Math 431 - Real Analysis I
Quest 1 Review Sheet

Logistics: Our quest will occur on Friday, September 21. It will be a 50 minute, no notes, no calculator test. Please bring blank paper on which you will write your solutions.

The successful quest-taker will have mastered the following concepts.

The Axiomatic Foundation of the Real Line

- Know the 10 real number axioms: 5 field axioms, 4 order axioms, and 1 completeness axiom
- Use the 10 real number axioms to prove well-known facts about the real numbers and their ordering.

Properties of the Integers

- Definition of an inductive set of real numbers
- Definition of $\mathbb{Z}_+$ as an inductive set.
- Bezout's Identity for the gcd of two integers
- Euclid's Lemma
- The role of primes in divisibility statements (e.g., if $p|ab$, then $p|a$ or $p|b$).
- The unique factorization theorem for integers

Properties of Rational Numbers

- Use proof by contradiction to prove that a number is irrational.
- Use the fact that $\mathbb{Q}$ is a field (and is thus closed under the four operations)

Bounds, Suprema, and Infima

- Know the definition of bounded above, bounded below, bounded, maximum, minimum
- Know the definition of a supremum/infimum; use the Completeness Axiom to prove that they exist.
- Prove that a number is a supremum or infimum for a given set.
- Use the approximation theorem for suprema in proofs
- Use the additive and comparison properties for suprema

Applications of the Completeness Axiom

- Know the proof that $\mathbb{Z}_+$ is an unbounded set and how to use it in proofs.
- Know the Archimedean Property and how to use it in proofs.

Euclidean Space and its Metric

- Know the definition of $\mathbb{R}^n$ as well as the vector space and metric properties of $\mathbb{R}^n$.
- Know how to compute the distance between two points in $\mathbb{R}^n$.

- Know the Caucy-Schwarz inequality and how to apply it in proofs and to obtain new inequalities.
- Know the various metric properties of $\mathbb{R}^n$ (e.g., triangle inequality, positive-definiteness, symmetry).

The Topology of $\mathbb{R}^n$

- Definition of the open n-ball $B(\mathbf{x}; r)$.
- Definition of an interior point of a set; know how to show a point is or is not interior to a set.
- Definition of an open set; know how to show a set is or is not open.
- Basic topological properties of open sets (closed under unions and finite intersections)
- Definition of a closed set; showing a set is closed; topological properties of closed sets.

Adherent Points and Accumulation Points

- Know the definition of an adherent point; show that that a point does or does not adhere to a set.
- Know the definition of an accumulation point; show that a point is or is not an accumulation point of a set.
- Know the subtle but important difference between an accumulation point and an adherent point. (e.g., isolated points)
- Show that a set is or is not discrete.
- Understand the relationship between closed sets, adherent points, and accumulation points.
- Given a set, find its closure and its derived set.