I. PRELIMINARIES– UNIT ROOTS AND SEASONALITY

Before we proceed to the identification stage of the modelling process, we need to investigate the stochastic properties of our data series. Generally, we look for trends in the data or outliers that might be due to data input errors etc. Trends are especially important to identify since, in general, standard estimation and inference using trending data will produce spurious results.

A. First Step- Look at the Data

The first step is to look at the data. Let's look at just the pre-intervention sample.

1. Seasonality

Clearly we have a series that has a very strong seasonal component.

2. Long Run

The series may also be characterized by a LR trend.
B. Modelling The Seasonality

To be able to correctly model hotel room revenues, we will need to model the seasonality in the series. First, we should define what we mean by seasonality.

1. **Definition** - Hylleberg (1992)

   *Seasonality is the systematic although not necessarily regular, intra-year movement caused by the changes of weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy. These decisions are influenced by endowments, the expectations and preferences of the agents, and the production techniques available in the economy.*
2. **Systematic intra-year movements: Models of Seasonality**

The question we need to answer first is just how systematic are the intra-year movements. The three main classifications of seasonality imply different degrees of systematic movements.

a) **Purely Deterministic Seasonal Process**

A purely deterministic seasonal process is a process generated by seasonal dummy variables such as:

\[ x_t = m_0 + m_1 S_{1t} + m_2 S_{2t} + m_3 S_{3t} \]

Notice that this process can be perfectly forecast and will never change its shape.- i.e. perfectly systematic.

b) **Stationary Seasonal Process**

A stationary seasonal process can be generated by an autoregressive process

\[ \phi(B)x_t = \varepsilon_t \quad \varepsilon_t \sim \text{iid}, \]

with all of the roots of \( \phi(B)=0 \) outside the unit circle, but with some complex pairs with seasonal periodicities.

i. **For example:**

The following AR model will exhibit seasonal behavior:

\[ x_t = \rho x_{t-4} + \varepsilon_t, \quad |\rho| < 1 \]

or

\[ (1-\rho B^4) x_t = \varepsilon_t \Rightarrow x_t = \sum_{i=0}^{\infty} \rho^i \varepsilon_{t-i} \]
Stationary Seasonal Process: Quarterly Data

\[ x(t) = 0.5x(t-4) + c(t) \]
c) **Integrated Seasonal Process**

An integrated seasonal process has a seasonal unit root in its autoregressive representation.

i. **Example:**

An example of an integrated quarterly process with unit roots at the seasonal frequencies and the LR frequency.

\[ x_t = x_{t-4} + \varepsilon_t. \]

or

\[ x_t = (1-B^4)^{-1} \varepsilon_t \]

To see that the above model has unit roots at each frequency, note that the familiar seasonal difference operator \((1-B^4)\) can be factorized as:

\[
(1-B^4) = (1-B)(1+B+B^2+B^3)
= (1-B)(1+B)(1+B^2)
= (1-B)S(B)
\]

ii. **Four roots with modulus one:**

- one is a zero frequency root: \(\omega=0\)
  
  \[(1-B)x_t = \varepsilon_t \quad \Rightarrow \quad x_t = x_{t-1} + \varepsilon_t\]

- one at two cycles per year: \(\omega=\pi\)
  
  \[(1+B)x_t = \varepsilon_t \quad \Rightarrow \quad x_t = -x_{t-1} + \varepsilon_t\]

- two complex pairs at one cycle per year: \(\omega = \pi/2\)
  
  \[(1+B^2)x_t = \varepsilon_t \quad \Rightarrow \quad x_t = -x_{t-2} + \varepsilon_t\]
iii. Show Graph of each of these periodic functions

d) Characteristics of Seasonally integrated series

We are all familiar with the properties of LR nonstationary series with \( \omega = 0 \)

\[
(1-B)x_t = \varepsilon_t \implies x_t = \frac{1}{1-B} \varepsilon_t = \sum_{i=0}^{\infty} \varepsilon_{t-i} .
\]

They have means and variances that change over time, their variances increase linearly with time, and their levels are a function of all past shocks.

Thus, innovations in the distance past have equal weight with recent innovations, i.e. shocks do not die out.

Although not as intuitively appealing as zero frequency integration, the properties of seasonally integrated series are very much the same as those of ordinary integrated processes.

i. Long memory

Seasonally integrated series will be represented by the infinite sum of past innovations at seasonal frequencies with unit roots

For instance, the particular solution to \( S(B)x_t = \varepsilon_t \) is given by:

\[
x_t = [S(B)]^{-1} \varepsilon_t = 1/2 \{ 1/(1+B) + (1-B)/(1+B^2) \}
\]

\[
= 1/2 \sum_{j=0}^{t-1} (-1)^j \varepsilon_{t-j} + 1/2 \sum_{j=0}^{\text{int}(t-1)/2} (-1)^j \Delta \varepsilon_{t-2j}
\]

Shocks last forever and may lead to permanent changes in the seasonal pattern. Thus, we would not expect that most seasonal series would be
characterized solely by seasonal unit roots. Otherwise, we would observe summer becoming winter!

ii. Variance
As you can tell from the above equations, the variances of seasonally integrated processes will have variances that tend to infinity as the process evolves.

iii. Time varying seasonality
A series generated by
\[ S(B)x_t = \varepsilon_t \]
will tend to have a seasonal peak that varies with time.
\[ x(t) = x(t-4) + e(t) \]

\[ y(1t) = S(B)x(t) = e(t) \]

\[ y(2t) = -(1-B)(1+B^2)x(t) \]

\[ y(3t) = -(1-B)(1+B)x(t) \]

\[ y(4t) = (1-B^4)x(t) \]

\[ y(5t) = (1-B)x(t) \]
3) **Problems from Seasonal Integration**

The problems that arise when using seasonally integrated are the same as those that occur when using regularly integrated series. Thus, we can expect to find spurious relationships, inconsistent parameter estimates, and nonstandard distributions for estimators.

One problem with seasonal unit roots is the testing for zero frequency roots. If we ignore the seasonal roots, our tests are likely to produce spurious results. (Bonham et. al. tested the hotel revenue series for zero freq. roots and rejected the null of a unit root)
4) Solutions
   i. Box and Jenkins
      Box and Jenkins suggested that we difference series that appear to be nonstationary at the zero frequency or seasonal frequency.

      Recall that the seasonal difference operator can actually be written as the product of filters at the zero and seasonal roots.

      \[(1-B^4) = (1-B)(1+B+B^2 +B^3) = (1-B)(1+B)(1+B^2)\]

      Thus, if we seasonally difference, we will remove all seasonal roots and the LR root. The mechanical application of the seasonal difference filter is likely to produce serious misspecification in many instances.

   iii. Dummies
      The use of seasonal dummies will of course not work if the seasonality is time varying as occurs when the processes are integrated.

   iv. Seasonal Adjustment
      The problem with seasonal adjustment procedures such as the X-11 procedure is that they alias the seasonal unit roots to the zero frequency. Thus, series that contain seasonal unit roots but no LR unit root will become nonstationary after seasonal adjustment, and we could even observe a form of "demodulation CI" among series which are LR stationary but seasonally integrated before seasonal adjustment.
C. Testing for Unit Roots

Fortunately, we can test for unit roots at all frequencies simultaneously. To get the basic idea here, remember that a series that has a LR unit root is stationary after first differencing. Also, stationary and nonstationary series will be uncorrelated asymptotically.

1. The standard unit root test

If \( x_t \) has a unit root, then \( \rho = 1 \) in the following equation

\[
    x_t = \rho x_{t-1} + \varepsilon_t
\]

Subtract \( x_{t-1} \) from both sides to get:

\[
    \Delta x_t = (\rho - 1)x_{t-1} + \varepsilon_t
\]

If \( x_t \sim I(1) \), then \( \Delta x_t = \varepsilon_t \sim I(0) \), and the coefficient on \( x_{t-1} \) will be equal to zero. We test for whether the coefficient is significantly less than zero as would occur for \( \rho < 1 \). Thus, we regress a stationary series on a series which has a unit root under the null hypothesis. The same process is followed for seasonal unit root tests.
2. **Seasonal Unit Root tests**

Recall that the quarterly difference operator can be parameterized as:

\[(1-B^4) = (1-B)(1+B)(1+B^2)\]

Thus, as series with a unit root at each frequency will be stationary after applying \((1-B^4)\), and will thus be uncorrelated with any series which is nonstationary.

The seasonal unit root test involves regressing \(y_{4t} = (1-B^4)x_t\) on three series, each with a different root removed:

\[y_{1t} = S(B)x_t\]
\[y_{2t} = -(1-B)(1+B^2)x_t\]
\[y_{3t} = -(1-B)(1+B)x_t\]

If the removed root is equal to unity, then the coefficient on that series will be equal to zero, otherwise it will be less than zero.
a) Using monthly data, We estimate the following regression due to Beaulieu and Miron (1992).

\[ y_{13,t} = \sum_{k=1}^{12} \gamma_k y_{k,t-1} + \sum_{k=2}^{12} m_k S_{kt} + \sum_{j=1}^{p} \phi_j y_{13,t-j} + m_0 \tau + m_1 + \varepsilon_t, \]

where

\[ y_{1,t} = (1+B+B^2 +B^3 +B^4 +B^5 +B^6 +B^7 +B^8 +B^9 +B^{10} +B^{11}) x_t = S(B)x_t, \]
\[ y_{2,t} = -(1-B+B^2 -B^3 +B^4 -B^5 +B^6 -B^7 +B^8 -B^9 +B^{10} -B^{11}) x_t, \]
\[ y_{3,t} = -(B-B^3 +B^5 -B^7 +B^9 -B^{11}) x_t, \]
\[ y_{4,t} = -(1-B^2 +B^4 -B^6 +B^8 -B^{10}) x_t, \]
\[ y_{5,t} = \frac{1}{2}(1+B^2+B^3+B^4-2B^5+B^6+B^7-2B^8+B^9+B^{10}-2B^{11}) x_t, \]
\[ y_{6,t} = \frac{\sqrt{3}}{2}(1-B+B^3-B^4+B^6+B^7+B^9-B^{10}) x_t, \]
\[ y_{7,t} = \frac{1}{2}(1-B^2-B^3+B^4+2B^5+B^6-B^7-2B^8-B^9+B^{10}+2B^{11}) x_t, \]
\[ y_{8,t} = -\frac{\sqrt{3}}{2}(1+B-B^3+B^4+B^6+B^7-B^9-B^{10}) x_t, \]
\[ y_{9,t} = -\frac{1}{2}(\sqrt{3} -B+B^3-\sqrt{3}B^4+2B^5-\sqrt{3}B^6+B^7-B^9+\sqrt{3}B^{10}-2B^{11}) x_t, \]
\[ y_{10,t} = \frac{1}{2}(1-\sqrt{3}B+2B^2-\sqrt{3}B^3+B^4-B^6+\sqrt{3}B^7-2B^8+\sqrt{3}B^9-B^{10}) x_t, \]
\[ y_{11,t} = \frac{1}{2}(\sqrt{3} +B-B^3-\sqrt{3}B^4-2B^5-\sqrt{3}B^6-B^7+B^9+\sqrt{3}B^{10}+2B^{11}) x_t, \]
\[ y_{12,t} = -\frac{1}{2}(1+\sqrt{3}B+2B^2+\sqrt{3}B^3+B^4-\sqrt{3}B^7-2B^8-\sqrt{3}B^9-B^{10}) x_t, \]
\[ y_{13,t} = (1-B^{12}) x_t. \]

In the above regression, \( \tau \) is a deterministic trend, \( m_1 \) is an intercept, and \( S_{k,t} \) are deterministic seasonal dummies. The number of lags of the dependent variable, \( p \), included in (2) should be sufficient to ensure serially uncorrelated residuals, \( \varepsilon_t \). \( \gamma_1 \) is the coefficient on \( y_{1,t-1} = S(B)x_{t-1} \), a series whose only root is a zero frequency root—all seasonal roots have been removed. Similarly, \( \gamma_2 \) is the coefficient on \( y_{2,t-1} \) which has been filtered to remove all roots except the root at frequency \( \omega = \pi \), i.e. 6 cycles per year.
The remaining coefficients, \( \gamma_3, \ldots, \gamma_{12} \) are on series which contain roots at frequencies: \( \pm \frac{\pi}{2}, \pm \frac{2\pi}{3}, \pm \frac{\pi}{3}, \pm \frac{5\pi}{6}, \) and \( \pm \frac{\pi}{6} \), corresponding to 3,9,8,4,2,10,7,5,1 and 11 cycles per year respectively.

### Table 1: Tests For Unit Roots at Long-Run and Seasonal Frequencies

<table>
<thead>
<tr>
<th>Coefficients tested</th>
<th>( y_{13}, t )</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \gamma_3, \gamma_4 )</th>
<th>( \gamma_5, \gamma_6 )</th>
<th>( \gamma_7, \gamma_8 )</th>
<th>( \gamma_9 )</th>
<th>( \gamma_{10} )</th>
<th>( \gamma_{11} )</th>
<th>( \gamma_{12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hotel Revenues</td>
<td>-1.23</td>
<td>-3.47</td>
<td>6.59</td>
<td>2.27</td>
<td>8.20</td>
<td>10.52</td>
<td>11.02</td>
<td>19.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rexchange rate-¥/$</td>
<td>-1.25</td>
<td>-2.59</td>
<td>NA</td>
<td>8.53</td>
<td>6.87</td>
<td>8.17</td>
<td>5.38</td>
<td>8.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rincome- JP</td>
<td>-1.57</td>
<td>-1.15</td>
<td>5.80</td>
<td>1.80</td>
<td>5.56</td>
<td>8.90</td>
<td>6.89</td>
<td>11.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment-CA</td>
<td>-1.30</td>
<td>-2.71</td>
<td>NA</td>
<td>3.05</td>
<td>8.25</td>
<td>11.56</td>
<td>10.83</td>
<td>9.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rincome- US</td>
<td>-2.13</td>
<td>-2.20</td>
<td>5.86</td>
<td>13.45</td>
<td>6.03</td>
<td>12.04</td>
<td>4.29</td>
<td>11.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rwealth-Japanese</td>
<td>1.33</td>
<td>-2.76</td>
<td>NA</td>
<td>1.22</td>
<td>6.63</td>
<td>5.07</td>
<td>2.70</td>
<td>6.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rbond rate-$</td>
<td>-0.88</td>
<td>-3.83</td>
<td>16.18</td>
<td>5.05</td>
<td>12.89</td>
<td>11.30</td>
<td>10.34</td>
<td>18.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rbond rate-¥</td>
<td>-2.27</td>
<td>-2.46</td>
<td>4.21</td>
<td>3.85</td>
<td>5.21</td>
<td>4.93</td>
<td>6.06</td>
<td>5.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rwage- HI</td>
<td>-2.61</td>
<td>-3.33</td>
<td>4.07</td>
<td>8.44</td>
<td>7.45</td>
<td>11.15</td>
<td>6.23</td>
<td>15.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Room price</td>
<td>-0.85</td>
<td>-4.34</td>
<td>8.65</td>
<td>10.40</td>
<td>3.47</td>
<td>8.40</td>
<td>10.41</td>
<td>17.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Visitor count</td>
<td>-3.28</td>
<td>-2.78</td>
<td>7.23</td>
<td>5.70</td>
<td>9.83</td>
<td>8.06</td>
<td>5.56</td>
<td>6.58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Numbers in table are one sided t-tests for \( \gamma_1, \gamma_2 \), and joint F-tests for the pairs, \( \gamma_{k-1} \) 4,6,8,10,12. All series are in logarithms. Critical Values (10%) from Beallieu and Miron (1993) are: for \( H_0 : \gamma_1 = 0 \) – \( t_1 = -2.99 \); for \( H_0 : \gamma_2 = 0 \) – \( t_2 = -2.47 \); \( H_0 : \gamma_{k-1} = \gamma_k = 0 \) – \( F_{k-1,k} = 5.25 \). This is the marginal significance level of the Ljung-Box Q-statistic for the null hypothesis that the autocorrelations are jointly equal to zero.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>( \pi )</th>
<th>( \pi/2 )</th>
<th>( 2\pi/3 )</th>
<th>( \pi/3 )</th>
<th>( 5\pi/6 )</th>
<th>( \pi/6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycles per year</td>
<td>0</td>
<td>6</td>
<td>3,9</td>
<td>4,8</td>
<td>2,10</td>
<td>7,5</td>
<td>1,11</td>
</tr>
</tbody>
</table>

\( b \) The real bond rate for Japan has a different sample than the other series.
D. Filtering Out the Seasonal Unit Roots

Because the series have different seasonal unit roots, we make no attempt to tests for seasonal cointegration, and simply remove the seasonal unit roots prior to examining the common LR behavior of our data. Table A1 shows the filtering we have done to each of our series, and we present graphs of the seasonally filtered data here as well.

<table>
<thead>
<tr>
<th>Series</th>
<th>Definition</th>
<th>Freq. of Seasonal Unit Root</th>
<th>Seasonal Filter</th>
</tr>
</thead>
</table>
| Hotel Revenues       | log \[
\frac{\text{Hotel tax base}}{1.0416^\ast \text{USCPI}}
\]                                                                      | 2\pi/3                      | 1+B+B^2         |
| Rexchange rate-Y/$   | log \[
\frac{\text{¥}/\text{ US CPI}}{\text{JPCPI}}
\]                                                                        | none                        |                 |
| Rincome- JP          | log \[
\frac{\text{JP disposable income}}{\text{JPCPI}}
\]                                                                        | \pi, 2\pi/3                 | 1+2B+2B^2+B^3   |
| Employment-CA        | log \[
\frac{\text{CA nonfarm empl.}}{\text{JPCPI}}
\]                                                                        | 2\pi/3                      | 1+B+B^2         |
| Rincome- US          | log \[
\frac{\text{US disposable income}}{\text{USCPI}}
\]                                                                        | \pi                         | 1+B             |
| Rwealth-Japanese      | log \[
\frac{\text{Nikkei index}}{\text{JPCPI}}
\]                                                                        | 2\pi/3, 5\pi/6,\pi/6       | 1+B-B^3+B^5+B^6 |
| Rbond rate-$         | US govt bond (7yr) - \pi                                                   | 2\pi/3                      | 1+B+B^2         |
| Rwage- HI            | log \[
\frac{\text{HI avg hotel wage}}{\text{USCPI}}
\]                                                                        | \pi/2                       | 1+B^2           |
| Room price           | log \[
\frac{\text{HI avgem room rate}}{\text{USCPI}}
\]                                                                        | \pi/3                       | 1-B+B^2         |
| Visitor count        | log \{number of visitors\}                                                  | none                        |                 |
II. LONG RUN ANALYSIS

When Byron and I first started working on building our TF model, I was working with the data in first differences/seasonally filtered form. In other words, I filtered out all of the unit roots we found at the seasonal and the LR frequency. What does that data look like?

Growth of Seasonally Filtered Real Room Revenues

This kind of stationary series is exactly what you would expect to work with when using ARIMA or TR models for short term forecasting ala Box and Jenkins (1976). However, we wanted to explain more than just the SR behavior of the data. If we simply difference all of our data and don't model the LR behavior, then we are assuming that no LR equilibrium relationship exists. Since the series are integrated, this assumption implies that the individual series can wander arbitrarily far away from each other. California
could fall into the ocean, and Japan could sink, and hotel revenues would continue to increase year after year!

But macroeconomists like to think about stable LR relationships among certain variables. We would like to think that the LR behavior of the income of east and west bound tourists would be closely tied to the LR behavior of hotel revenues.
Real Hotel Room Revenues and Japanese Disposable Income

![Graph showing real hotel room revenues and Japanese disposable income over time](image)
A. Engle-Granger Cointegration

Thus, we might expect a LR equilibrium relationship between HRevenues, CA income, JP income, and the real yen/dollar exchange rate. We would like to know if some linear combination of these variables is stationary. For instance,

$$\pi \cdot x_t = \pi_1 H_{Revenues_t} + \pi_2 CA_{EMP_t} + \pi_3 JP_{INC_t} + \pi_4 REXR.$$ 

One way to test for CI is simply to pick a normalization, i.e. normalize by setting \(\pi_1 = -1\), and estimate:

$$H_{Revenues_t} = \pi_2 CA_{EMP_t} + \pi_3 JP_{INC_t} + \pi_4 REXR + \varepsilon_t.$$ 

Cointegration implies \(\varepsilon_t \sim I(0)\)

Non cointegration implies \(\varepsilon_t \sim I(1)\).

We can test the null of no cointegration by testing for a unit root in the "equilibrium error", \(\varepsilon_t\).

B. Problems With Static CI Tests

1. Power

The power of these test is very low, so we have a difficult time rejecting the null of a unit root. Therefore detecting cointegrating relationships among variables—even when they truely exist—is relatively hard.

2. Multiple CI vectors

The static test assumes that there is only one cointegrating vector. However, we have four time series, and potentially three linear combinations which are stationary. Thus, the static model might be estimating some average of these three CI vectors.
3. **Dynamics**

The static CI test ignores all dynamics, and Monte Carlo studies show that methods which model the SR dynamics are more powerful.

C. **CI Testing in VECM**

We follow the VECM methodology of Johansen (1989) and Johansen and Juselius (1990) to test for the number of cointegrating vectors.

1. **VAR(1) model**

Let \( x_t = \pi_1 \cdot x_{t-1} + \varepsilon_t \), or \( x_t = (1-\pi_1)^{-1} \varepsilon_t \)

where \( x_t \) is an \((n x 1)\) vector, and \( \pi_1 \) is a \((n x n)\) matrix of coefficients

2. **Integrated and not Cointegrated rank(\(\pi_1\)) = 0**

Suppose that each element in \( x_t \) is \(I(1)\), and that the elements are mutually independent so that \( \pi = I \). Thus, \((I-\pi)\) has a rank of zero and no LR equilibrium exists. Each of the series in \( x_t \) is integrated, and there are no stationary linear combinations.

\[ \pi_1 \cdot x_t = x_t \sim I(1) \]
3. **Stationary variables**  \( \text{rank}(\pi_1) = n \)

Suppose that \( x_t \sim I(0) \). In other words, \( \pi_1 \neq I \), but \( \pi_1 \cdot x_t \sim I(0) \), and the LR solution to the VAR(1) model is:

\[
x_t = (I - \pi_1)^{-1} \varepsilon_t.
\]

For \( (I - \pi_1)^{-1} \) to exist, each of the eigenvalues of \( \pi \) must be less than one in absolute value.

3. **Integrated and Cointegrated**  \( \text{rank}(\pi_1) = r < n \)

Now assume that \( x_t \sim I(1) \), and \( \pi_1 \neq I \). Thus, not every linear combination is stationary as in example 1 when \( (I - \pi_1) \) is of full rank, and \( x_t \sim I(0) \).

Here, \( (I - \pi_1) \) does not have full rank, and \( (I - \pi_1)^{-1} \) does not exist.

But some linear combination of \( x_t \) might be stationary, and the number of CI vectors is determined by the rank of \( (I - \pi_1) \).
4. **Tests**

The objective of CI analysis is to find an \((n \times n)\) matrix \(\Pi = (I - \pi_1)\) such that \(\Pi x_t\) decomposes \(x_t\) into stationary and nonstationary components.

The first \(r\) rows of \(\Pi\) make up a submatrix, \(\beta' (r \times n)\) with rank \(r\) and \(\beta' x_t \sim I(0)\).

The cointegrating vectors, i.e., the columns of \(\hat{\beta}\), are significant only if the associated eigenvalue is significantly different from zero. Johansen and Juselius (1989) show that a test of the hypothesis that there are at most \(r\) cointegrating vectors, \(H_0: \lambda_i = 0, i = r+1, \ldots, n-1\), can be based on the likelihood ratio test:

\[
(15a) \quad LR(n-r) = -2\ln(Q) = -T \sum_{i=r+1}^{n} \ln(1-\hat{\lambda}_i),
\]

where \(\hat{\lambda}_i, i = r+1, \ldots, n\) are the \((n-r)\) smallest eigenvalues. The eigenvalues are ordered so that

Error!

\[
(15b) \quad LR(n-r) = -2\ln(Q) = -T \ln(1-\hat{\lambda}_{r+1}^2)
\]

**TABLE 2: COINTEGRATION TESTS**

<table>
<thead>
<tr>
<th>Trace Test- Eq(15a)</th>
<th>Maximum Eigenvalue Test- Eq(15b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r=0)</td>
<td>(r \leq 1)</td>
</tr>
<tr>
<td>112.34*</td>
<td>40.72*</td>
</tr>
</tbody>
</table>

Cointegrating Vector Normalized on Real Room Revenues
<table>
<thead>
<tr>
<th>Employment- CA</th>
<th>Rincome- JP</th>
<th>Rexchange rate- ¥/$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.388*</td>
<td>0.616*</td>
<td>-2.137*</td>
</tr>
<tr>
<td>8.967*</td>
<td>3.042*</td>
<td>-3.739*</td>
</tr>
</tbody>
</table>

Note: r is the number of cointegrating vectors, the rank of the matrix $\Pi$ in eq. (12). * indicates statistical significance at the 5% level. Critical values for CI tests are from Osterwald-Lenum (1992, Table 1). Tests of null hypothesis that elements of cointegrating vector are equal to zero are distributed as a Chi-square with 2 degrees of freedom.

For the trace test reported above, the usual procedure is to begin by testing the hypothesis that there are no cointegrating vectors, i.e., r=0. If we cannot reject this hypothesis, we proceed to model the real revenue series in differences with no long run information included since there are no long run restrictions on the dynamic behavior of the elements of $x_t$. If we reject $H_0: r=0$, we can sequentially test the hypotheses that $r \leq 1$, $r \leq 2$, $\cdots$, $r \leq n-1$. In each case, the alternative hypothesis is that there are no cointegrating vectors. If we cannot reject the null that $r \leq r^*$, but we reject the null that $r \leq r^*-1$, then we conclude that the rank of $\beta$, i.e. the number of cointegrating vectors, is equal to $r^*$. For the maximum eigenvalue test, we test the null that $r=r^*$, against the explicit alternative that $r=r^*+1$, for $r^* = 0, 1, 2, 3$. Critical values for these tests are tabulated in Osterwald-Lenum (1992). Using the trace test, we reject the null of no cointegrating vectors, as well as the null of one or fewer cointegrating vectors. Finally we cannot reject the null of 2 cointegrating vectors at the 5% level. Results from the maximum eigenvalue test indicate one fewer cointegrating vectors. While we reject the null of zero cointegrating vectors in favor of the alternative $r=1$, we cannot reject the null that $r=1$ in favor of $r=2$. Hence on the basis of the maximum eigenvalue test we would conclude that only one cointegrating vector exists, while the trace test indicates two cointegrating vectors. The bottom panel of Table 2 presents the cointegrating vectors (eigenvectors) associated with the two largest eigenvalues. Using these vectors, we can write two possible equilibrium relationships between our four series:

(1) \[ HR_t = 1.388 \cdot CAEMP_t + 0.616 \cdot JPY_t - 2.137 \cdot EXCH_t + z_t^1 \]
\[ HR_t = 8.967 \cdot \text{CAEMP}_t + 3.042 \cdot \text{JPY}_t - 3.739 \cdot \text{EXCH}_t + z_t^2, \]

where \( HR \) is the real hotel revenue series, CAEMP is California non-farm employment, JPY is real disposable Japanes income, and EXCH is the real yen/dollar exxchange rate (all series are in logarithms). If actual hotel revenues deviate from the long run equilibrium level implied by (1) or (2), the error correction terms, \( z_t^1 \) and \( z_t^2 \) will be nonzero, and from equation (4), \( \Delta HR_t \) adjust toward the LR equilibrium. The speed of adjustment toward equilibrium is given by the coefficients in the \( (nxr) \) matrix \( \alpha \). For the first row of \( x_t \), i.e. the row corresponding to the real revenue equation, \( \hat{\alpha}_{11} = -0.635 \), and \( \hat{\alpha}_{12} = -0.021 \). Thus, if \( HR_t \) is above its long run equilibrium level as implied by (1), then \( \Delta HR_t \) will fall by sixty percent of the equilibrium error, \( z_t^1 \), each month. This is a very rapid speed of adjustment, implying that west and east bound travelers adjust their expenditures on Hawaii hotels very rapidly in response to changes in their employment and income status.