the multipath range observations

2) M-Step (Model Parameter Estimation): Given a proba-
ibilistic classification ($\gamma_{\theta_j}(k)$), the maximization step identifies
the model by estimating the multipath model parameters $\Phi$.

The challenge is to find the maximum likelihood estimate of
the parameters, given the range observations and their classifi-
cations. Since the data model assumes a mixture of Gaussian
and uniform distributions (which can be viewed as a Gaussian
with a large variation), the least-squares merit function, the
residual error between the estimated and observed range values,
is appropriate as the likelihood function. Because the only un-
known model parameters are those of the multipath portion
of the model, the error metric includes only the multipath
associations. The least-squares residual is a function of the
difference between the observed range data ($z(k)$) and the
estimated multipath range $\hat{z}_{\text{mp}}(k)$ for each data point $k$

$$\chi^2 = \frac{1}{\hat{\sigma}_{\text{mp}}} \sum_{k=1}^{N} [(z(k) - \hat{z}_{\text{mp}}(k))^2 \hat{\sigma}_{\text{mp}}(k)] .$$ (8)

We use the classification probability ($\gamma_{\text{mp}}(k)$) as a weighting
function to reinforce the clustering necessary for the algorithm
to converge. Model identification consists of finding the height
of the reflecting plane ($H$) to minimize the residual ($\chi^2$ in (8)).

To implement the least-squares model-fitting, we calculate the
$H$ for each observation ($H(k)$) and combine the individual
estimates using the weighted average

$$\hat{H} = \frac{\sum_{k=1}^{N} [H(k) \gamma_{\text{mp}}(k)]}{\sum_{k=1}^{N} \gamma_{\text{mp}}(k)} .$$ (9)

The maximum likelihood estimate of the variance in these
measurements is a similarly weighted average of the residuals

$$\hat{\sigma}_{\text{mp}} = \frac{\sum_{k=1}^{N} [(z(k) - \hat{z}_{\text{mp}}(k))^2 \gamma_{\text{mp}}(k)]}{\sum_{k=1}^{N} \gamma_{\text{mp}}(k)} .$$ (10)

The EM algorithm alternates e-steps and m-steps until the
model parameters converge. We determine convergence by
the consistency of the parameters in successive iterations. This
iterative solution relies on the consistency of the multipath data
to identify both the height of the reflecting plane and the variance
in the multipath measurement while converging on a classifi-
cation of the range observations. This presentation of EM clas-
cification and identification is brief. For a detailed example and
discussion of the operation and characteristics of the algorithm’s
performance, refer to [32], [33].

C. Representation: From Classification to Hypothesis Grids

With the range observations classified, the next challenge is
to create a compact representation. We divide the 2-D survey area
into rectangular cells—a grid-based decomposition—where
each cell represents an aggregation of the constituent observ-
eations from that region. The following illustrates a simple
example; an evenly spaced grid divides the environment into
equally sized region. Each cell contains a subset of the observ-
ations made from within a particular region. Location estimates
associate each range measurement with a particular cell in the
hypothesis grid.\textsuperscript{2} This approach is compact, allows for efficient interrogation, and accurately models the data with quantified uncertainty.

To determine the appropriate grid spacing, we propose a set of metrics to balance the statistical significance with the consistency within each cell. Using too fine a mesh reduces the number of observations associated with each cell—reducing the confidence in the resulting probability estimate. Using too coarse a mesh reduces the information by averaging properties of the environment. The tradeoff is articulated by considering the following metrics.

\begin{itemize}
  \item Cell count: The number of range measurements associated with each cell. A high cell count indicates a statistically significant result.
  \item Cell variance: The disparity in association probabilities within a cell. A small variance indicates consistency in the estimate for a particular cell.
  \item Grid variance or grid spread: The disparity in cell probabilities between the cells within the grid. A large disparity indicates a significant variation in range measurement behavior as a function of estimator position.
\end{itemize}

IV. CASE STUDY: HYPOTHESIS GRIDS OF ABE DIVE 58

To illustrate the process and test the feasibility of Hgrids, we implement the synthesis algorithm using navigation data from dive number 58 of the autonomous benthic explorer (ABE58). From November 5 to December 4, 2001 a research team studied midocean ridge geology to understand the fundamental processes involved in the creation of new ocean crust through high-resolution mapping of the near-bottom magnetic field. The team used a towed system (DSL-120A), a manned submersible (Alvin), and an autonomous vehicle (ABE) to study the East Pacific Rise. ABE (see Fig. 6) surveyed a total of 14.3 km\textsuperscript{2} of the sea-floor over 11 individual dives at two separate locations. At an altitude of 20–30 m the vehicle performed parallel track lines 40–60 m apart. The payload sensors aboard ABE included a magnetometer, a 675 kHz Imagex pencil beam sonar, a digital still camera, and a CTD, producing high-resolution maps of the oceanographic phenomena \cite{34}.

A. The ABE58 Survey: Long Baseline Data Overview

The long baseline (LBL) positioning data from ABE58 is ideal for illustrating and evaluating the hypothesis grid algorithm. During dive 58 the vehicle made range measurements to four transponders at known locations using standard 7–17 kHz acoustic transponders.\textsuperscript{3} These ranges, shown in Fig. 7, include direct-path, multipath, outliers, and null returns. The strong multipath returns are the result of acoustic signals reflecting off the ocean surface. The proposed multipath model (Fig. 5) captures this geometry with the horizontal reflecting plane set at the air-water interface. Because the vehicle and transponder depths are known, this data serves as a test-case to verify the proposed EM algorithm converges on a physically meaningful solution. In many environments the cause of multipath is not observable and the algorithm estimates a physical parameter not otherwise measurable.

To explain the nature of this data and how the non-Gaussian observations present a challenge for autonomous navigation, we briefly discuss the range data from one particular transponder in Fig. 7. From the data from transponder number 3 (red markers), the direct-path is evident in the sawtooth pattern of first return ranges oscillating between 700–2200 m. Another similar sawtooth pattern between ranges of 3000–4000 m indicates the first multipath return. A weaker second multipath is evident at roughly 5100 m. Returns corresponding to neither of these three paths clutter the data, especially at two particular times—around 2000 and 3500 on the data index axis. Lastly null returns, ranges with a value of zero, are present in the data, but should not be used in localization. This qualitative examination highlights the non-Gaussian line-of-sight range errors. While the human eye is adept at this classification, autonomously classifying and identifying this type of data is difficult.

The hypothesis grid approach is dependent on two preprocessing steps: Rough vehicle position estimates and statistical characterization of the direct-path range and outlier range measurements. Fig. 8 illustrates the ABE58 survey by plotting the estimated positions of the vehicle projected into the horizontal

\textsuperscript{2}A simple extension calculates an observation’s membership within a cell based on the estimate and the uncertainty in the estimate.

\textsuperscript{3}6000 series transponders from Benthos, Inc., Falmouth, MA.
Fig. 8. ABE58 survey in the X-Y plane. The Northings and Eastings are a simple Mercator projection from an arbitrary latitude/longitude origin. Approximately 5000 data points are plotted from the survey. The LBL transponder locations are shown as labeled red markers. The surveyed depths for transponders 1–4 are 2392, 2491, 2395, and 2472 m respectively. The mean depth of ABE during the survey is 2567 m. (Color version available online at http://ieeexplore.ieee.org.)

Fig. 9. (a) Long baseline acoustic range values: Transponder 3, ABE 58. (b) Classification of long baseline acoustic range values: Transponder 3, ABE 58. The association probabilities are indicated by the color levels—red indicates high probability of direct-path; blue indicates high probability of multipath; and green indicates high probability of an outlier.
Fig. 10. Hypothesis grid with the ABE58 survey positions.

Fig. 11(a) and (b) are two visualizations of the hypothesis grid for transponder number 3 in ABE58. Mapping the probabilities to the blue-red-green color-map in Fig. 11(c) illustrates the entire hypothesis grid with a single illustration. For this particular transponder the disparity between the northwest corner and the southeast corner illustrates an important spatial characterization of the sensed environment. In the southeast corner and along the southern row of the grid, the blue color dominates indicating that most of the observations correspond to direct-path measurements. Fig. 8 shows the placement of the transponder number 3, in the south and east of the survey. Near the transponder, the direct-path is more reliably available. In the northwest corner of the grid and along the most northern row, the red colors dominate indicating more multipath returns. Also, in the most northern row, a green grid cell indicates a region particularly prone to outlier ranges. Fig. 11(b) illustrates the hypothesis grid numerically using the actual probability values for each cell.

1) Representation Evaluation: Grid Metrics: The evaluation metrics proposed in Section III-C balance confidence, derived from having many measurements in each cell, and consistency, derived from having few measurements in each cell. To evaluate the metrics, we repeatedly grid the survey area using a increasingly fine grid (more cells) and rebuild the hypothesis grid using the same EM classification results. The cell count metric decreases geometrically as the 5000 data points are spread over more and more grid cells. The cell variance holds fairly steady for between 4–196 cells. The grid variance increases with finer grids with a local maximum at 25 cells—a $5 \times 5$ grid. These metrics quantify the tradeoff between significance and information content. We chose a $5 \times 5$ grid based on

<table>
<thead>
<tr>
<th>Bayesian Term</th>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Posterior</td>
<td>$P(\theta</td>
<td>z, \Phi)$</td>
</tr>
<tr>
<td>Generative</td>
<td>$P(z</td>
<td>\theta, \Phi)$</td>
</tr>
<tr>
<td>Prior</td>
<td>$P(\theta</td>
<td>\Phi)$</td>
</tr>
<tr>
<td>Normalization</td>
<td>$\rho$</td>
<td>Sum of the probabilities to enforce the mutual exclusive constraint</td>
</tr>
</tbody>
</table>

TABLE II
MULTIPATH IDENTIFICATION RESULTS: ABE DIVE 58. HRP—HEIGHT OF REFLECTION PLANE

<table>
<thead>
<tr>
<th>Transponder#</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>2392</td>
<td>2500</td>
<td>2404</td>
<td>2486</td>
</tr>
<tr>
<td>$\sigma_{mp}$ (m)</td>
<td>4.2</td>
<td>10.5</td>
<td>14.0</td>
<td>6.9</td>
</tr>
<tr>
<td>Transponder Depth(m)</td>
<td>2392</td>
<td>2491</td>
<td>2395</td>
<td>2472</td>
</tr>
</tbody>
</table>
Fig. 11. (a) Hypothesis grid for transponder 3 with the ABE58 survey positions. The probabilistic associations are mapped into blue, red, and green for direct-path, multipath, and outlier. The small black markers indicate the post processed vehicle positions for reference. (b) Hypothesis grid values for transponder 3. The numbers in each cell are the association probabilities for direct path (blue), multipath (red), and outlier (green). (c) Three-value color-map for visualizing the probabilistic associations. The probabilistic classifications are mapped to blue, red, and green for direct-path, multipath, and outliers.

these metrics to maximize the information, indicated by a high grid variance, while keeping the grid coarse so probabilities within each cell are the result of as many measurements as possible—in this case an average of over 200 observations per cell. Significantly, a coarse representation, generalizing about large portions of the survey area, sufficiently and succinctly represents the environment.

D. Summary and Results

Since four transponders where used for ABE58, we build four hypothesis grids to characterize each range sensor. Rather than present the illustration in Fig. 11 for each hypothesis grid we summarize the results using the boxplots in Fig. 12. (The grid...
based representations for each transponder are available in [32] and [33].) Fig. 12(a) shows the distribution of direct-path probabilities over each Hgrid. The Hgrid for transponder number 3 has the largest variance indicating the most disparity between cells. The same is true for characterizing the probability of multipath ranges in Fig. 12(b). From the data illustrated in Fig. 12, we conclude that there is significant variability in the prior probabilities for each Hgrid, i.e., for each transponder. The individual Hgrids capture the spatial contribution to this variability, presenting it in a succinct form for use in estimation and decision.

Examining these results, one can imagine how a human operator would make use of the information—minimizing time spent in particular regions where direct-path information is scarce or using multipath information were the maps indicate a tendency for multipath returns. Hypothesis grids provide a representation for an autonomous platform to use this information for managing sensing and control.

V. CONCLUSION: FEASIBILITY AND UTILITY

The hypothesis grids representation captures the prior association probability as a function of the observer location, providing a compact representation of the environment for efficient interrogation. For autonomous estimation and decision Hgrids provide an accurate model of the empirical data, quantifying the uncertainty in the relationship between sensor behavior and vehicle position. This paper develops the concept of a hypothesis grid representation through three steps: Application of a mixed-distribution model, simultaneous identification and classification of the multipath parameters with expectation-maximization, and construction of a grid-based representation. The example, using LBL range data from the ABES8 dive, illustrates creating the representation and shows the grid resolution necessary to represent a survey environment.

1) Feasibility: Hypothesis grids are predicated on the spatial dependence of range sensing, i.e., regions in a survey area exhibit particular behaviors, and the relationship between that behavior and the estimator location is quantifiable probabilistically. Empirical evidence supports the notion that there are positions where the range observations have characteristics which can be inferred from past evidence. For example, in one corner of the survey environment ranges may be reliable while in other locations observations are in large part spurious. Sections of the survey may contain measures of the line-of-sight path while other regions contain mostly multipath returns. This paper illustrates the feasibility of quantifying this intuition in a form appropriate for autonomous operation—hypothesis grids.

2) Utility: Hypothesis grids are predicated on the concept of prior probabilities and their importance for probabilistic navigation methods. Bayesian techniques employ prior probabilities that are not easily estimated. By explicitly modeling the prior probabilities, hypothesis grids operate alongside estimation and navigation techniques, forming a base layer of information for higher level algorithms. For example, multiple hypothesis tracking (MHT) uses Bayes rule to maintain many estimators in parallel. Each estimator uses the prior association probabilities in determining correspondence. A more accurate map of these probabilities would enable more accurate correspondence and better tracking.

Multipath range observations are often discarded in acoustic positioning. Incorporating multipath measurements can enhance the localization capability in noisy or acoustically shadowed environments [18]. Hypothesis grids enable a predictive model for determining the source of future measurements, making more information available for navigation based on both line-of-sight and multipath observations.

In addition to adding accuracy to general navigation techniques, hypothesis grids extend the capabilities of autonomous survey. Exploration algorithms couple robot motion to the resulting uncertainty. Sensor management employs active sensor control to reduce uncertainty. Hypothesis grids are useful for both types of algorithms because they provide the environmental information to coordinate the survey path.

3) Future Work: This work presents the feasibility of the hypothesis grid approach through application to one particular dataset. To increase the utility of the approach a few natural extensions are underway.

- Creating hypothesis grid representations is currently a postprocessing exercise appropriate for instrumented areas subject to multiple dives. This batch processing method is not the most useful for operational use. Real-time implementation could be achieved in two ways.
  - Classification could be done during a single mission to build a Hgrid representation during a dive.
  - Kalman filter based approach might be used to track the multipath parameters and capture the spatial variations without using the formal grid structures.
- The two-dimensional, rectangular, time-invariant grid is sufficient to illustrate the feasibility of the approach but should be extended.
  - Vehicle depth is strongly correlated to the quality of the acoustic range data. A three-dimensional (3-D) representation is a natural extension.
  - The grid representation is rectangular and not a function of the actual data. Multiresolution gridding or data-driven segmentation should be investigated as a succinct representation.
  - The sensor data is also time-dependent. The quality of range measurements should be considered as a function of both space and time, i.e., the hypothesis grids could be adaptive in real-time.
- The multipath model should be extended to capture different phenomena. Any model that parameterizes the multipath environment as a vector $\Phi$ is appropriate for the estimation step in creating the Hgrids.

ACKNOWLEDGMENT

The authors would like to thank the ABE group lead by Dr. D. Yoerger at Woods Hole Oceanographic Institution (WHOI) for generously supplying the data for this work and J. Gendron for the conceptual illustration of Fig. 1.
REFERENCES


Hypothesis Grids: Improving Long Baseline Navigation for Autonomous Underwater Vehicles

Brian Bingham, Member, IEEE, and Warren Seering

Abstract—Navigation continues to fundamentally limit our ability to understand the underwater world. Long baseline navigation uses range measurements to localize a remote vehicle using acoustic time-of-flight estimates. For autonomous surveys requiring high precision navigation, current solutions do not satisfy the performance or robustness requirements. Hypothesis grids represent the survey environment capturing the spatial dependence of acoustic range measurement, providing a framework for improving navigation precision and increasing the robustness with respect to non-Gaussian range observations. Prior association probabilities quantify the measurement quality as a belief that subsequent observations will correspond to the direct-path, a multipath, or an outlier as a function of the estimated location. Such a characterization is directly applicable to Bayesian navigation techniques. The algorithm for creating the representation has three main components: Mixed-density sensor model using Gaussian and uniform probability distributions, measurement classification and multipath model identification using expectation-maximization (EM), and grid-based spatial representation. We illustrate the creation of a set of hypothesis grids, the feasibility of the approach, and the utility of the representation using survey data from the autonomous benthic explorer (ABE).

Index Terms—Author, please supply your own keywords or send a blank e-mail to keywords@ieee.org to receive a list of suggested keywords.

I. INTRODUCTION

Navigation, the combination of localization and guidance, remains a fundamental challenge to field robotics. For many robotic missions on land and in the air the global positioning system (GPS) [1] has revolutionized this problem, but not all applications can rely on a GPS position estimate. Sensor networks operate in places where GPS is not practical reasons of power and size [2], and robotic planetary exploration requires self contained navigation solutions independent of GPS. Saltwater is impenetrable to most forms of electromagnetic radiation, including GPS signals, challenging underwater remote-sensing and navigation schemes [3].

Navigation for underwater survey is sensor-limited. A localization solution is most often the product of combining complimentary sensing modalities emphasizing the strengths of each instrument. Long baseline (LBL) provides a low-update-rate, fixed-reference, bounded-error estimate of location based on acoustic travel times between transponders in fixed locations on the seafloor [4]. In contrast, a Doppler velocity log (DVL) or inertial navigation system (INS) provides a high-update-rate, internal-reference, unbounded-error position estimate based on integrating velocity or acceleration. Combining these types of measurements can provide a solution with the advantages of each, but requires an external reference, i.e., instrumenting the environment with transponders, a time consuming task not appropriate for all operations [5], [6].

Underwater navigation continues to improve while autonomous underwater vehicles (AUVs) prove their utility as operational assets. The challenge of autonomous underwater navigation, operating without a human in the loop, requires a new level of robustness to sensor operation. This paper presents a step in this direction—an algorithm compensating for the unique characteristics of LBL navigation. The hypothesis grid (Hgrid) approach represents the sensor environment of an underwater robot navigating with fixed acoustic transponders. The approach builds a probabilistic representation of the environment based on empirical evidence, quantifying the quality of subsequent time-of-flight range measurements based on the estimated observer position.

Acoustic range measurement error cannot be fully characterized with a Gaussian distribution. Acoustic multipaths cause consistently erroneous range observations; whales and outboard motors cause storms of false data; transponders refuse to reply; and measurements are often just wrong. Capturing all the possible sources of error in range measurement is not possible; there is always a new type of failure. This paper argues that by modeling range sensors with mixtures of simple distributions and inferring probabilistic characteristics from past evidence autonomous systems realize more performance and gain new navigation capabilities.

Hypothesis grids quantify the quality of an acoustic range measurement using prior association probability, the belief in the particular source of a subsequent range observation: Direct-path, multipath, or outlier. The representation captures the relationship between the probabilistic source and the estimated location of the observer, i.e., the spatial dependence of the range measurement quality. As an environmental representation capturing the prior probability, Hgrids increase the precision of Bayesian inference-based navigation algorithms: Multiple hypothesis tracking (MHT), Bayesian filters, particle filters, or Kalman filtering approaches to simultaneous localization and mapping (SLAM). Fig. 1 illustrates implementation of a hypothesis grid within a precision acoustic ranging environment. The figure shows the two-dimensional (2-D) hypothesis grid based
on empirical observations from the mobile autonomous platform. The vehicle operates within this grid, using the prior probability for estimation and decision. This environmental awareness increases the navigation precision and raises the robustness with respect to non-Gaussian sensor behavior.

A. Problem Statement

To state the problem clearly and succinctly requires three basic assumptions.

1) There are three exhaustive, mutually exclusive, hypothetical associations for each range measurement: The direct-path (DP), a multipath (MP), and an outlier (OL), i.e., the association hypothesis ($\theta$) has three possible values, $\theta = \{DP, MP, OL\}$.

2) An environmental representation estimates the prior association probabilities ($P(\theta)$), the probability that a particular measurement is from one of the three sources without knowledge of the measurement.

3) This prior association probability depends on the estimated location of the observer ($\hat{X}$).

With this formalism, the challenge is representing the sensor environment.

• Given range observations (evidence) and corresponding position estimates, approximate the dependence between the prior association probabilities and the estimated location, i.e., approximate the function, $f(\hat{X})$, in the relationship $P(\theta) = f(\hat{X})$.

B. Approach

Guiding this approach are two observations from experience. First, the statistics of range errors are not captured by a well-behaved Gaussian distribution. Range measurements contain spurious outliers, multipath returns, and systematic errors. Second, the quality of range data is often dependent on the location within the survey area, e.g., a particularly reliable corner of a survey near a transponder with good line of sight or a problematic region where observations are prone to error. Hypothesis grids are an effort to formalize this operator experience, making it available to autonomous navigation systems. Hypothesis grids summarize past experience through Bayesian inference, succinctly representing the belief that future measurements from a particular location will be from a particular source.

The hypothesis grid method builds this representation using three steps. First, we use a mixed Gaussian and uniform distribution sensor model to parameterize the sensor model. Second, we apply expectation-maximization (EM) to find parameters fitting model to empirical data. Expectation-maximization is a process of iterative identification and classification to simultaneously estimate the parameters of the mixed distribution model and probabilistically associate each observation with a particular source. Third, we represent the sensor environment with a grid representation. Each rectangular region of the grid contains three association probabilities corresponding to the three potential observation sources—DP, MP, and OL. Fig. 2 illustrates this Hgrid representation.

II. CLOSELY RELATED WORK AND BACKGROUND

A. Closely Related Work

The Hgrid approach is not a navigation algorithm, but a method for representing the environment. Hgrids provide a framework for utilizing the well-studied techniques from robot exploration [7],[8] and map-based navigation [9],[10].

Bayesian techniques are the basis for many estimation techniques [12]. Bayes filters, occupancy grids, Kalman filters, etc. are each based on Bayesian inference. Thrun’s summary develops the techniques from the fundamentals and draws connections between the disparate approaches [13]. Hypothesis grids operate in concert with such Bayesian estimation techniques. By explicitly modeling prior probabilities, the algorithm forms a foundation for other navigation techniques and supplies an informed estimate of values that are typically approximated heuristically.

Occupancy grids are similar to hypothesis grids because they share a rectangular representation of a sensor environment [14],[16]. An occupancy grid divides the environment into small cells. The information from sensor readings (typically laser-line-scanners) leads to an estimate of whether each cell contains an object or is unoccupied. This technique has been applied to underwater applications where a scanning sonar builds a geometric map from a localized platform [17]. The differences are greater than the similarities. Hgrids model the probability of association; occupancy grids capture the physical geometry of the environment. Hgrids consider multiple ($\geq 3$) possible hypotheses; occupancy grids use only two hypotheses—free or occluded space. Simple Hgrids, dividing
the space into modest number of cells, are shown to capture the critical sensor behavior; dense occupancy grids, requiring complex computation, are necessary to model even simple environments.

B. Background

Three areas summarize the important background for the development of the hypothesis grid approach: Deffenbaugh’s work using multipath range data for navigation [18], Leonard’s application of multiple hypothesis tracking to underwater navigation [19], and Blimes’ basics of the expectation-maximization algorithm [20]. These three areas—multipath modeling, Bayesian tracking and navigation, and data association—are each distinct research topics that are brought together within the proposed hypothesis grid method.

A general model of the acoustic communication channel is an open research question. At the coarsest level there are two basic multipath models: A probabilistic reflector model prevalent in applications to electromagnetic wave communications [21], [22] and a deterministic ray-tracing model typical in the ocean acoustics community [23], [24]. The latter methods are based on the physics of large scale acoustic propagation and have originated from military sonar research. Ray-tracing techniques demand knowledge of the source-receiver geometry, the geometry of the environment, and the sound-speed structure of the medium. Since these quantities are not often completely known, the challenge is to simultaneously identify the model and use it to classify the measurements.

A fundamental challenge of this paper is the association of range observations with their individual sources. Our approach follows the probabilistic association techniques in radar tracking where measurements are classified based on the multiple sources of a particular return [25], [26]. This challenge is also related to the correspondence problems in robot mapping and machine vision. Correspondence in robot mapping is the problem of matching features across disparate sensor scans, i.e., to find a feature known from a previous measurement in the current observation [27]. Correspondence in machine vision is the difficulty is finding the same portions within multiple images.

Background on EM ranges from mathematical fundamentals [20], [28] to applications such as simultaneous localization and mapping [29] and image segmentation [30], [31]. The operation of this batch processing algorithm for simultaneous model identification and observation classification will become clear in the following development and example.

III. ALGORITHM DEVELOPMENT

Hypothesis grids are both a representation and a synthesizing algorithm. The algorithm, illustrated in Fig. 3, consists of three components: A mixed distribution measurement model, an EM algorithm for simultaneous measurement classification and model identification, and a cellular decomposition of the survey area.

Two restrictions are important. First, hypothesis grids complement navigation and do not produce a localization directly. Uncertain localization estimates are available corresponding to each range observation. This does not mean that we assume perfect navigation; instead the framework incorporates uncertain estimates of location for classification and representation. Second, building Hgrids is a batch process requiring a full data set of range measurements. This does not constrain the application to postmission processing; initialization may happen once enough evidence is available, followed by continued accumulation of observations.

A. Measurement Model: Mixing Gaussian and Uniform Distributions

For any particular acoustic range observation, $z(k)$ at time $k$, three classifications of measurement are possible—a direct-path (DP) range, a multipath (MP) range, or an outlier (OL). The measurement model captures this structure by expanding an additive noise model where three hypotheses, $\theta(k)$, exist for each observation

$$z(k) = \left\{ \begin{array}{ll}
\tilde{x}(k) - x_0(k) + \nu_{\text{dp}} & \text{if } \theta(k) = \text{DP} \\
\tilde{x}(k) - x_0(k) + \nu_{\text{mp}} & \text{if } \theta(k) = \text{MP} \\
\nu_{\text{ol}} & \text{if } \theta(k) = \text{OL}
\end{array} \right.$$

(1)

The direct-path measurement is the Cartesian distance metric between the estimated observer position vector, $\tilde{x}(k)$, and the known transponder location, $x_0(k)$. The additive zero-mean Gaussian noise has a standard deviation of $\sigma_{\text{dp}}$. The multipath observation is a function, $f_{\text{mp}}$, of the observer location, the transponder location, and the multipath model parameter vector $\Phi$. The outlier observations are modeled as uniformly distributed over a specified range. Fig. 4 illustrates this combination of probability density functions. The prior probability of a particular association, $P(\theta)$, is the belief, without evidence, that a measurement will be from one of the three sources. For example, a value of $P(\text{MP}) = 0.25$ predicts a 25% chance of subsequent measurements corresponding to multipath ranges.

We specify the multipath geometry with a ray-trace model. For small spatial scales we assume a constant speed of sound and consider a single specular reflector to generate an alternate acoustic path—a single multipath. Fig. 5 illustrates this geometry in a vertical plane containing the vehicle and transponder. Because we know the estimated position of the vehicle and the surveyed position of the transponder, the height of the reflecting plane $H$ determines the length of the multipath ray. This simple geometry defines the relationship between the parameter $H$ and the estimated multipath range ($z_{\text{mp}}$)

$$H = \frac{1}{2} \left[ \frac{\Delta d}{\tan(\arcsin(\Delta r / \Delta_{\text{mp}}))} \right]$$

(2)

$$z_{\text{mp}} = \frac{\Delta r}{\sin(\theta)}$$

(3)

1Time-of-flight is actually measured and the range is estimated based on this measurement and an uncertain estimate of the speed of sound.
Fig. 3. Algorithm flowchart for building hypothesis grids.

Fig. 4. Probability density functions for the three associations. DP = direct-path; MP = multipath; OL = outlier. (Color version available online at http://ieeexplore.ieee.org.)

where

$$\theta = \arctan \left( \frac{\Delta r}{2H - \Delta d} \right).$$  \hspace{1cm} (4)

The relative location of the vehicle and transponder are known
- $\Delta r$—the estimated horizontal distance between the vehicle and transponder position;
- $\Delta d$—the estimated vertical distance between the vehicle and transponder depth.

This multipath model completes the sensor model (1). The proposed geometry specifies the functional relationship $f_{\text{MP}}$ (1) with a single unknown geometric parameter $\Phi = H$, the height of the reflecting plane. To simplify this discussion, the geometry presented here includes a single multipath reflector. We can use the same model for multiple reflectors, each corresponding to a particular classification of the range data. In the case-study application which follows we use a separate reflector model for each of the four transponder transponders.
B. Expectation-Maximization: Classifying and Identifying

The next challenge is to probabilistically classify past range measurements by determining the probability that each observation is a direct-path measure, a multipath measure, or an outlier. If the multipath geometry were known, this step would be straightforward. However, since we do not assume prior knowledge of the environment, we rely on empirical data for both the model identification and data classification. By iteratively determining the classification (the expectation or e-step) and identifying the model based on that classification (the maximization or m-step), EM simultaneously determines a set of model parameters to fit the data and associates the observations with source hypotheses.

The unknown parameters in the sensor model are those of the multipath model—the height of the reflecting plane \( H \) and the standard deviation of the multipath range observations \( \sigma_{\text{mp}} \). The direct-path and outlier parameters of the model are static; their parameters do not change with successive iterations.

1) E-Step (Observation Classification): Given a set of model parameters \( \Phi \), the estimation step classifies the observations. For each data point, the association probabilities \( P(\theta(k) = \theta_j \| \theta_j = \{\text{DP, MP, OL}\}) \) classify the data using Bayes Rule

\[
\gamma_{\theta_j}(k) = P(\theta(k) = \theta_j \| z(k), \Phi) = \frac{P(z(k)\| \theta(k) = \theta_j, \Phi)P(\theta(k) = \theta_j \| \Phi)}{\sum_{i=1}^M [P(z(k)\| \theta(k) = \theta_i, \Phi)P(\theta(k) = \theta_i \| \Phi)]}
\]

\[
= \frac{P(z(k)\| \theta(k) = \theta_j, \Phi)P(\theta(k) = \theta_j)}{\sum_{i=1}^M [P(z(k)\| \theta(k) = \theta_i, \Phi)P(\theta(k) = \theta_i)]}
\]

\[
= \frac{P(z(k)\| \theta(k) = \theta_j, \Phi)P(\theta(k) = \theta_j)}{\sum_{i=1}^M [P(z(k)\| \theta(k) = \theta_i, \Phi)P(\theta(k) = \theta_i)]}, \quad \quad (5)
\]

- time index: \( k = 1, \ldots, N \) for \( N \) data points;
- class index: \( j = 1, \ldots, M \) for \( M \) hypotheses.

The EM algorithm calculates the generative term for a particular observation based on the probability density function of the particular classification. For the Gaussian DP and MP associations, the estimated mean and variance values characterize the distributions. For the OL association, the probability of a particular measurement is constant. The output of the e-step is a posterior probability of each hypothesis \( \{\text{DP, MP, OL}\} \) for each range observation—\( \gamma_{\theta_j}(k) \).

2) M-Step (Model Parameter Estimation): Given a probabilistic classification \( \gamma_{\theta_j}(k) \), the maximization step identifies the model by estimating the multipath model parameters \( \Phi \).

The challenge is to find the maximum likelihood estimate of the parameters, given the range observations and their classifications. Since the data model assumes a mixture of Gaussian and uniform distributions (which can be viewed as a Gaussian with a large variation), the least-squares merit function, the residual error between the estimated and observed range values, is appropriate as the likelihood function. Because the only unknown model parameters are those of the multipath portion of the model, the error metric includes only the multipath associations. The least-squares residual is a function of the difference between the observed range data \( z(k) \) and the estimated multipath range \( \hat{z}_{\text{mp}}(k) \) for each data point \( k \)

\[
\chi^2 = \frac{1}{\sigma_{\text{mp}}^2} \sum_{k=1}^{N} [(z(k) - \hat{z}_{\text{mp}}(k))^2 \sigma_{\text{mp}}^2(k)]. \quad \quad (8)
\]

We use the classification probability \( \gamma_{\text{mp}}(k) \) as a weighting function to reinforce the clustering necessary for the algorithm to converge. Model identification consists of finding the height of the reflecting plane \( H \) and estimating the multipath range \( \hat{z}_{\text{mp}}(k) \) for each observation \( (H(k)|k) \) and combining the individual estimates using the weighted average

\[
\hat{H} = \frac{\sum_{k=1}^{N} [H(k)\gamma_{\text{mp}}(k)]]}{\sum_{k=1}^{N} \gamma_{\text{mp}}(k)} \quad \quad (9)
\]

The maximum likelihood estimate of the variance in these measurements is a similarly weighted average of the residuals

\[
\hat{\sigma}_{\text{mp}} = \frac{\sum_{k=1}^{N} [(z(k) - \hat{z}_{\text{mp}}(k))^2 \gamma_{\text{mp}}(k)]}{\sum_{k=1}^{N} \gamma_{\text{mp}}(k)} \quad \quad (10)
\]

The EM algorithm alternates e-steps and m-steps until the model parameters converge. We determine convergence by the consistency of the parameters in successive iterations. This iterative solution relies on the consistency of the multipath data to identify both the height of the reflecting plane and the variance in the multipath measurement while converging on a classification of the range observations. This presentation of EM classification and identification is brief. For a detailed example and discussion of the operation and characteristics of the algorithm’s performance, refer to [32, 33].

C. Representation: From Classification to Hypothesis Grids

With the range observations classified, the next challenge is to create a compact representation. We divide the 2-D survey area into rectangular cells—a grid-based decomposition—where each cell represents an aggregation of the constituent observations from that region. The following illustrates a simple example; an evenly spaced grid divides the environment into equally sized region. Each cell contains a subset of the observations made from within a particular region. Location estimates associate each range measurement with a particular cell in the
hypothesis grid. This approach is compact, allows for efficient interrogation, and accurately models the data with quantified uncertainty.

To determine the appropriate grid spacing, we propose a set of metrics to balance the statistical significance with the consistency within each cell. Using too fine a mesh reduces the number of observations associated with each cell—reducing the confidence in the resulting probability estimate. Using too coarse a mesh reduces the information by averaging properties of the environment. The tradeoff is articulated by considering the following metrics:

- **Cell count**: The number of range measurements associated with each cell. A high cell count indicates a statistically significant result.
- **Cell variance**: The disparity in association probabilities within a cell. A small variance indicates consistency in the estimate for a particular cell.
- **Grid variance or grid spread**: The disparity in cell probabilities between the cells within the grid. A large disparity indicates a significant variation in range measurement behavior as a function of estimator position.

### IV. CASE STUDY: HYPOTHESIS GRIDS OF ABE DIVE 58

To illustrate the process and test the feasibility of H grids, we implement the synthesis algorithm using navigation data from dive number 58 of the autonomous benthic explorer (ABE58). From November 5 to December 4, 2001 a research team studied midocean ridge geology to understand the fundamental processes involved in the creation of new ocean crust through high-resolution mapping of the near-bottom magnetic field. The team used a towed system (DSL-120A), a manned submersible (Alvin), and an autonomous vehicle (ABE) to study the East Pacific Rise. ABE (see Fig. 6) surveyed a total of 14.3 km$^2$ of the sea-floor over 11 individual dives at two separate locations. At an altitude of 20–30 m the vehicle performed parallel track lines 40–60 m apart. The payload sensors aboard ABE included a magnetometer, a 675 kHz Imaginex pencil beam sonar, a digital still camera, and a CTD, producing high-resolution maps of the oceanographic phenomena [34].

#### A. The ABE58 Survey: Long Baseline Data Overview

The long baseline (LBL) positioning data from ABE58 is ideal for illustrating and evaluating the hypothesis grid algorithm. During dive 58 the vehicle made range measurements to four transponders at known locations using standard 7–17 kHz acoustic transponders. These ranges, shown in Fig. 7, include direct-path, multipath, outliers, and null returns. The strong multipath returns are the result of acoustic signals reflecting off the ocean surface. The proposed multipath model (Fig. 5) captures this geometry with the horizontal reflecting plane set at the air-water interface. Because the vehicle and transponder depths are known, this data serves as a test-case to verify the proposed EM algorithm converges on a physically meaningful solution. In many environments the cause of multipath is not observable and the algorithm estimates a physical parameter not otherwise measurable.

To explain the nature of this data and how the non-Gaussian observations present a challenge for autonomous navigation, we briefly discuss the range data from one particular transponder in Fig. 7. From the data from transponder number 3 (red markers), the direct-path is evident in the sawtooth pattern of first return ranges oscillating between 700–2200 m. Another similar sawtooth pattern between ranges of 3000–4000 m indicates the first multipath return. A weaker second multipath is evident at roughly 5100 m. Returns corresponding to neither of these three paths clutter the data, especially at two particular times—around 2000 and 3500 on the data index axis. Lastly null returns, ranges with a value of zero, are present in the data, but should not be used in localization. This qualitative examination highlights the non-Gaussian line-of-sight range errors. While the human eye is adept at this classification, autonomously classifying and identifying this type of data is difficult.

The hypothesis grid approach is dependent on two preprocessing steps: Rough vehicle position estimates and statistical characterization of the direct-path range and outlier range measurements. Fig. 8 illustrates the ABE58 survey by plotting the estimated positions of the vehicle projected into the horizontal plane.
Fig. 8. ABE58 survey in the $X$-$Y$ plane. The Northings and Eastings are a simple Mercator projection from an arbitrary latitude/longitude origin. Approximately 5000 data points are plotted from the survey. The LBL transponder locations are shown as labeled red markers. The surveyed depths for transponders 1–4 are 2392, 2491, 2395, and 2472 m respectively. The mean depth of ABE during the survey is 2567 m. (Color version available online at http://ieeexplore.ieee.org.)

Fig. 9. (a) Long baseline acoustic range values: Transponder 3, ABE 58. (b) Classification of long baseline acoustic range values: Transponder 3, ABE 58. The association probabilities are indicated by the color levels—red indicates high probability of direct-path; blue indicates high probability of multipath; and green indicates high probability of an outlier.

B. Multipath Identification Results

Applying the EM algorithm classifies of the range data from Fig. 7. The results, summarized in Table II, identify the four multipath models by determining the height of the reflecting plane relative to each transponder and the variance in the acoustic measurements. The final row of the table lists the surveyed transponder depths which agree with the location of the reflecting planes determined by the EM method. These results show the multipath phenomena as the result of the acoustic signal reflecting off the air-water interface on either the outgoing or incoming path and traveling the line-of-sight path in the other direction—the triangle path in Fig. 5.

The iterative EM algorithm converges on the solution in Table II from an initial condition were $H = 4000 \text{ m}$ and $\sigma_{\text{map}} = 3000 \text{ m}$ for each of the four transponders. The only consideration in choosing these arbitrary initial conditions is to select initial values much greater than the expected final solutions. Experience with the EM algorithm to this application have shown the convergence to be insensitive to the particular choice of initial conditions.

Fig. 9(a) shows a subset of the acoustic range data from Fig. 7—the ranges from acoustic transponder number 3. To illustrate the performance of the EM classification and identification Fig. 9(b) shows the same set of ranges with the probabilistic data association indicated by color. Since there are three potential hypotheses (direct-path, multipath, and outlier associations) a red, blue, green color-map indicates the probability of each hypothesis conditioned on the observation. The ability of the EM algorithm to effectively sort the range measurements is evident in Fig. 9(b) where the direct-path associations are shown with red markers; the multipath associations are shown in blue; and outlier associations are green. The color-map for the figure is the posterior probabilities [Table I and (7)] mapped directly onto the red, green, and blue levels used for each marker color. For more detail on these results and an illustrative example see [32], [33].

C. Grid Representation

The grid representation sections the survey into rectangular regions. In two dimensions, this grid structure is a regularly spaced mesh in Cartesian coordinates. Fig. 10 shows the particular grid used for the ABE58 survey. The 5-by-5 representation, chosen based on the metrics previously introduced, bal-
Fig. 10. Hypothesis grid with the ABE58 survey positions.

ances the consistency of the information within each cell. The ABE58 survey is roughly north-south so the grid aligns with the cardinal directions.

The EM algorithm classifies each range measurement probabilistically, i.e., it determines probability that each range is a direct-path, a multipath, or an outlier. Using the independent localization estimates, the decomposition step divides the data set based on the estimated location of observation. Aggregating the association probabilities, from the output of the EM step within a particular region, determines the prior probabilities for that cell. The resulting compact representation assigns probabilities for each region based on past evidence.

Fig. 11(a) and (b) are two visualizations of the hypothesis grid for transponder number 3 in ABE58. Mapping the probabilities to the blue-red-green color-map in Fig. 11(c) illustrates the entire hypothesis grid with a single illustration. For this particular transponder the disparity between the northwest corner and the southeast corner illustrates an important spatial characterization of the sensed environment. In the southeast corner and along the southern row of the grid, the blue color dominates indicating that most of the observations correspond to direct-path measurements. Fig. 8 shows the placement of the transponder number 3, in the south and east of the survey. Near the transponder, the direct-path is more reliably available. In the northwest corner of the grid and along the most northern row, the red colors dominate indicating more multipath returns. Also, in the most northern row, a green grid cell indicates a region particularly prone to outlier ranges. Fig. 11(b) illustrates the hypothesis grid numerically using the actual probability values for each cell.

1) Representation Evaluation: Grid Metrics: The evaluation metrics proposed in Section III-C balance confidence, derived from having many measurements in each cell, and consistency, derived from having few measurements in each cell. To evaluate the metrics, we repeatedly grid the survey area using a increasingly fine grid (more cells) and rebuild the hypothesis grid using the same EM classification results. The cell count metric decreases geometrically as the 5000 data points are spread over more and more grid cells. The cell variance holds fairly steady for between 4 – 196 cells. The grid variance increases with finer grids with a local maximum at 25 cells—a 5 x 5 grid. These metrics quantify the tradeoff between significance and information content. We chose a 5 x 5 grid based on

<table>
<thead>
<tr>
<th>Bayesian Term</th>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Posterior</td>
<td>( P(\theta</td>
<td>z, \Phi) )</td>
</tr>
<tr>
<td>Generative</td>
<td>( P(z</td>
<td>\theta, \Phi) )</td>
</tr>
<tr>
<td>Prior</td>
<td>( P(\theta</td>
<td>\Phi) )</td>
</tr>
<tr>
<td>Normalization</td>
<td>( \rho )</td>
<td>Sum of the probabilities to enforce the mutual exclusive constraint</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transponder#</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>2392</td>
<td>2500</td>
<td>2404</td>
<td>2486</td>
</tr>
<tr>
<td>( \sigma_{mp} ) (m)</td>
<td>4.2</td>
<td>10.5</td>
<td>14.0</td>
<td>6.9</td>
</tr>
<tr>
<td>Transponder Depth(m)</td>
<td>2392</td>
<td>2491</td>
<td>2395</td>
<td>2472</td>
</tr>
</tbody>
</table>
Fig. 11. (a) Hypothesis grid for transponder 3 with the ABE58 survey positions. The probabilistic associations are mapped into blue, red, and green for direct-path, multipath, and outlier. The small black markers indicate the post processed vehicle positions for reference. (b) Hypothesis grid values for transponder 3. The numbers in each cell are the association probabilities for direct path (blue), multipath (red), and outlier (green). (c) Three-value color-map for visualizing the probabilistic associations. The probabilistic classifications are mapped to blue, red, and green for direct-path, multipath, and outliers.

These metrics to maximize the information, indicated by a high grid variance, while keeping the grid coarse so probabilities within each cell are the result of as many measurements as possible—in this case an average of over 200 observations per cell. Significantly, a coarse representation, generalizing about large portions of the survey area, sufficiently and succinctly represents the environment.

D. Summary and Results

Since four transponders where used for ABE58, we build four hypothesis grids to characterize each range sensor. Rather than present the illustration in Fig. 11 for each hypothesis grid we summarize the results using the boxplots in Fig. 12. (The grid-
based representations for each transponder are available in [32] and [33]. Fig. 12(a) shows the distribution of direct-path probabilities over each Hgrid. The Hgrid for transponder number 3 has the largest variance indicating the most disparity between cells. The same is true for characterizing the probability of multipath ranges in Fig. 12(b). From the data illustrated in Fig. 12, we conclude that there is significant variability in the prior probabilities for each Hgrid, i.e., for each transponder. The individual Hgrids capture the spatial contribution to this variability, presenting it in a succinct form for use in estimation and decision.

Examining these results, one can imagine how a human operator would make use of the information—minimizing time spent in particular regions where direct-path information is scarce or using multipath information were the maps indicate a tendency for multipath returns. Hypothesis grids provide a representation for an autonomous platform to use this information for managing sensing and control.

V. CONCLUSION: FEASIBILITY AND UTILITY

The hypothesis grids representation captures the prior association probability as a function of the observer location, providing a compact representation of the environment for efficient interrogation. For autonomous estimation and decision Hgrids provide an accurate model of the empirical data, quantifying the uncertainty in the relationship between sensor behavior and vehicle position. This paper develops the concept of a hypothesis grid representation through three steps: Application of a mixed-distribution model, simultaneous identification and classification of the multipath parameters with expectation-maximization, and construction of a grid-based representation. The example, using LBL range data from the ABES8 dive, illustrates creating the representation and shows the grid resolution necessary to represent a survey environment.

1) Feasibility: Hypothesis grids are predicated on the spatial dependence of range sensing, i.e., regions in a survey area exhibit particular behaviors, and the relationship between that behavior and the estimator location is quantifiable probabilistically. Empirical evidence supports the notion that there are positions where the range observations have characteristics which can be inferred from past evidence. For example, in one corner of the survey environment ranges may be reliable while in other locations observations are in large part spurious. Sections of the survey may contain measures of the line-of-sight path while other regions contain mostly multipath returns. This paper illustrates the feasibility of quantifying this intuition in a form appropriate for autonomous operation—hypothesis grids.

2) Utility: Hypothesis grids are predicated on the concept of prior probabilities and their importance for probabilistic navigation methods. Bayesian techniques employ prior probabilities that are not easily estimated. By explicitly modeling the prior probabilities, hypothesis grids operate alongside estimation and navigation techniques, forming a base layer of information for higher level algorithms. For example, multiple hypothesis tracking (MHT) uses Bayes rule to maintain many estimators in parallel. Each estimator uses the prior association probabilities in determining correspondence. A more accurate map of these probabilities would enable more accurate correspondence and better tracking.

Multipath range observations are often discarded in acoustic positioning. Incorporating multipath measurements can enhance the localization capability in noisy or acoustically shadowed environments [18]. Hypothesis grids enable a predictive model for determining the source of future measurements, making more information available for navigation based on both line-of-sight and multipath observations.

In addition to adding accuracy to general navigation techniques, hypothesis grids extend the capabilities of autonomous survey. Exploration algorithms couple robot motion to the resulting uncertainty. Sensor management employs active sensor control to reduce uncertainty. Hypothesis grids are useful for both types of algorithms because they provide the environmental information to coordinate the survey path.

3) Future Work: This work presents the feasibility of the hypothesis grid approach through application to one particular dataset. To increase the utility of the approach a few natural extensions are underway.

- Creating hypothesis grid representations is currently a postprocessing exercise appropriate for instrumented areas subject to multiple dives. This batch processing method is not the most useful for operational use. Real-time implementation could be achieved in two ways.
  - Classification could be done during a single mission to build a Hgrid representation during a dive.
  - Kalman filter based approach might be used to track the multipath parameters and capture the spatial variations without using the formal grid structures.
- The two-dimensional, rectangular, time-invariant grid is sufficient to illustrate the feasibility of the approach but should be extended.
  - Vehicle depth is strongly correlated to the quality of the acoustic range data. A three-dimensional (3-D) representation is a natural extension.
  - The grid representation is rectangular and not a function of the actual data. Multiresolution gridding or data-driven segmentation should be investigated as a succinct representation.
- The sensor data is also time-dependent. The quality of range measurements should be considered as a function of both space and time, i.e., the hypothesis grids could be adaptive in real-time.
- The multipath model should be extended to capture different phenomena. Any model that parameterizes the multipath environment as a vector $\Phi$ is appropriate for the estimation step in creating the Hgrids.

ACKNOWLEDGMENT

The authors would like to thank the ABE group lead by Dr. D. Yoerger at Woods Hole Oceanographic Institution (WHOI) for generously supplying the data for this work and J. Gendron for the conceptual illustration of Fig. 1.
REFERENCES


[27] Brian Bingham (M’XX) AU: YEAR OF MEMBERSHIP? received the Ph.D. degree in mechanical engineering from the Massachusetts Institute of Technology (MIT), Cambridge, in 2003. He is an Assistant Professor at the Franklin W. Olin College of Engineering, Needham, MA. His current interests focus on expanding the capabilities of autonomous underwater vehicles for exploration and science.

[28] Warren Seering received the Ph.D. degree from Stanford University, Stanford, CA, in 1978. AU: PLEASE PROVIDE DEGREE OF FIELD OF STUDY.

The same year he joined the Systems and Design Division at Massachusetts Institute of Technology (MIT), Cambridge, where he is now the Weber-Shaughness Professor of Mechanical Engineering and Engineering Systems. His research and teaching interests are in the areas of system dynamics, design, and product development.

Prof. Seering is a Fellow of the American Society of Mechanical Engineers and a member of Board of Management of the Design Society.