Math 431 - Real Analysis I
Test 2 Review Sheet

Logistics: Our test will occur on Monday, November 5. It will be a 110 minute, no notes, no calculator test. Please bring blank paper on which you will write your solutions.

The successful test-taker will have mastered the following concepts.

Sequences in General Metric Spaces

- Show that a sequence in a metric space does or does not converge using the $\varepsilon - N$ definition of convergence.
- Show that a sequence x_n in a metric space converges to a by showing that $d(x_n, a) \to 0$.

Cauchy Sequences

- Understand that in any metric space, a convergent sequence is Cauchy
- Understand that in $\mathbb{R}$ (and other complete metric spaces), Cauchy sequences converge.
- Show that a sequence is Cauchy using its $\varepsilon - N$ definition.
- Show that a sequence converges by showing it's Cauchy (even though you may not know the limit).

Limits and Continuous Functions

- Prove that for a function $f : \mathbb{R} \to \mathbb{R}$ that $\lim_{x \to a} f(x) = L$ using the$\varepsilon - \delta$ definition of the limit.
- Show that a function is continuous at a point using the $\varepsilon - \delta$ definition of continuity.
- Show that a function is not continuous at a point using the $\varepsilon - \delta$ definition of continuity.
- Know the various properties of continuous functions from $\mathbb{R}$ to $\mathbb{R}$.

Continuity in General Metric Spaces

- Use various properties of pre-images to compute the pre-image of a given set under a given function
- Use the fact that $f : S \to T$ is continuous if and only if for every open $U \subset T$, $f^{-1}(U)$ is open in S.
- Use the similar fact to the above, but with closed sets instead of open sets.

Continuity and Convergence

- Show that a function $f : S \to T$ is not continuous at a by finding a sequence $x_n \to a$ in S but that $f(x_n) \not\to f(a)$ in T.

Continuous Functions on Compact Sets

- Use the fact that continuous images of compact sets are compact.
- Use the fact that if $f : S \to \mathbb{R}$ is a continuous function, then f attains a maximum and a minimum.

Bolzano's Theorem and the Intermediate Value Theorem

- Use Bolzano's Theorem to show that a function $f : \mathbb{R} \to \mathbb{R}$ that changes sign has a root.
- Use the Intermediate Value Theorem to show that a continuous function takes on all intermediate values.