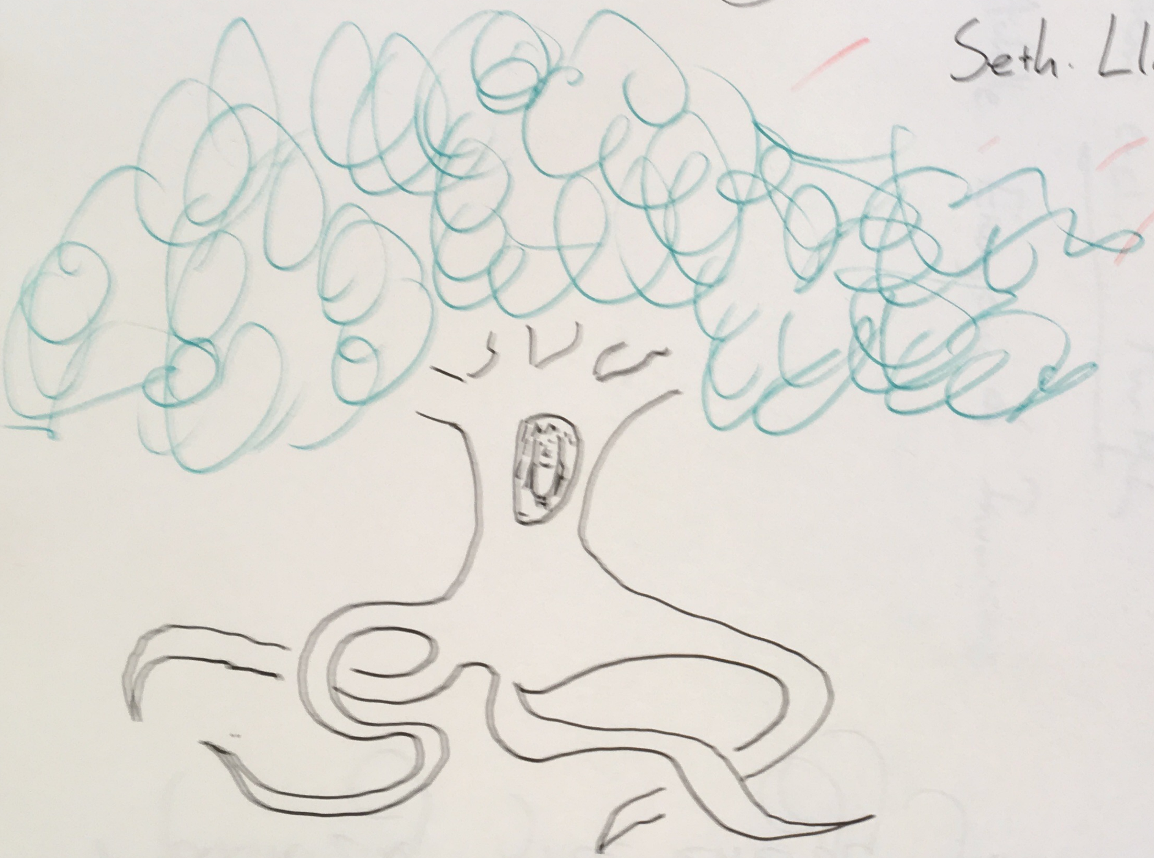
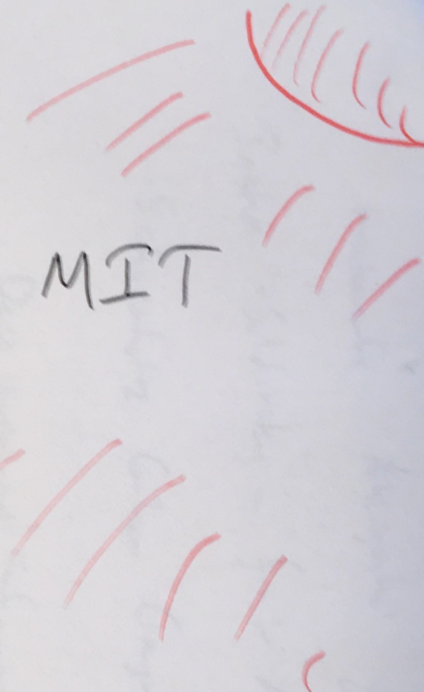
The free energy game

Seth Lloyd, MIT





Free energy:

$$F = \langle E \rangle - TS = \sum_x p(x) E(x) - T\left(-\sum_x p(x) \ln p(x)\right)$$

energy − temperature × entropy

Extractable work:

$$W^{ex} = F - F_{thermal}$$

$$P_{thermal}(E) \propto e^{-E/T}$$

$$= T\left(-\sum_x p(x) \ln \frac{P_{th}(x)}{p(x)}\right)$$

$$= T\, D(p \| P_{th})$$

Free Energy Increase

Stochastic process: $p(x_1|x_0)$

$$p(x_1) = \sum_{x_0} p(x_1|x_0)\,p(x_0)$$

$$\Delta F = F_1 - F_0$$

$$\Delta W^{ex} = W^{ex\,1} - W^{ex\,0}$$

Goal: choose $p(x_0)$ to <u>maximize</u> $\Delta F, \Delta W^{ex}$

Theorem: Let $q_0(x)$ be the initial distribution that maximizes ΔF, and let $r_0(x)$ be another, potentially sub-optimal initial distribution. Then the difference between the optimal increase and the sub-optimal increase is

$$\Delta F(q) - \Delta F(r) = T\left(D(r_0 \| q_0) - D(r_1 \| q_1)\right)$$

Theorem:

$D(r_0 \| q_0) - D(r_1 \| q_1)$ is convex in r_0.

$\Rightarrow$ optimal free energy harvesting state can be found by convex optimization.

Theorem:

$D(r_0\|q_0) - D(r_1\|q_1)$ is convex in r_0.

$\Rightarrow$ optimal free energy harvesting state can be found by convex optimization.