**Math 232 - Calculus IV**  
**Homework due January 14**

**Question 1.** Compute all of the first and second partial derivatives of the following multi-variable functions and verify that mixed partial derivatives are equal.

(a) \( f(x, y) = \ln(2x + 3y) \)

(b) \( f(x, y) = \frac{1}{x + y} \)

(c) \( f(x, y) = (2x - 3y)^2 \)

(d) \( f(x, y) = (xy - 1)^2 \)

**Question 2.** Consider the two 3-vectors \( \mathbf{u} = \langle 2, -1, 4 \rangle \) and \( \mathbf{v} = \langle 0, -1, 5 \rangle \).

(a) \( \mathbf{u} \cdot \mathbf{v} \)

(b) \( \mathbf{u} \times \mathbf{v} \)

**Question 3.** Consider the curve given by \( \mathbf{r}(t) = \langle 2 \cos t, 2 \sin t, \sqrt{5}t \rangle \) for \( 0 \leq t \leq \pi \).

(a) Compute the velocity vector \( \mathbf{r}'(t) \).

(b) Compute the arc length of this curve.

**Question 4.** Compute the following double integrals.

(a) \( \int_1^2 \int_0^4 2xy \, dx \, dy \)

(b) \( \int_0^2 \int_{-1}^1 x - y \, dy \, dx \)

(c) \( \int_0^{\ln 2} \int_{\ln 1}^{\ln 5} e^{2x+y} \, dy \, dx \)
Question 5. Compute the following double integrals by using $dxdy$ and $dydx$. Verify that you obtain the same answer.

(a) $\int \int_R 6y^2 - 2x \, dA$ where $R$ is given by $0 \leq x \leq 1$ and $0 \leq y \leq 2$.

(b) $\int \int_R e^{x-y} \, dA$ where $R$ is given by $0 \leq x \leq \ln 2$ and $0 \leq y \leq \ln 2$. 