A vector field in $\mathbb{R}^2$ is a function $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ written as

$$F(x, y) = \langle M(x, y), N(x, y) \rangle.$$ 

Figure 1: The vector field is given by $F(x, y) = \langle 2x, 2y \rangle$. Note that this vector field is also equal the gradient $\nabla f(x, y)$ where $f(x, y) = x^2 + y^2$. 
Figure 2: The vector field is given by \( \mathbf{F}(x, y) = (\sin x, \sin y) \). Note that this vector field is also equal the gradient \( \nabla f(x, y) \) where \( f(x, y) = -\cos x - \cos y \).
Figure 3: The vector field is given by $\mathbf{F}(x, y) = (-y, x)$. This vector field is *not* the gradient of any function. How can we tell this?
Figure 4: This is a picture of iron filings under the magnetic force of a dipole magnet. The filings give a sort of vector field that reflects the magnetic field.
We can also define vector fields in $\mathbb{R}^3$. These are functions $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ written as $\mathbf{F}(x, y, z) = \langle M(x, y, z), N(x, y, z), P(x, y, z) \rangle$.

Figure 5: This is the vector field $\mathbf{F}(x, y, z) = \langle -y, x, z \rangle$. For every point $(x, y, z)$, the vector field $\mathbf{F}(x, y, z)$ associated a vector.
How do we integrate vector fields over paths?

Figure 6: We wish to integrate the vector field \( \mathbf{F}(x, y) \) along our path \( C \). This is called a line integral and is notated by

\[
\int_C \mathbf{F}(x, y) \cdot ds
\]