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An Extension of the Life-Cycle Model
and Its Application to Population
Growth and Aggregate Saving

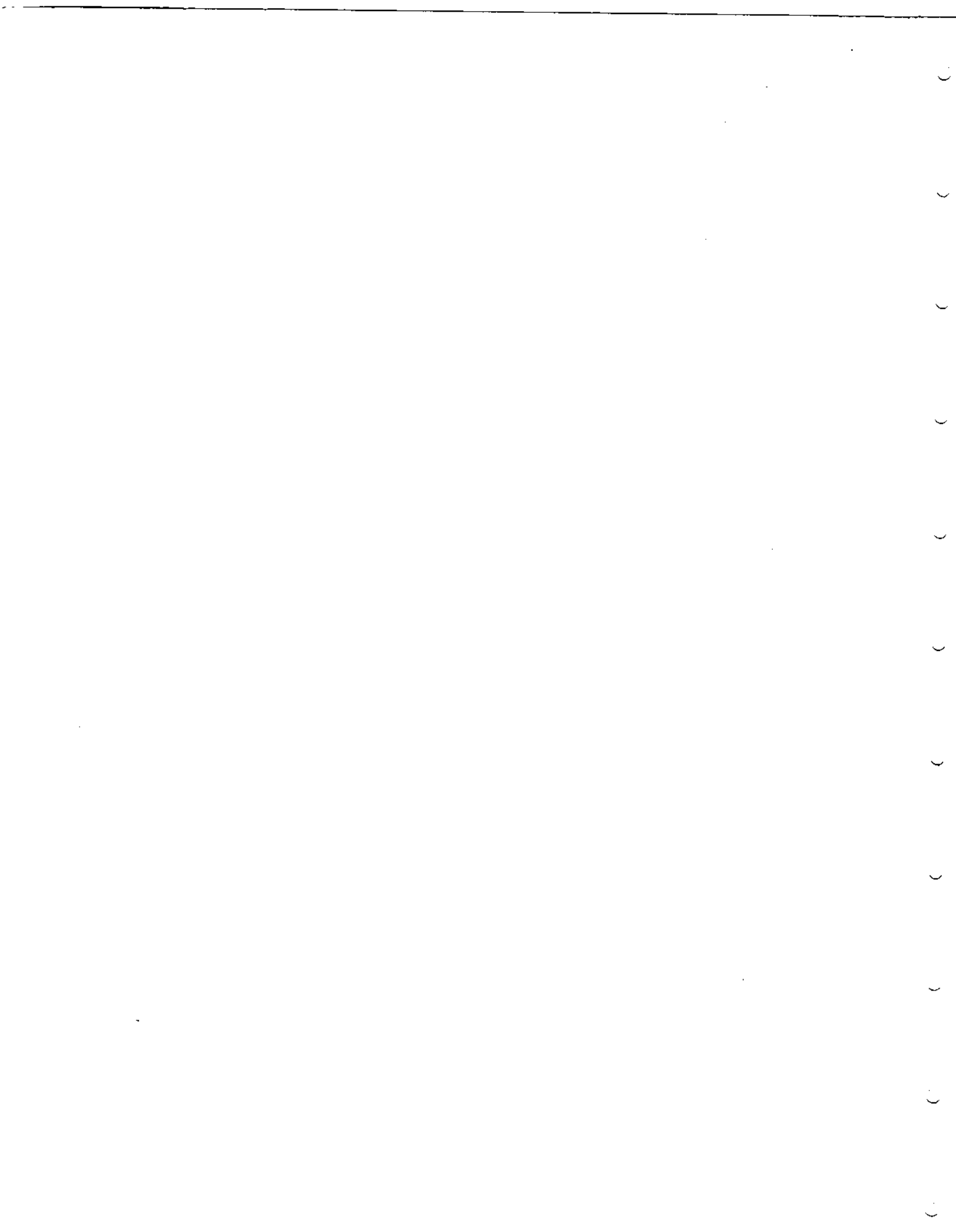
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ABSTRACT

The life-cycle saving model is extended to cases in which both the level and the timing of consumption by households change. The resultant model is estimated from a cross-section of countries of the world. There is no evidence that the level of lifetime consumption by households depends on population growth. The empirical results are consistent with the view that in countries with rapid population growth, consumption is more heavily concentrated at young ages. The so-called timing effect implies that countries with rapid economic growth, e.g., Japan and the Republic of Korea, have experienced higher aggregate saving rates as a result of declining population growth. The saving rate of countries with more typical rates of economic growth is not unduly influenced by population growth.



The principal objective of this paper is to model and to estimate the relationship between aggregate saving and the population growth rate. The population growth rate affects the aggregate saving rate in two ways. The first, usually referred to as the rate of growth effect, is implicitly included in many estimates of the aggregate rate of saving, s .¹ Typically, s is regressed on the rate of growth of total income, g , and other independent variables,

$$(1) \quad s = \beta_0 + \beta_1 g + \dots$$

In each of these studies, saving is found to be positively related to the growth of total income. As the rate of growth of total income equals the rate of growth of per capita income plus the rate of growth of population, an increase in either raises saving and by identical amounts. Thus, an implication of this research is that population growth encourages saving. Few of these studies, however, discuss this relationship or test the hypothesis that population growth has the same effect on saving as growth of per capita income.

The theoretical justification for including the rate of growth of total income in the saving function varies from study to study, but we will confine our analysis to an extension of the life-cycle saving model developed primarily by Modigliani and his colleagues.² The model analyzes a population in which young, working-age households are, on average, savers while old, retired households are, on average, dis-savers. An increase in the rate of growth of per capita income raises the lifetime wealth of young households relative to old households. An increase in the population growth rate raises the number of young households relative to old households. In either case, the relative importance of young (saving) households will rise and, consequently, the aggregate rate of saving.

The second way in which population growth influences aggregate saving is called the dependency effect.³ As the population growth rate increases, the average number of children each household supports during its lifetime also increases. There are two distinct ways that saving by the typical household can be affected. First, the level of household saving may be influenced. Second, the timing of household saving may be influenced. For example, if households shift consumption from one period of their life to another, leaving total lifetime consumption constant, the timing but not the level of saving changes. By extending the life-cycle model to include dependency effects, we demonstrate the importance of distinguishing between changes in the level and changes in the timing of household saving. Changes in the level of saving directly affect aggregate saving. However, the effect of changes in the timing of consumption depends on the rate of growth of total income.

The research described below has three important implications for research on aggregate saving. First, the extension of the life-cycle model to include timing effects has important implications for all research on aggregate saving. Any factor, the rate of interest being an obvious example, may affect the timing as well as the level of household saving. Failure to allow for timing effects in the specification of the aggregate saving function may seriously bias parameter estimates. The empirical evidence presented below is consistent with the hypothesis that population growth does affect the timing of household saving. Contrary to what is typically assumed, the rate of growth effect (β_1 in equation (1)) is not constant. Rather, it is inversely related to the rate of population growth. The estimated rate of growth effect for zero growth populations is nearly twice that of populations growing at three percent annually.

The second contribution of this research is new evidence confirming a rarely tested and previously unconfirmed hypothesis of the life-cycle model. As discussed above, one of the implications of the model is that, controlling for the dependency effect, population growth and growth of per capita income have identical effects on saving. Simply put, evidence presented below confirms that it is appropriate to regress aggregate saving on the rate of growth of total income, g , rather than include growth in per capita income and population separately.

Finally, the relationship between population growth and saving implied by the life-cycle saving model and estimated from a cross-section of countries differs considerably from that found in previous research. In particular, it is shown that only in countries which enjoy rapid growth in per capita or total income will a decline in population growth lead to significantly higher rates of saving. In countries with low levels of economic growth, a decline in population growth has no important positive effect on saving and may even reduce the aggregate saving rate.

The paper is organized in the following manner. Section I describes the life-cycle model and the procedure followed to separate the rate of growth and dependency effects. Section II presents qualitative implications of the rate of growth effect. As these results do not differ from those derived elsewhere, they are not discussed in detail. In section III, the dependency effect is considered in detail and theoretical results are derived. In section IV, the aggregate saving function is specified and estimates are presented. Results are discussed in section V.

I. THE LIFE-CYCLE SAVING MODEL

The life-cycle saving model and its accompanying rate of growth effect are sufficiently well documented in the literature that only a cursory review will be undertaken here.⁴ The assumptions underlying the life-cycle model analyzed here are similar to the assumptions typically made elsewhere.

Each household is omniscient with respect to income at age a , Y_a , its childbearing and its childrearing. The household's expected lifetime resources, V , are:

$$(2) \quad V = \int_0^W p_a Y_a da,$$

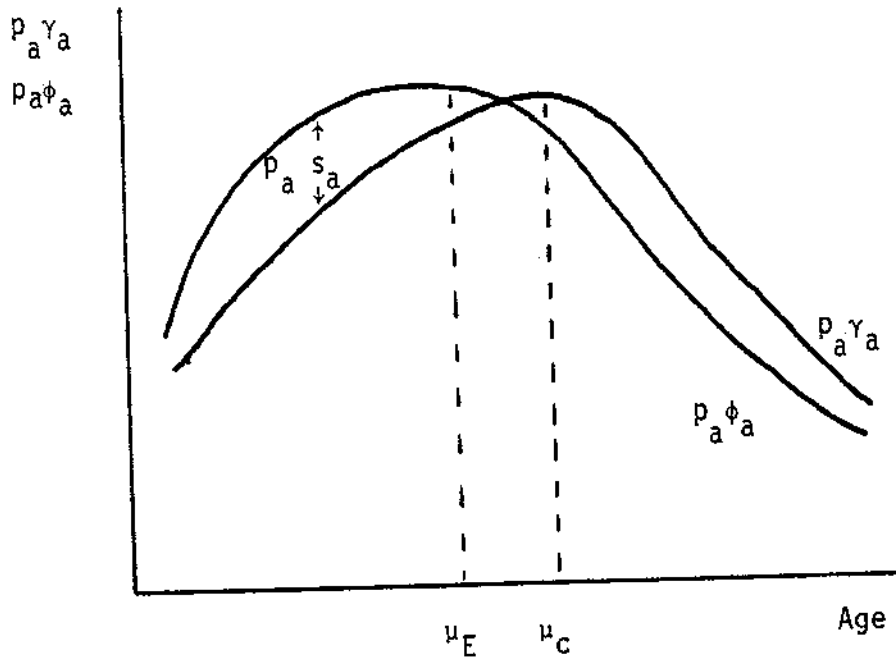
where p_a is the probability that the household will survive to age a .⁵ The interest rate is assumed to equal zero. The fraction of resources the household earns at each age, $\phi_a = Y_a/V$, is independent of the level of household resources, V , and the rate of growth of per capita income. Likewise, the fraction of resources the household consumes at each age, $\gamma_a = C_a/V$, is independent of its lifetime resources and the rate of growth of per capita income. It follows, of course, that the fraction of lifetime resources saved at each age, $s_a = \phi_a - \gamma_a$, is independent of V and the growth rate of per capita income.

The effect of demographic factors on the life-cycle patterns of consumption, earning and saving is central to this paper. There are two ways in which household consumption may be affected. First, the expected fraction of lifetime resources the 'average' household consumes, $\Gamma = \int p_a \gamma_a da$, may depend on the number of children reared. In other words, the household's consumption distribution, illustrated by Figure 1, may shift up or down. Second, the timing of household consumption may change. The timing of consumption can be measured by the mean age of consumption, μ_C , where:

$$(3) \quad \mu_C = \int a p_a \gamma_a da / \int p_a \gamma_a da.$$

The mean age of consumption, shown in Figure 1, is an average age weighted by consumption. If the households reallocate consumption to younger ages from older ages, the mean age of consumption declines.

Figure 1. Household Distributions of Consumption and Earning



By definition the level of household earning $\int p_a \phi_a da = 1$; hence, only the timing of household earning can change. The timing of earning by the household will be measured by the mean age of earning, μ_E , where:

$$(4) \quad \mu_E = \int a p_a \phi_a da / \int p_a \phi_a da$$

The interpretation of μ_E is identical to μ_C .

Of course, the households pattern of saving is determined by its consumption and earning. A rise in the level of consumption, Γ , implies a reduction in the fraction of lifetime resources saved. A decline in the mean age of consumption (Γ held constant) implies a decline in saving at young age and compensating rise at older ages. Likewise, a decline in the mean age of earning implies a rise in saving at young ages and compensating decline at older ages. Finally, $\mu_E - \mu_C$ defines whether household saving is concentrated

more heavily in young ages or old ages. If $\mu_E < \mu_C$, the household generally earns its money before it spends. Hence, saving is concentrated at young ages.

Letting $s_a = \phi_a - \gamma_a$ represent the average rates for all households aged a , the aggregate saving rate, s , at any point in time is:

$$(5) \quad s = \int_0^W v_a s_a da$$

where v_a is the ratio of average lifetime resources of all households currently aged a to total national income and s is the ratio of total saving to total national income.

The population and the economy are assumed to be in full equilibrium, i.e., the population growth rate, n , and the growth rate of per capita (household) income, λ , are constant. It is shown in the appendix (I.A.) that:

$$(6) \quad v_a = e^{-ga} p_a v_0$$

where $g = \lambda + n$ is the growth rate of total income. Equation (6) follows from the fact that the relative wealth of successive generations depends on the rate of growth of total income and survival.

In all but a few respects the model described above does not differ from previous formulations of the life-cycle model and all caveats are applicable. There are four respects in which this model is distinct from most previous formulations. First, the unit of analysis is the household, not the individual. This introduces no difficulties as long as the conditions governing household formation do not change. In full equilibrium the growth rate of each age group equals the population growth rate. Consequently, the growth rate of households and of individuals are identical. Likewise the growth rate of per capita and per household income are identical.

Second, households do not have fixed lifetimes.⁶ Consequently some households will 'die' earlier than anticipated; others will 'die' later than anticipated. Those who die early will leave larger than anticipated bequests. Those who die late will leave smaller than anticipated bequests.

Third, households do not necessarily consume all of their resources. Bequests are governed by a 'golden rule'. Each household passes on to its heirs bequests received plus what it does not consume out of its own resources. It is straightforward to show that the ratio of bequests received to own lifetime resources is constant over time.

Finally, household earning and household consumption are not independent of the rate of population growth. This is, of course, the central issue addressed below. Referring to the saving function, equation (5), most previous formulations of the life-cycle model have addressed the effect of the rate of growth of total income, g , on the intergenerational distribution of lifetime resources, v_a . With this model it is possible to analyze the effect of population growth on household saving, s_a , as well.

Two aspects of saving are considered separately. Taking the total derivative of the saving rate given by equation (5):

$$(7) \quad ds = \int_0^W dv_a s_a da + \int_0^W v_a ds_a da$$

The first term isolates changes in the distribution of resources across generations and captures the rate of growth effect. The second term isolates changes in household saving and is called the dependency effect.

II. THE RATE OF GROWTH EFFECT

An increase in the rate of growth of total income, g , increases the lifetime resources of young households relative to old households. The effect on aggregate saving depends upon the life-cycle pattern of household saving. If saving is more heavily concentrated at young household ages, as is usually hypothesized, aggregate saving will rise.

The rate of growth effect is formally determined by taking the partial derivative of v_a with respect to g , holding s_a constant. It is shown in the appendix (parts II and V) that:

$$(8) \quad \frac{\delta S}{\delta g} = \int_0^w \frac{\delta v_a}{\delta g} s_a da = (1-s) (\mu_C - \mu_E)$$

(To simplify notation the limits of integration will no longer be indicated unless they are other than 0, w.) The necessary and sufficient condition for a positive rate of growth effect is that the mean age of consumption exceeds the mean age of earning.⁷

The aggregate saving function in the absence of a dependency effect is found by integrating equation (8) with respect to g :

$$(9) \quad -\ln(c) = \ln\left(\frac{1}{1-s}\right) = b_0 + b_1 g \\ = -\ln \Gamma + (\mu_C - \mu_E)g$$

It is straightforward to demonstrate that $b_0 = -\ln \Gamma$ where Γ is the fraction of resources consumed by the household over its entire lifetime, i.e., the level of consumption. As Γ should be close to but less than one, b_0 is hypothesized to be small and positive. The coefficient of g , $\mu_C - \mu_E$ is hypothesized to be greater than zero. Finally, g equals $\lambda + n$, the growth rate of per capita income plus the population growth rate. If λ and n are entered separately, their coefficients should be significantly greater than zero and not

significantly different from each other. The empirical evidence presented below is consistent with each of these hypotheses.

Equation (9) is identical to equation (1), the saving function typically estimated, except for the use of a slightly different dependent variable. The equations are simply alternative approximations and except for very high saving rates $\ln(1/1-s)$ is approximately equal to s . The advantage of the approximation employed here, equation (9), is that the parameters, b_0 and b_1 , are estimates of the level and timing of household consumption and earning.

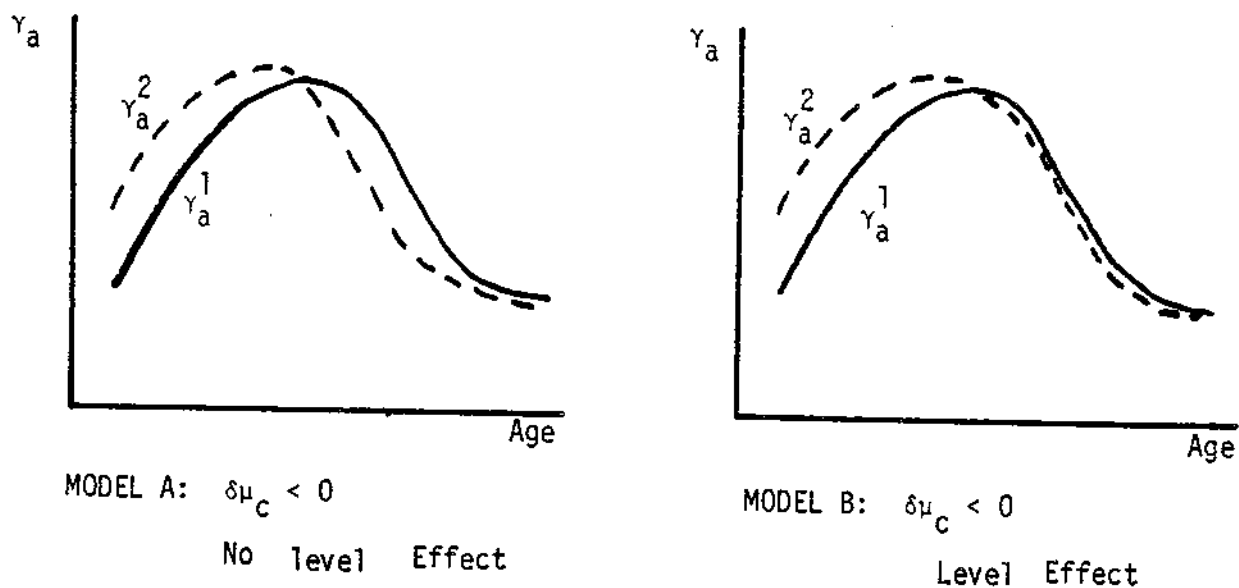
III. THE DEPENDENCY EFFECT

If the patterns of household consumption and earning are not fixed, equation (9) is an incomplete specification of the aggregate saving function. Changes in the aggregate saving rate also will depend on changes in household saving, s_a , as described by the last term of equation (7). The implications of changes in household consumption is considered first. Analysis is then extended to household earning.

Changes in the Level and Timing of Consumption

Figure 2 illustrates alternative hypotheses regarding the affect of demographic factors on household consumption. In Model A, household consumption shifts toward young ages (μ_c declines); however, the average level of consumption, Γ , is constant. In Model B, consumption shifts toward young ages (μ_c declines) and the average level of consumption, Γ , rises. Of course a third alternative would have μ_c constant and Γ changing. It is implausible that a change in childbearing would change consumption at each age by a constant percentage. The third alternative is ruled out on a priori grounds. However, such a hypothesis can be easily handled by the model.

Figure 2. Alternative Hypothesized Shifts in Household Consumption.



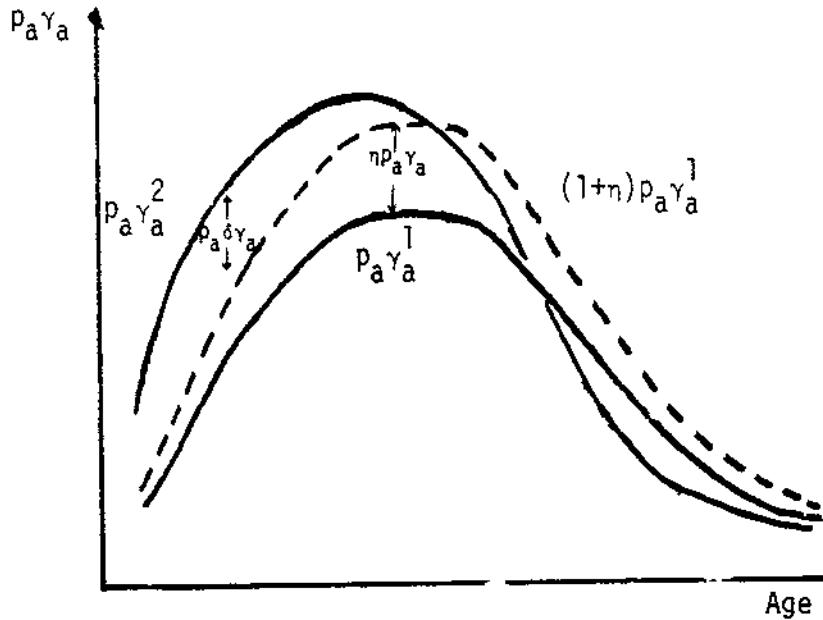
It is possible to separate any shift in the household's consumption function into changes in the level of consumption and changes in the timing of consumption. The procedure is illustrated by Figure 3. Consumption by a household which survives to age a has shifted from γ_a^1 to γ_a^2 and expected consumption from $p_a \gamma_a^1$ to $p_a \gamma_a^2$. The change in expected consumption is $p_a \delta \gamma_a = p_a (\gamma_a^2 - \gamma_a^1)$. The fraction of resources the household expects to consume over its lifetime has risen by η where:⁸

$$(10) \quad \delta \ln \Gamma = \eta = \frac{\int p_a \delta \gamma_a da}{\int p_a \gamma_a da}$$

As shown in Figure 3, the level of consumption has risen to $(1+\eta) p_a \gamma_a^1$ at each age.

Figure 3. Separation of Shift of Consumption Profile into Change in Level and Change in Timing.

$$p_a \delta \gamma_a = n p_a \gamma_a + p_a \delta \gamma_a^*$$



In addition, the household redistributes consumption toward younger ages.

The timing change, $\delta \gamma_a^*$, is defined as:

$$(11) \quad \delta \gamma_a^* = \delta \gamma_a - n \gamma_a$$

Multiplying both sides of equation (11) by p_a and integrating, $\int p_a \delta \gamma_a^* da = 0$, i.e., $\delta \gamma_a^*$ measures only changes in the distribution of consumption.

From the definition of $s_a = \phi_a - \gamma_a$ and from equation (5) the effects of a change in household consumption on aggregate saving is:

$$(12) \quad \frac{\delta S}{\delta \gamma} = - \int v_a \delta \gamma_a da$$

Substituting for v_a from equation (6) and replacing $\delta\gamma_a$ with $n\gamma_a + \delta\gamma_a^*$:

$$(13) \quad \frac{\delta S}{\delta Y} = -v_0 \int e^{-ga} p_a (n\gamma_a + \delta\gamma_a^*) da$$

It is shown in the appendix (I,C.) that $v_0 = (1-s) / \int e^{-gx} p_x \gamma_x dx$. Substituting for v_0 , expanding the integral and simplifying:

$$(14) \quad \frac{\delta S}{\delta Y} = - (1-s) \left(n + \frac{\int e^{-ga} p_a \delta\gamma_a^* da}{\int e^{-gx} p_x \gamma_x dx} \right)$$

Equation (14) shows the separate effects on aggregate saving of changes in the level and changes in the timing of household consumption. An increase in the level of consumption by n percent reduces aggregate saving by $(1-s)n$. It is possible to estimate n directly as will be shown below. The effect of changes in the timing of consumption depends on the rate of growth of total income, g , and the characteristics of $\delta\gamma_a^*$. We discuss two aspects of the timing effect in turn. First, we demonstrate that a shift in consumption toward younger ages has no effect on aggregate saving if g equals zero and reduces aggregate saving if g is greater than zero. Second, we provide an approximation of the timing effect that is amenable to estimation.

If the growth rate of total income is zero, changes in the timing of consumption have no effect on aggregate saving. Setting n and g to zero:

$$(15) \quad \frac{\delta S}{\delta Y} = - (1-s) \frac{\int p_a \delta\gamma_a^* da}{\int p_x \gamma_x dx} = 0$$

since by definition $\int p_a \delta\gamma_a^* da = 0$.

If the growth rate of total income is positive, shifts of household consumption toward younger ages reduces the aggregate saving rate. Suppose that the change in consumption is continuous and that there exists some age,

a^* , below which consumption rises and above which consumption falls.

In other words:

$$\delta\gamma_a^* > 0 \text{ if } a < a^*$$

$$\delta\gamma_a^* < 0 \text{ if } a > a^*$$

Setting η to zero the timing effect is:

$$(16) \quad \frac{\delta S}{\delta \gamma} = - (1-s) \frac{\int_0^{a^*} e^{-ga} p_a \delta\gamma_a^* da + \int_{a^*}^w e^{-ga} p_a \delta\gamma_a^* da}{\int_0^w e^{-gx} p_x \gamma_x dx}$$

Applying the mean value theorem for integrals:

$$(17) \quad \frac{\delta S}{\delta \gamma} = - (1-s) \frac{e^{-ga_1} \int_0^{a^*} p_a \delta\gamma_a^* da + e^{-ga_2} \int_{a^*}^w p_a \delta\gamma_a^* da}{e^{-ga_3} \int_0^w p_x \gamma_x dx}$$

$$0 < a_1 < a^* < a_2 < w$$

$$0 < a_3 < w$$

From the definition of $\delta\gamma_a^*$:

$$(18) \quad \int_{a^*}^w p_a \delta\gamma_a^* da = - \int_0^{a^*} p_a \delta\gamma_a^* da$$

Substituting into equation (17) and collecting terms:

$$(19) \quad \frac{\delta S}{\delta \gamma} = (1-s) \frac{e^{-ga_1} - e^{-ga_2}}{e^{-ga_3}} \frac{\int_0^{a^*} p_a \delta\gamma_a^* da}{\int_0^w p_x \gamma_x dx} \begin{array}{l} < 0 \text{ if } g > 0 \\ = 0 \text{ if } g = 0 \\ > 0 \text{ if } g < 0 \end{array}$$

For a growing economy a shift in consumption toward young ages reduces the aggregate saving rate.

As explained above, the timing of household consumption can be represented by the mean age of consumption, μ_c . Obviously, a shift in consumption toward younger ages reduces the mean age of consumption. As is shown in the appendix (Parts IV, VI.A), the timing effect can be approximated as:

$$(20) \quad \frac{\delta S}{\delta \gamma} \approx (1-s) g \delta \mu_c.$$

The total effect of a change in household consumption can be approximated by:

$$(21) \quad \frac{\delta S}{\delta \gamma} \approx - (1-s) (n - g \delta \mu_c)$$

Changes in the Timing of Earning

The relationship of household earning to the aggregate saving rate is quite similar to that derived for household consumption with one important exception. By definition only the timing, but not the level, of household earning can change. The rate of household earning is defined as the fraction of lifetime resources earned in each period and by definition $\int p_a \phi_a da = 1$. Likewise $\int p_a \delta \phi_a da = 0$.

The functional form of the timing effect of household earning is identical, though of opposite sign, to the timing effect of household consumption. The effect of a change in household earning is given by:

$$(22) \quad \frac{\delta S}{\delta \phi} = v_0 \int e^{-ga} p_a \delta \phi_a da$$

It is shown in the appendix (Part III) that:

$$(23) \quad \frac{\delta S}{\delta \phi} = (1-s) \frac{\int e^{-ga} p_a \delta \phi_a da}{\int e^{-gx} p_x \phi_x dx}$$

The functional equivalence to the timing effect of a change in consumption is apparent by comparing with equation (14). Further analysis of earning is unnecessary. The effect of a change in household earning is approximated by:

$$(24) \quad \frac{\delta S}{\delta \phi} = - (1-s) (g \delta \mu_E)$$

where $\delta \mu_E$, the change in the mean age of household earning, approximates the shift in the timing of household consumption.

Synthesis: Rate of Growth and Dependency Effects

Combining the effect of changes in household consumption, equation (21), and household earning, equation (24), gives the dependency effect, as defined by equation (5):

$$(25) \quad \int_0^W v_a ds_a da = (1-s) [-n + g (\delta \mu_C - \delta \mu_E)]$$

The rate of growth effect, from equation (8) is:

$$(26) \quad \int_0^W dv_a s_a da = (1-s) [(\mu_C - \mu_E) \delta g]$$

Substituting $\delta(\mu_C - \mu_E)$ for $\delta \mu_C - \delta \mu_E$, the total change in aggregate saving is the sum of the dependency and rate of growth effects:

$$(27) \quad \delta s = (1-s) [-n + g \delta(\mu_C - \mu_E) + (\mu_C - \mu_E) \delta g].$$

The effect of any independent variable, x, on aggregate saving depends upon the relationship of x to the level of household consumption and to the mean age of household consumption minus the mean age of household earning. There is no a priori basis by which to determine the appropriate functional form of these

two relationships. Consequently, analytically convenient functions are used. The level of household consumption is approximated by:

$$(28) \quad \Gamma = \int p_a \gamma_a da = e^{-\beta_0 - \beta_3 x}$$

The percentage increase in the level of consumption due to a change in x is given by $-\beta_3 = \eta$. For $x = 0$, Γ is determined by β_0 .

The difference between the mean age of consumption and the mean age of earning is approximated by:

$$(29) \quad \mu_C - \mu_E = \beta_1 + \beta_2 x$$

The effect on the timing of household consumption and earning of a change in x is β_2 , while $\beta_1 = \mu_C - \mu_E$ for $x = 0$.

The aggregate saving function is found by integrating equation (27) or, equivalently, by substituting for Γ and $\mu_C - \mu_E$ in equation (9):

$$(30) \quad \ln \left(\frac{1}{1-s} \right) = \beta_0 + \beta_3 x + (\beta_1 + \beta_2 x)g \\ = \beta_0 + \beta_1 g + \beta_2 gx + \beta_3 x$$

The validity of the specification is readily confirmed by taking the partial derivative of equation (30) with respect to g and comparing with the derived rate of growth effect, equation (26), and taking the partial derivative with respect to x and comparing with the derived dependency effect, equation (25).

The Affect of Population Growth on Household Saving

It is important to realize that the aggregate saving function and its interpretation is in no way limited to analysis of demographic factors. The unidentified variable, x , can be any independent variable (or vector of

variables) deemed to bear on the level and/or timing of household consumption. Indeed, an extension of the model to include the rate of interest in addition to demographic factors has proved quite successful (Fry and Mason, 1980).

A substantial amount of research has examined the relationship between number of children and household consumption.⁹ Although the studies are by no means unanimous, a number do provide support for the hypothesis that the number of children affect the timing of household consumption. These studies find that current household consumption increases with number of children. Given the concentration of childrearing at early household ages, this implies that the mean age of consumption is inversely related to the rate of population growth.

This evidence does not imply a change in the level of household consumption, because it is not determined whether increased consumption during childrearing years will result in higher lifetime consumption or lower post-childrearing consumption. Furthermore, there is no theoretical basis for hypothesizing an effect. An increase in the level of consumption will, of course, reduce bequests of which children are the principal recipients. One might appeal to the 'new home economics' and argue that expenditures per child (including bequests) and number of children are substitutes. However, this implies that bequests per child, not total bequests, decline with family size. We take an agnostic position on the issue and will retain a bequests effect (see Model B, above) only if its inclusion contributes to the explanatory power of our model or has important qualitative implications for the relationship between population growth and saving.

The effect of population growth on aggregate saving through household earning has not received much attention. However, a substantial amount of research has demonstrated that household earning is effected by demographic factors. Three separate mechanisms have been identified. First, children

may contribute directly to household income. Second, employment and earning by women depends on the number of children reared. Third, the age-earnings profile may depend on the age distribution of the labor force. No clear relationship between earning and population emerges from reviewing these effects, however. Consider the diverse effects of a decline in population growth.

The family may lose the earnings of the marginal child, but the effect on the distribution of earnings is ambiguous and undoubtedly small. Fertility decline is typically concentrated among older women, those aged thirty and above. The loss in earnings is delayed another ten to fifteen years when women are forty to fifty years old. The mean age of earning typically falls into the same age interval and will not be effected to any important extent.¹⁰

Second, labor force participation by women, particularly of late childbearing age, may rise reducing the mean age of earning. However, the wage of these women may rise at later ages by virtue of their increased labor market experience. This will increase earning during older household years offsetting the decline in the mean age at earning.¹¹

Third, as population growth declines the ratio of young to old workers declines. Wages of young workers may rise relative to old workers, i.e., the mean age of earning declines.¹² This reinforces the effect of population growth on consumption. The other two mechanisms are ambiguous in their effect. In total, there is little reason to suppose that mean age of earning is particularly sensitive to the rate of population growth.

In any case, it is not important to distinguish the effect of population growth on consumption from the effect on earning. Only the net effect, $\delta(\mu_C - \mu_E)$, is of interest. We hypothesize that the consumption effect dominates the earning effect, i.e., the difference between μ_C and μ_E declines with a rise in population growth. An increase in population growth will reduce the mean age of consumption by more than the mean age of earning.

IV. EMPIRICAL RESULTS: THE EFFECT OF POPULATION GROWTH

As discussed in Section III, two alternative models are estimated. Model A is the appropriate specification if the population growth rate or number of children reared affects only the timing of consumption and earning by households. Model B is the appropriate specification if both timing of consumption and earning and the level of consumption are affected. The results presented below show that the inclusion of the level effect does not contribute to the explanatory power of the model nor does it effect the estimates of other parameters of predicted saving rates in any meaningful way. Consequently, Model A is retained and subject to additional analysis.

Two alternative population variables are employed below: the population growth rate and the dependency ratio (the population aged less than fifteen divided by the population aged fifteen to sixty-five.) The dependency ratio (D^*) has been rescaled to have an expected value of zero for a population growth rate of zero and an expected value of 0.03 for a population growth rate of 0.03.¹³ The rescaling, of course, has no effect on statistical tests, the coefficient of determination or predicted values. It does facilitate comparison of alternative specifications and comparisons of the rate of growth and dependency effects.

The saving function to be estimated is:

$$(31) \quad \ln \left[\frac{1}{1-s} \right] = \beta_0 + \beta_1 g + \beta_2 gD^* + \beta_3 D^*$$

where the last term is included only under the conditions described above and the population growth rate, n , may be substituted for D^* . The following hypotheses are tested. First, $r(0)$, the level of consumption for $D^*=0$, should be close to one. Consequently, $\beta_0 = -\ln(r(0))$ should be a small positive value. Second, β_1 is equal to $\mu_C - \mu_E$ for $D^* = 0$ and is hypothesized to be

greater than zero. Third, the effect of D^* on $\mu_C - \mu_E$, β_2 , is hypothesized to be negative. No hypothesis regarding β_3 is offered.

Data and Estimation Procedures

The saving function is estimated from a cross-section of countries whose labor force exceeded one million workers in 1960. The data are taken from World Bank, World Tables, 1975. The saving rate is measured by the average of the total rate of saving for 1960 and 1970. The growth rate of total income and population are measured by the average annual rate of growth between 1960 and 1970. The dependency ratio is measured by the average of the 1960 and 1970 dependency ratios.

Two aspects of the data require some discussion. First, average values over the 1960 to 1970 period are deemed more appropriate to estimating a long-run saving model. Of course, there are important short-run variations in aggregate saving which are averaged away, but explaining these short-run changes are beyond the scope of this model.

Second, our measure of saving includes government and business saving in addition to household saving. At first glance, household saving may appear to be a more appropriate variable. However, government and business saving may be a substitute for household saving and thereby influenced by the same factors as household saving.¹⁴

The saving function is estimated by two stage least squares. The dependence of the growth of total income on the rate of saving is obvious and requires no elaboration. The growth rate of gross domestic product is regressed on average per capita income (y), average annual growth rate of the labor force (\dot{L}/L), the average ratio of net factor income from foreign sources to gross national product (NFI), the average annual growth rate of energy consumption per capita (\dot{NRG}/NRG) and the average percent of the labor force in agriculture (F/L) for the 1960 to 1970 decade.

The first stage parameter estimates and standard errors are:

$$\begin{aligned}
 (32) \quad g &= -0.019 + 0.0105 \ln y + 0.171 \dot{L}/L \\
 &\quad (0.017) \quad (0.0036) \quad (0.132) \\
 &+ 0.00086 \text{ NFI} + 0.0524 \dot{\text{NRG}}/\text{NRG} + 0.0141 \ln (F/L) \\
 &\quad (0.00047) \quad (0.0534) \quad (0.0050) \\
 N &= 69 \quad R^2 = .364
 \end{aligned}$$

Estimated Saving Function

The parameter estimates and standard errors for the alternative specifications of Model A are:

$$\begin{aligned}
 (33) \quad \ln \left(\frac{1}{1-s} \right) &= - .0239 + 5.6546 \hat{g} - 69.898 \hat{g} n \\
 &\quad (.0353) \quad (.6991) \quad (15.731) \\
 N &= 69 \quad R^2 = 0.53
 \end{aligned}$$

$$\begin{aligned}
 (34) \quad \ln \left(\frac{1}{1-s} \right) &= .0183 + 4.953 \hat{g} - 81.356 \hat{g} D^* \\
 &\quad (.0322) \quad (.5982) \quad (11.977) \\
 N &= 69 \quad R^2 = 0.64
 \end{aligned}$$

In all respects, the use of the dependency ratio, D^* , to measure the dependency effect proves superior. The overall explanatory power of the second specification is greater and the standard errors of each parameter estimate are smaller. The parameter estimates from either specification are consistent with hypothesized values. Although the constant terms are not significantly different from zero, this is not a relevant hypothesis test. We cannot reject the proper hypothesis, that β_0 is a small positive number.

The coefficient of \hat{g} is significantly greater than zero, as hypothesized. The coefficient of the interaction term is significantly less than zero. OLS results, not shown, were quite similar to the 2SLS results.

Before examining these results in detail, we turn to Model B. As discussed above Model A is based on the assumption that the level of household consumption, Γ , is independent of the population growth rate. Model B is the appropriate specification if the level of consumption depends on the rate of population growth or the dependency ratio. This elaboration is accomplished by including n or D^* as an independent variable. The parameter estimates and standard errors for alternative specifications of Model B are:

$$(35) \quad \ln \left(\frac{1}{1-s} \right) = - .0221 + 5.6219 \hat{g} - 68.245 n \hat{g} - .0898 n$$
$$\quad \quad \quad (.089) \quad (1.625) \quad (75.601) \quad (4.014)$$
$$\quad \quad \quad N = 69 \quad \quad R^2 = 0.53$$

$$(36) \quad \ln \left(\frac{1}{1-s} \right) = .0414 + 4.5426 \hat{g} - 60.495 D^* \hat{g} - 1.119 D^*$$
$$\quad \quad \quad (.0682) \quad (1.2258) \quad (55.6) \quad (2.910)$$
$$\quad \quad \quad N = 69 \quad \quad R^2 = 0.64$$

Obviously, considerable care must be applied in deciding between the two models. The inclusion of D^* or n can have an important bearing on the relationship between the population growth rate and saving. In addition, the multicollinearity of Model B undoubtedly reduces the precision of our estimates. However, neither of the estimates of Model B support the hypothesis that the level of consumption depends on the rate of population growth. The coefficients of n and D^* are not significantly different from zero. Nor does the inclusion of n or D^* increase the explanatory power over that of Model A. Finally, the qualitative relationship between saving and the rate of

population growth or the growth of total income is not effected by the inclusion of the additional term. The partial effect of an increase in population growth has been calculated for both models for a number of values of n and g . In no case are there any differences in the signs of the partial derivatives, nor are there important differences in the magnitudes of the partial derivatives. Likewise, the partial effect of an increase in g does not differ in any important way between Models A and B. In the absence of any evidence supporting Model B, Model A is judged superior and further analysis is restricted to the simpler model.

The final hypothesis to be tested is that the partial effect of an increase in the population growth should be identical to the partial effect of an increase in the growth rate of per capita income (holding D^*g fixed.) The estimates presented above impose this restriction on the aggregate saving function. It is possible to test the restriction by regressing the saving variable on the growth rate of per capita income (λ), the rate of population growth and the interaction term, D^*g . The estimated relationship and standard errors obtained are:

$$(38) \quad \ln(1/1-s) = - .0116 + 5.2839 \hat{\lambda} + 7.0672 n - 105.89 D^*g$$

	(.039)	(.6440)	(1.6889)	(21.868)
N = 69			$R^2 = 0.65$	

The coefficients of λ and n are each significantly greater than zero, but not significantly different from each other. The value of the F statistic is 1.788. This provides new evidence in support of the life-cycle saving model.

V. DISCUSSION OF RESULTS

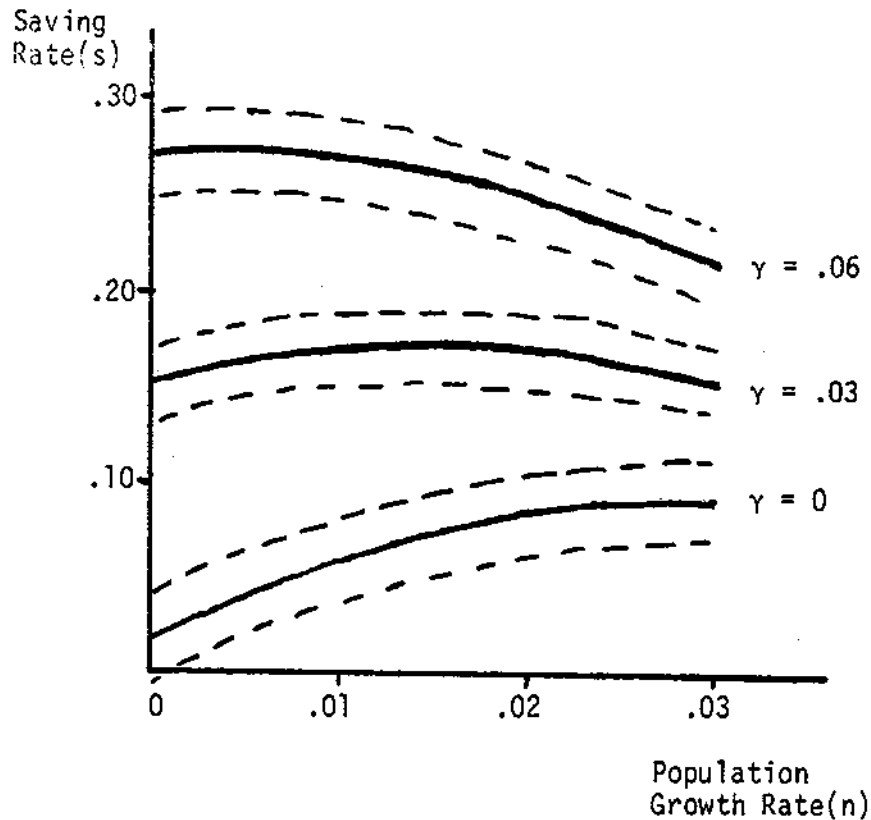
The Rate of Growth Effect

The existence and importance of the rate of growth effect is confirmed by the results presented above. For all plausible values of D^* , an increase in g raises aggregate saving. The estimated relationship is distinct from previous research as the dependency ratio determines the magnitude of the rate of growth effect. Setting g equal to its sample mean (.0488), $\delta s/\delta g$ ranges from 3.8 down to 2.2 for dependency ratios (or population growth rate) of zero and 0.03, respectively. The substantial variation in the rate of growth effect has been neglected by previous research which typically assumes a constant rate of growth effect. (See Mikesell and Zinser, 1973.)

The Effect of Population Growth

The relationship of population growth to aggregate saving, holding the growth rate of per capita income constant, depends on the relative strengths of the rate of growth effect and the dependency effect. As shown in Figure 4, the rate of growth effect dominates for small values of per capita income growth and saving is positively related to population growth. For countries experiencing moderate levels of growth in per capita income, neither effect dominates; hence, there is no significant relationship between population growth and aggregate saving.¹⁵ For higher rates of growth of per capita income, the dependency effect dominates and an inverse relationship between aggregate saving and population growth emerges.¹⁶

Figure 4. Predicted Saving Rates and the Population Growth Rate; $\lambda = 0.0$, 0.03 and 0.06.



For most countries population growth may not be an important determinant of the rate of saving, except for its influence on the rate of growth effect. Countries with extremely low rates of growth of per capita income have had little success in reducing their population growth rates. On the other hand countries with high rates of growth of per capita income have very nearly achieved zero population growth.

There are, however, exceptions to this generalization. Several countries, Japan and the Republic of Korea being notable examples, have simultaneously experienced rapid declines in population growth and high rates of growth in

income. Between 1955 and 1970, the annual growth rate of GDP averaged nearly ten percent in Japan while the dependency ratio declined from 0.545 to 0.360. Over the same period the saving rate rose from twenty-six to forty percent of GDP.¹⁷ Application of the cross-section estimates imply that the decline of the dependency ratio accounts for over half of the increase in saving. In Korea the rate of growth of GDP rose from four percent in 1963 to over nine percent in 1979. The dependency ratio was relatively constant at 0.8 until 1967 but had declined to 0.56 by 1979. From 1963 to 1979 the saving rate rose by twenty percentage points.¹⁸ The cross-section estimates imply that roughly a third of the increase in saving was attributable to the decline in the dependency ratio. Although these results are tentative, they illustrate the potential importance of population growth as a determinant of saving.

Conclusion

The extension of the life-cycle model to cases in which the level and timing of household saving varies systematically with independent variables opens several avenues of research in addition to the effect of population growth on aggregate saving. For example, the timing of household saving may well depend upon the rate of interest and the size of publicly funded pension programs. The empirical evidence presented above indicates the important effect that changes in the timing of saving can have on the rate of growth effect. Previous research which is based on the assumption that the rate of growth effect is constant overlooks substantial variation related to the rate of population growth.

The direct affect of population growth on aggregate saving may be considerably less important than has been suggested in previous research. In particular, the results do not support the view that countries with low rates of economic growth and rapid population growth will experience higher rates of saving in response to reduced rates of population growth. Only in countries

with rapid economic growth is there an inverse relationship between population growth and aggregate saving.

Although the analysis carried out here focusses on the decline in population growth that has accompanied economic development, there are other demographic processes which may also have an important bearing on saving. The substantial changes in the age-structure of the United States population have had a demonstrable effect on the age-earnings profile (see note 12). The possibility that aggregate saving rates have also been influenced warrants investigation. A dynamic formulation of the life-cycle model is clearly required, however, to analyze the U.S. case.

MATHEMATICAL APPENDIX

I. CHARACTERISTICS OF v_a

A. According to equation (6):

$$(6) \quad v_a = e^{-ga} p_a v_0$$

PROOF: By definition

$$(A.1) \quad v_a = \frac{H_{a,t} V_{a,t}}{GNP_t}$$

where: $H_{a,t}$ is the number of households aged a at time t , and $V_{a,t}$ is the expected lifetime resources of households aged a at time t , i.e.,

$$(A.2) \quad V_{a,t} = \int_0^w p_x Y_{x,t-a+x} dx$$

By assumption, the population is in long-run equilibrium. This implies:

$$(A.3) \quad H_{a,t} = p_a H_{0,t-a} = p_a e^{-na} H_{0,t}$$

Likewise, per capita income is assumed to be growing at a constant rate. This implies:

$$(A.4) \quad Y_{x,t-a+x} = e^{-\lambda a} Y_{x,t+x}$$

Substituting into (A.2):

$$(A.5) \quad V_{a,t} = e^{-\lambda a} \int_0^w p_x Y_{x,t+x} dx = e^{-\lambda a} V_{0,t}$$

Substituting into (A.1) for $H_{a,t}$ from (A.3) and $V_{a,t}$ from (A.5):

$$(A.6) \quad v_a = \frac{e^{-(\lambda+n)a} p_a H_{0,t} V_{0,t}}{GNP_t} = e^{-ga} p_a v_0$$

B. The value of v_0 is given by:

$$(A.7) \quad v_0 = 1/\int e^{-ga} p_a \phi_a da$$

PROOF: By definition:

$$(A.8) \quad v_0 = \frac{H_{0,t} V_{0,t}}{GNP_t}$$

where GNP_t is defined as:

$$(A.9) \quad GNP_t = \int H_{a,t} Y_{a,t} da$$

By definition:

$$(A.10) \quad Y_{a,t} = \phi_a V_{a,t}$$

From (A.5):

$$(A.11) \quad Y_{a,t} = \phi_a e^{-\lambda a} V_{0,t}$$

Substituting into (A.9) from (A.11) and (A.3):

$$(A.12) \quad GNP_t = H_{0,t} V_{0,t} \int e^{-ga} p_a \phi_a da$$

Substituting into (A.8) and cancelling like terms:

$$(A.13) \quad v_0 = 1/\int e^{-ga} p_a \phi_a da.$$

C. An alternative representation of v_0 will prove useful below:

$$(A.14) \quad v_0 = (1-s) / \int e^{-ga} p_a \gamma_a da$$

PROOF: From equations (5) and (6) and the definition of $s_a = \phi_a - \gamma_a$, s is given by:

$$s = v_0 \int e^{-ga} p_a (\phi_a - \gamma_a) da$$

Substituting for s in (A.14) and expanding terms:

$$(A.15) \quad v_0 = \frac{1 - v_0 (\int e^{-ga} p_a \phi_a da - \int e^{-ga} p_a \gamma_a da)}{\int e^{-ga} p_a \gamma_a da}$$

Substituting for v_0 from (A.13) and cancelling like terms:

$$(A.16) \quad v_0 = \frac{1 - 1 + v_0 \int e^{-ga} p_a \gamma_a da}{\int e^{-ga} p_a \gamma_a da} = v_0$$

II. The Rate of Growth Effect:

Equation (8) gives an approximation of the rate of growth effect. We first derive an exact representation of the rate of growth effect. Derivation of equation (8) is completed in part V:

The rate of growth effect equals:

$$(A.17) \quad \int \frac{\delta v_a}{\delta g} s_a da = (1-s) (A_C - A_E)$$

$$\text{where: } A_C = \frac{\int a e^{-ga} p_a \gamma_a da}{\int e^{-ga} p_a \gamma_a da}$$

$$A_E = \frac{\int a e^{-ga} p_a \phi_a da}{\int e^{-ga} p_a \phi_a da}$$

PROOF: Substituting for v_a from (A.6) and differentiating with respect to g :

$$(A.18) \quad \frac{\delta v_a}{\delta g} = \frac{\delta}{\delta g} e^{-ga} p_a v_0$$

$$= e^{-ga} p_a \frac{\delta v_0}{\delta g} - a e^{-ga} p_a v_0$$

The rate of growth effect is given by:

$$(A.19) \quad \int \frac{\delta v_a}{\delta g} s_a da = \frac{\delta v_0}{\delta g} \int e^{-ga} p_a s_a da - v_0 \int a e^{-ga} p_a s_a da$$

Substituting for v_0 from (A.13) and solving for $\delta v_0/\delta g$:

$$(A.20) \quad \frac{\delta v_0}{\delta g} = \frac{\delta}{\delta g} \left[\frac{1}{\int e^{-gx} p_x \phi_x dx} \right]$$

$$= v_0 \frac{\int x e^{-gx} p_x \phi_x dx}{\int e^{-gx} p_x \phi_x dx} = v_0 A_E$$

Substituting for $\frac{\delta v_0}{\delta g}$ in (A.19) the first right hand side term is:

$$(A.21) \quad \frac{\delta v_0}{\delta g} \int e^{-ga} p_a s_a da = A_E v_0 \int e^{-ga} p_a s_a da$$

$$= A_E s$$

Substituting $\phi_a - \gamma_a$ for s_a in the second term of (A.19) and expanding the integral:

$$(A.22) \quad -v_0 \int a e^{-ga} p_a s_a da = -v_0 \int a e^{-ga} p_a \phi_a da + v_0 \int a e^{-ga} p_a \gamma_a da$$

Substituting for the first right hand side v_0 from (A.13) and for the second right hand side v_0 from (A.14):

$$(A.23) \quad -v_0 \int a e^{-ga} p_a s_a da = - \frac{\int a e^{-ga} p_a \phi_a da}{\int e^{-ga} p_a \phi_a da} + (1-s) \frac{\int a e^{-ga} p_a \gamma_a da}{\int e^{-ga} p_a \gamma_a da}$$

$$= - A_E + (1-s) A_C$$

Substituting from (A.21) and (A.23) into (A.19):

$$(A.24) \quad \int \frac{\delta v_a}{\delta g} s_a da = A_E s - A_E + (1-s) A_C$$

Collecting terms:

$$(A.25) \quad \int \frac{\delta v_a}{\delta g} s_a da = (1-s) (A_C - A_E).$$

III. The Dependency Effect:

The effect of a change in household consumption, equation (12), is easily derived given (A.14). Equation (23) gives the effect of a change in the distribution of earnings as:

$$(23) \quad \frac{\delta S}{\delta \phi} = (1-s) \frac{\int e^{-ga} p_a \delta \phi_a da}{\int e^{-gx} p_x \phi_x dx}$$

PROOF: By definition, the dependency effect of a change in the distribution of earnings is given by:

$$\begin{aligned} (A.26) \quad \frac{\delta S}{\delta \phi} &= \frac{\delta}{\delta \phi} v_0 \int e^{-ga} p_a s_a da \\ &= \frac{\delta}{\delta \phi} \frac{\int e^{-ga} p_a (\phi_a - \gamma_a) da}{\int e^{-gx} p_x \phi_x dx} \\ &= \frac{\int e^{-ga} p_a \delta \phi_a da}{\int e^{-gx} p_x \phi_x dx} - \frac{\int e^{-ga} p_a \delta \phi_a da}{\int e^{-gx} p_x \phi_x dx} s \end{aligned}$$

$$(A.27) \quad \frac{\delta S}{\delta \phi} = (1-s) \frac{\int e^{-ga} p_a \delta \phi_a da}{\int e^{-gx} p_x \phi_x dx}$$

IV. Approximating Rate of Growth and Dependency Effects

The rate of growth and dependency effects each include non-linear functions in g . In order to estimate the aggregate saving function, it is necessary to derive linear approximations to the functions. In each case, the non-linear functions can be represented generally by:

$$(A.28) \quad Z(g) = \frac{F(g)}{H(g)} = \frac{\int e^{-gx} f(x) dx}{\int e^{-gx} h(x) dx}$$

$Z(g)$ can be approximated by a Taylor series expansion as:

$$(A.29) \quad Z(g) = Z(0) + gZ'(0) + \frac{g^2}{2} Z''(0) + \dots$$

where $Z'(0)$ and $Z''(0)$ are the first and second derivatives of Z with respect to g evaluated at g equal to zero. It requires some algebra, but is straightforward to show that:

$$(A.30) \quad Z(0) = \frac{\int f(x) dx}{\int h(x) dx}$$

$$Z'(0) = \frac{\int x f(x) dx}{\int h(x) dx} + Z(0) \mu$$

$$Z''(0) = \frac{\int (x-\mu)^2 f(x) dx}{\int h(x) dx} - Z(0) \sigma^2$$

where $\mu = \frac{\int x h(x) dx}{\int h(x) dx}$, and

$$\sigma^2 = \frac{\int (x-\mu)^2 h(x) dx}{\int h(x) dx}$$

the mean and variance, respectively, of $h(x)$. We apply (A.29) and (A.30) to each of the non-linear functions to determine the first two terms of the linear approximation.

V. Approximating the Rate of Growth Effect

A. A_c can be approximated by:

$$(A.31) \quad A_c \approx \mu_c - g \sigma_c^2$$

PROOF:

Letting $f(x) = a p_x \gamma_x$ and $h(x) = p_x \gamma_x$

$$(A.32) \quad Z(0) = \frac{\int x p_x \gamma_x dx}{\int p_x \gamma_x dx} = \mu_c$$

$$(A.33) \quad Z'(0) = \frac{\int x^2 p_x \gamma_x dx}{\int p_x \gamma_x dx} + \mu_c^2$$

$$\begin{aligned} Z'(0) &= - \frac{\int (x-\mu_c)^2 p_x \gamma_x dx}{\int p_x \gamma_x dx} - 2\mu_c \frac{\int x p_x \gamma_x dx}{\int p_x \gamma_x dx} \\ &\quad + \mu_c^2 \frac{\int p_x \gamma_x dx}{\int p_x \gamma_x dx} + \mu_c^2 \end{aligned}$$

The last three terms cancel, leaving:

$$(A.34) \quad Z'(0) = - \frac{\int (x-\mu_c)^2 p_x \gamma_x dx}{\int p_x \gamma_x dx} = -\sigma_c^2$$

Substituting for $Z(0)$ and $Z'(0)$ in (A.29) and dropping the third term (we are carrying only two non-zero terms) gives (A.31).

B. A_E can be approximated by:

$$A.35 \quad A_E \approx \mu_E - g \sigma_E^2$$

PROOF:

Letting $f(x) = a p_a \phi_a$ and $h(x) = p_a \phi_a$, (A.35) is derived in analogous fashion to (A.31).

C. The rate of growth effect is approximated by:

$$A.36 \quad \frac{\delta S}{\delta g} = (1-s) [(\mu_C - \mu_E) + g (\sigma_E^2 - \sigma_C^2) + \dots]$$

The corresponding aggregate saving function is:

$$\ln[1/(1-s)] \approx \beta_0 + (\mu_C - \mu_E) g + (\sigma_E^2 - \sigma_C^2) g^2$$

The higher order term, g^2 , was included in preliminary estimates of the saving function; the estimated coefficient was not significantly different from zero and was dropped from the final estimates.

VI. Approximating the Dependency Effect

A. The effect of a change in household consumption is given in equation (14) as:

$$(14) \quad \frac{\delta S}{\delta Y} = (1-s) \left[\eta + \frac{\int e^{-gx} p_x \delta \gamma_x^* dx}{\int e^{-gx} p_x \gamma_x dx} \right]$$

A linear approximation is given by:

$$(A.37) \quad \frac{\delta S}{\delta Y} = (1-s) (\eta - g \delta \mu_c + g^2 \delta \sigma_c^2)$$

PROOF:

Letting $f(x) = p_x \delta \gamma_x^*$ and $h(x) = p_x \gamma_x$

$$(A.38) \quad Z(0) = \int p_x \gamma_x dx = 0$$

$$(A.39) \quad Z'(0) = - \frac{\int x p_x \delta \gamma_x^* dx}{\int p_x \gamma_x dx} = - \delta \mu_x$$

$$(A.40) \quad Z''(0) = \frac{\int (x-\mu_c)^2 p_x \delta \gamma_x^* dx}{\int p_x \gamma_x dx} = \delta \sigma_c^2$$

Substituting into (A.29) gives the linear approximation (A.37).

B. The linear approximation of the earnings effect is analogous:

$$(A.41) \quad \frac{\delta S}{\delta \phi} = (1-s) (-g \delta \mu_E + g^2 \delta \sigma_E^2)$$

C. Combining (A.40) and (A.41) the dependency effect is:

$$(A.42) \quad \frac{\delta S}{\delta \phi} - \frac{\delta S}{\delta \gamma} = (1-s) (n + g (\delta \mu_E - \delta \mu_C) + g^2 (\delta \sigma_C^2 - \delta \sigma_E^2))$$

The last term in (A.42) has been dropped from the final specification of the aggregate saving function. The empirical analysis described above found no significant relationship between the higher order term and aggregate saving.

NOTES

1. Mikesell and Zinser (1973) review much of the literature. See also Papanek (1973) and Fry (1978, 1980).
2. Modigliani and Brumberg (1954), Modigliani and Ando (1957), Modigliani (1965, 1966), Fisher (1956), and Farrell (1970).
3. Tobin (1967) and Arthur and McNicoll (1978) have modelled the effect of population growth on equilibrium wealth using a life-cycle framework which includes both dependency and rate of growth effects. Conroy (1979) has recently used the Tobin framework to analyze the effect of population growth on equilibrium saving. The principal difference between these models and the model developed here is in the treatment of children. The unit of analysis is the individual so that children consume out of their own resources rather than from the resources of their parents. Tobin does work out a few examples using a household unit of analysis and alternative assumptions about the effect of children on consumption. No attempt is made to derive wealth or saving functions amenable to estimation. Empirical work on the dependency effect and aggregate saving includes studies by Leff (1969), Gupta (1975), and Bilsborrow (1973, 1979). At the micro-level Fisher (1956) and Modigliani and Ando (1957) modify the life-cycle model to include the effect of variations in family size on consumption by households within age brackets.
4. See Modigliani and Brumberg (1954), Modigliani (1966), Fisher (1956), and Farrell (1970).
5. The age of a household is the number of years since it was formed. Conceptualizing what is meant by the formation or birth of a household is not of crucial importance to the analysis carried out. For simplicity's sake,

we will assume that at some point in time individuals leave home, get married, face their own budget constraint and make their own decisions (jointly with their spouse). The age of household is the number of years since marriage.

6. Strictly speaking, life-cycle models, this one included, are incomplete because mortality rates are held constant, limiting formal analysis to variations in fertility. Mortality is of course an important determinant of the rate of population growth. However, the effect of a change in population growth should not depend crucially on the extent to which mortality as opposed to fertility is responsible. Either a decline in mortality or a rise in fertility result in a younger population, although the effects of fertility are somewhat stronger (Coale, 1964). In addition, cross-sectional differences in population growth reflect differences in fertility to a much larger extent than differences in mortality. For an analysis of the effect of mortality changes per se see Conroy (1979).
7. A similar condition is derived by Farrell (1970) and by Arthur and McNicoll (1978).
8. Analysis of the dependency effect requires the evaluation of shifts in the entire distribution of consumption, designated by δY , consisting of incremental changes at each age δY_a . For an introduction to the application of modern functional analysis to demographic function see W.B. Arthur (1979). For the most part, however, the rules for functional differentials are straight-forward extensions of those for standard differentials. A lack of familiarity with functional analysis should not impede the readers' understanding of analysis of the dependency effect.

9. For a review and discussion of literature on consumption and household consumption see Kleiman (1966) and Mueller (1976). See also Somermeyer and Bannink (1973), Espenshade (1975), and Kelley (1972).
10. See Mueller (1976).
11. See McCabe and Rosenzweig (1976), Nerlove and Schultz (1970), Mauer et al., (1973), and Mincer and Polachek (1974).
12. Research on the effect of cohort size on relative earnings has been limited to industrialized countries (Freeman, 1979, and Welch, 1979, for the U.S. and Martin, 1980, for Japan). These studies tentatively suggest that education is an important determinant of the substitutability of labor. Hence, the importance of this effect may be limited to industrialized countries.
13. The dependency ratio is rescaled by regressing the DR on the population growth rate (standard errors in parentheses):

$$\begin{aligned} \text{DR} &= \exp (-1.1770 + 46.840 n - 392.79 n^2) \\ &\quad (.0831) \quad (9.576) \quad (250.67) \\ R^2 &= 0.72 \end{aligned}$$

The rescaled dependency ratio, D^* , is :

$$\begin{aligned} D^* &= .03 (\text{DR} - \hat{\text{DR}}(0)/\hat{\text{DR}}(.03) \\ D^* &= .0523 (\text{DR} - .3082) \end{aligned}$$

14. Modigliani (1965) and Leff (1969) justify the use of total saving rather than household saving in their empirical analyses by pointing to the substitutability of corporate and public sector saving for private saving. Fry (1979) has demonstrated the importance of substitution in Turkey. For an analysis of separate components of saving see Bilsborrow (1979).

15. Although the aggregate saving rate hits a maximum for the intermediate growth case, a horizontal saving rate is consistent with the ninety-five percent confidence interval. Of course, for some value of λ a non-monotonic relationship exists. It cannot be confirmed, however, for any particular value of λ .
16. The relationship between population growth and saving obtained here is considerably different from that found by Leff (1969). As Kelly (1973) has pointed out, Leff's estimated saving function implies a U-shaped relationship between population growth and saving. In important respects, our study is in keeping with the position of Bilsborrow (1973, 1979) that the effect of population growth on saving for most developing countries has been overstated. As Gupta (1975) has pointed out, a full analysis of the effect of population growth requires estimation of a complete model of population and development. In particular, a decline in the rate of population growth may raise the rate of growth of per capita income. Consequently, the effect of population growth on saving will be greater. It is straight-forward to assess the implications of Gupta's findings for the saving function estimated here. The decline in the rate of growth in total income will be less than the decline in the rate of population growth. Consequently, the rate of growth effect will be less important and the saving functions shown in Figure 4 would be rotated clockwise.
17. Japanese data from Statistical Yearbook of Japan various years.
18. Korean data from Handbook of Korean Economy 1979.

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