Question 1. In class, we learned that we can easily translate from polar \((r, \theta)\) coordinates to Cartesian \((x, y)\) coordinates using the equations

\[ x = r \cos \theta, \quad y = r \sin \theta. \]

In this question, we will investigate how to reverse this process.

(a) If you are given a point in \(\mathbb{R}^2\) in Cartesian form (that is, \((x, y)\) form), how can you solve for \(r\)? In other words, find an equation for \(r\) in terms of \(x\) and \(y\) only.

(b) If you are given a point in \(\mathbb{R}^2\) in Cartesian form, how can you solve for \(\theta\)? In other words, find an equation for \(\theta\) in terms of \(x\) and \(y\) only. Hint: draw a triangle like in class and use it solve for the angle \(\theta\).

(c) Use your above observations to convert the following cartesian points into polar form. For each, draw each point in the \(\mathbb{R}^2\). Use radians for \(\theta\) and round to the nearest hundredth.

\((2, 1), (4, 5), (-3, 2), (-4, 0), (0, 2)\)

Question 2. Graph the sets of points whose polar coordinates satisfy the below equations and inequalities.

(a) \(\theta = \pi/3, \quad -1 \leq r \leq 3\)

(b) \(0 \leq \theta \leq \pi, \quad 0 \leq r < 2\)

(c) \(-\pi/2 \leq \theta \leq \pi/2, \quad 1 \leq r \leq 2\)

(d) \(0 \leq \theta < \pi, \quad r = 2\)

Question 3. Replace the below polar equations by the equivalent Cartesian equations. Use the Cartesian form to graph the equation in \(\mathbb{R}^2\).

(a) \(r \sin \theta + 3r \cos \theta = 1\)

(b) \(r = 4 \csc \theta\)

(c) \(r = 3 \sec \theta\)

(d) \(r = \frac{5}{\sin \theta - 2 \cos \theta}\)

(e) \(r^2 = 4r \sin \theta\)
**Question 4.** Use a polar change of coordinates to compute the following integrals. Don’t forget that for this substitution, you will need the Jacobian $J(r, \theta) = r$. Also, sketch the region of integration $R$ in the $xy$-plane.

(a) $\iint_R 1 \, dA$ where $R$ is the semicircular portion of the unit disk about the origin of radius 1 below the $x$-axis.

(b) $\iint_R x^2 + y^2 \, dA$ where $R$ is the quarter of the unit disk about the origin in the fourth quadrant.

**Question 5.** Consider the integral

$$\int_0^2 \int_0^x y \, dy \, dx.$$  

(a) Sketch the given region in the $xy$-plane.

(b) Compute the integral without using a substitution (that is, using $dy \, dx$).

(c) Use a polar change of coordinates to re-compute the integral (don’t forget to include your Jacobian).