Logistics: This exam will be a 110-minute, closed book, closed notes exam. Successful test-takers will, at the least, know the below concepts well.

Major Concepts:

Darboux Sums

- Given a bounded function \( f \) on \([a, b]\) and a partition \( P = \{a = t_0 < t_1 < \cdots < t_n = b\} \), compute the “heights” of the rectangles \( m(f, [t_{k-1}, t_k]) \) and \( M(f, [t_{k-1}, t_k]) \). Use this to compute the upper and lower Darboux sums \( U(f, P) \) and \( L(f, P) \).

- Understand the relationship between the Darboux sums and the role that the partitions play. For example, for any partitions \( P \) and \( Q \), \( L(f, Q) \leq U(f, P) \); if \( P_1 \subset P_2 \), then \( L(f, P_1) \leq L(f, P_2) \) and \( U(f, P_2) \leq U(f, P_1) \).

Darboux Integrals

- Compute the upper and lower Darboux integrals \( U(f) \) and \( L(f) \)

- Use the fact that for any function, \( L(f) \leq U(f) \).

- Prove that \( f \) is or is not integrable by showing that \( U(f) \) and \( L(f) \) are or are not equal, respectively.

- Use the \( \varepsilon - P \) condition to prove that a function is Darboux integrable.

- Compute the Darboux integral by finding a sequence of partitions \( P_n \) such that \( \lim_{n \to \infty} U(f, P_n) = \lim_{n \to \infty} L(f, P_n) \).

Riemann Integration

- Understand the concept of the mesh of a partition

- Understand the definition of a Riemann sum and why it is not specified even if \( f \) and \( P \) are specified.

- Understand the definition of Riemann integrability.

- Use the fact that the Riemann integral and the Darboux integral are equivalent.

Integrable Functions

- Know that any continuous or monotonic function is automatically integrable

- Given two integrable functions, know that any linear combination of these functions is also integrable.

- Use the fact that if \( f \) is integrable, then \( |f| \) and \( f^2 \) is integrable.

- Use the fact that the product of two integrable functions are integrable.

- Use the fact that any piecewise continuous or piecewise monotonic function is integrable.

- Use the fact that if \( f \) is an integrable function and \( g \) is a function agreeing with \( f \) everywhere except for finitely many points, then \( g \) is also integrable and their integrals are equal.
Fundamental Theorem of Calculus

- Understand how the first FTC allows you to evaluate integrals of continuous functions whose derivative is integrable
- Know how to derive the formula for Integration by Parts using the FTC part 1
- Given an integrable function $f$ on $[a, b]$, compute $F(x) = \int_a^x f(t) \, dt$.
- Understand how the second FTC relations the integrability and continuity of $f(x)$ to the continuity and differentiability of $F(x)$.
- Know how to derive the Change of Variable formula
- Understand how to compute the integral of the inverse of a function