1. Introduction

Strategies used to manage invasive species include prevention, eradication, suppression, containment, and adaptation. A central question in the invasive species discussion is how to execute these strategies optimally. Policymakers generally use a combination of these tools to ward off economically detrimental species. While the literature has explicitly addressed optimal prevention, eradication, suppression, and adaptation, the economics of containment remain uncharted.

This study has three objectives. First, to elucidate the confusing taxonomy related to the management of invasive species. Second, we take one level of this categorization, containment, and develop an economic model for determining how this strategy can be implemented optimally. Finally, we will use the results of the model to suggest improvements to current invasive species policy.

Section 2 proposes and illustrates an invasive species management taxonomy, followed by a review of literature related to this taxonomy in Section 3. Section 4 discusses potential advantages of containment over suppression, and motivates the case with an example from Hawaii. Section 5 defines the biological and economic variables, the model is presented in Section 6, and Section 7 derives and interprets results. Implications for policy, limitations of the model, and directions for future research are offered in Section 8.

2. Invasive Species Management Taxonomy

Management of invasive species usually falls under one of three broad categories: prevention, control, or adaptation. We define and discuss each below.

2.1 Prevention

Prevention includes activities that reduce the likelihood of a pest’s entry. Instruments commonly used for prevention are highly regulatory in nature and include red (prohibited) listing, green (allowed) listing, inspection, quarantine, public education, and risk assessments. Species that are placed on a red list are banned from being imported into a particular region, due to a proven or postulated threat the species poses to an area. Typically red lists are reactionary responses to species that have already established in the region and revealed their destructive nature. Green listing takes the alternative approach – only the species who make it on to this roster are allowed to enter the region, and no others. Green listing establishes a guilty until proven innocent system for species introductions. This more cautious approach is seen far less frequently than its reactionary counterpart.
Port of entry inspection is another common instrument intended to catch invasive species before they enter a region. Agency personnel and dogs are trained to inspect cargo for the presence of potentially harmful species. Import permits, letter of authorization, quarantine and health certificates are other instruments used to prevent unwanted organisms from entering a region. Permit requirements, length of quarantine, and ease of obtaining all of the above will determine the success of these types of tools.

Prevention can also include a mechanism widely known as early detection and rapid response (EDRR). The objective is to catch the invader after it enters the region but before it becomes established. An example of EDRR is the Brown Treesnake rapid response teams in Hawaii. Specially trained personnel have been organized specifically to respond to credible Brown Treesnake sightings. If the teams catch the refugee snake, an invasion may have been prevented.

2.2 Control

Control entails reducing the impact a species has on the environment and economy. This strategy decreases or eliminates the population of an invader. Under the control strategy there are another three instruments that are commonly used. These are eradication (complete elimination of population), suppression (reduction of population), or containment (keeping the invasion within some predetermined area). Manual, chemical, and biological control are the most commonly used mechanisms to control invasive species.

While eradication is often the desired outcome, it is often the least realistic.\(^1\) Complete removal of a species is complicated by strong species resiliency, high search costs, seed bank longevity, and incomplete information regarding the total population. Most successful eradication projects occur when the population is very small or in the very early stages of establishment.

Suppression seeks to draw the population down, without eliminating it entirely, and without concern for location. The suppression strategy removes a proportion of the population in each time period. Often annual or biannual reconnaissance will be conducted, and removal will occur based on ease of access, proximity to valuable “at-risk” resources, or number of individuals.

Finally, the objective of containment is to keep the species within some specified area. While this strategy can be utilized for both plant and animal invasions, we will address plant invasions specifically in this paper. Containment of an animal species requires fencing or some type of natural barrier (e.g., high mountains or water) to keep the population contained. For a plant species, containment can make use of natural boundaries, such as rainfall or elevation limits, as well as harvesting around the perimeter of the invasion.

\(^1\) Although particular success has recently been documented on islands (Krajick 2005)
2.3 Adaptation

Adaptation requires changes in behavior, be it publicly or privately, in order to lessen the impact of an invasive species. This objective of this strategy is to reduce the adverse consequences caused by the species. We further consider adaptation as separable into actions that are passive, where the success of the new species is accommodated, and actions that are active, i.e. intervention that invests directly in reducing the damages of the new species or in the creation of substitutes. Active adaptation includes planting native trees, water catchments, flood control, and others, while passive adaptation involves avoidance activities, such as creating incentives for individuals to seek substitutes for the lost benefits. Passive adaptation also includes the choice of doing nothing.

We illustrate the proposed taxonomy in Figure 1.

Figure 1. Invasive species management taxonomy

3. Literature

The theoretical literature on how to best execute invasive species management is small but growing. Most studies have considered each strategy independently and derived optimal conditions for implementation. Shogren (2000) began the discussion by distinguishing between mitigation (reduction of the chance that a bad event occurs) and adaptation (reducing the consequences when a bad event occurs). The static model’s
optimization reflects the standard result that one should equate the marginal cost of influencing probability (or the severity of the bad event) to marginal wealth acquired. Horan et al. (2002) follow with an analysis of optimal prevention. Under uncertainty, they show that it is optimal to devote more resources to high-damage events that are considered possible even if the probability is low. Few or no resources should be allocated to events that are considered less possible, regardless of the expected damages of those events.

The largest body of theoretical literature focuses on the optimal execution of control. A dynamic model of control is developed in Eiswerth and Johnson (2002). Taking into account the effectiveness of control, they derive the condition that the marginal cost of effective invasive species control should be equal to the marginal benefit of a decrease in stock. Perrings (2002) collapses several types of control measures (exclusion, eradication, containment, mitigation, adaptation) into single index of control, $\beta(t)$, a choice variable which measures “intensity” of control effort. First order conditions are used to derive the “optimal control rate,” $\beta_*$. Olson and Roy (2002) also investigate control, where the growth and spread of the invader is a random process. They present conditions under which eradication is optimal. Stochastic dynamic programming and fuzzy logic are employed by Eiswerth and van Kooten (2002) to find the optimal set of control strategies, which range from doing nothing to controls such as chemicals and revegetation.

Adaptation is the strategy of interest in Barbier and Shogren (2004). They consider the implications when households self protect by allocating income to reduce the potential damages from a biological invasion.

Few models have allowed for the option of multiple management strategies. Pitafi and Roumasset (2005) find that optimal management of an invader over time may require a cycle of both prevention and control strategies. Burnett et al. (2005) model the choice of prevention and control as endogenous, and illustrate how various parameters in the model affect the optimal combination of strategies. For an established invader, control alone is found to be optimal. For a potential invader, the optimal program will sometimes involve an integration of prevention and control. As relayed in Perrings (2005), whenever it is possible to explicitly include probability of establishment in the model, optimal policy will consider both strategies.

As detailed in Section 2.2, control of an invasive species can include at least three distinct strategies: eradication, suppression, and containment. Suppression is the control mechanism assumed by most of the literature above. To date, the literature has neglected the possibility of containment. In order to accurately consider this strategy, it is necessary to introduce space explicitly into the model. We do this in a simple model below.

4. Containment of an Invasive Plant

4.1 Miconia calvescens

---

2 Although Olson and Roy (2002) consider eradication as a special case
*Miconia calvescens*, a striking purple and green tree native to South America, was intentionally introduced as an ornamental plant to Hawaii in the 1960’s. The tree now threatens the economy with damages in the form of reduced quantity of groundwater, impaired quality of surface water, and biodiversity losses. The invasive tree has large populations on the islands of Hawaii and Maui, and smaller populations on the islands of Kauai and Oahu.

Current management strategies for Miconia consist of a combination of the approaches discussed in Section 2. The major preventative measure is Miconia’s inclusion on *Hawaii’s Noxious Weed* list³, thereby banning the sale and/or distribution of the plant in the state of Hawaii. Other preventative mechanisms include public outreach campaigns⁴ on the possible implications of a large population of Miconia, in order to solicit help in locating the plants.

Major suppression strategies for Miconia have been in place since the early 1990’s, after the plant was identified as a contributor to the demise of Tahiti’s native forests and biodiversity. Suppression is the main strategy currently utilized on all four invaded islands. Reconnaissance surveys are routinely performed by helicopter and on foot, and County Invasive Species Committees formulate annual plans for removal. Managers clearly need guidance in setting management strategies. This is articulated in a 1997 report by Lloyd Loope, who suggests “in formulation of strategies for combating Miconia in Hawaii, it has been difficult to settle on the term for the goal to be achieved – whether it is eradication or containment (p.2).”

### 4.2 Containment vs. Suppression

So far in the literature, control has been characterized only by the population suppression mechanism, without regard for space. Especially for species like Miconia, where resource managers are concerned with “core” populations versus offspring “satellite” populations, explicit modeling of space is crucial to understand the tradeoffs involved in different size invasions.

There may be both biological and economic advantages to considering the containment strategy over suppression. Biologically, there will be fewer disturbances inside the center or core of the invasion, since containment only removes around, not within the invaded area. When soil is disturbed, there may be increased potential for plants to begin growth. Containment also limits the chance of seed transfer (by field crews or equipment) to uninvaded areas, since containment does not work directly in the center of the population, only around the perimeter.

Containment also allows us to make economic sense of cost and damage functions. It is tricky to relate removal cost to population, as some removals will be located in more inaccessible spots. If the initial population can be isolated, high search costs may be

---


⁴ [http://www.hear.org/miconiainhawaii/MiconiaTreatItYourselfPoster.pdf](http://www.hear.org/miconiainhawaii/MiconiaTreatItYourselfPoster.pdf), [http://www.hear.org/miconiainhawaii/micpostr.htm](http://www.hear.org/miconiainhawaii/micpostr.htm)
avoided. One of the most costly ventures in Miconia control is that of reconnaissance, usually done from above in helicopters.

Without considering space, it is difficult to determine how damage is related to population. For Miconia, a single tree will not cause a great deal (if any) damage by itself, and even thousands of trees, as long as they are spread out, may do little damage. What biologists are more certain about is that a very dense stand of trees will result in considerable damage, namely in the form of reduced groundwater recharge and surface water quality, and a loss of biodiversity.

The inherent tradeoff in the model is high damage costs versus slowing growth as the invasion expands. In the steady state, containment will require a specified level of harvest around the perimeter every period. As the initial population is in a phase a rapid growth, there may be some benefit from waiting until the growth subsides, although this benefit will need to be measured against the increasing damage accruing as the perimeter increases. The following attempts to address this tradeoff with optimal policy responses.

5. Variable Definition

In this section we define the variables in the containment model, and illustrate the spatial dimensions in Figure 2 below. Because we are most interested in the tradeoff between increasing damage and decreasing growth, we make several simplifying assumptions regarding the invaded area. First, we model the invasion as a perfect circle. The harvesting that takes place each period will equal the circumference of the circle. Secondly, population density is constant throughout the interior, so we consider only the size of the invasion, rather than the population of trees. Finally, we assume that the population grows symmetrically from the center of the circle. The size of the radius therefore follows the size of the total population.

Figure 2. Spatial dimensions of the invasion

---

5 This discussion is referring to the standing stock of trees. Obviously the reproductive capacity of a single tree is an issue.
5.1 Biological variables

\[ F_{\text{max}} \]  
Maximum circumference of invasion, created by natural boundaries (e.g., rainfall, elevation, shoreline of island)

\[ F_t \]  
Circumference of invasion at time \( t \)

\[ r_t \]  
Radius of invasion at time \( t \)

\[ r_{\text{max}} \]  
Maximum radius

\[ x_t \]  
Segment of the radius removed at time \( t \). The total amount harvested will be equal to \( 2\pi r \).

\[ n_t \]  
Population of the invasive species. Because we assume constant density throughout the invasion, we can define population as the invaded area, or \( \pi r^2 \).

\[ g(n_t) \]  
Biological growth function of population. The population expands in the following way. In the center of the invasion, growth is vigorous because the species is in its prime habitat, since this is where the invasion began. The rate of growth increases as the invasion expands, up to some maximum level, at which the invasive runs into less favorable habitat (e.g., another invasive species, less rainfall, steep terrain). At this point, the growth rate begins to decline, until finally it reaches zero at the maximum perimeter, \( F_{\text{max}} \). The logistic growth model, for example, follows this pattern. Because \( n_t = \pi r^2 \), the growth function can also be written \( g(r_t) \). Assuming logistic growth of the population\(^6\), we compare the shape of the two functions below.

---

\(^6\) The assumed logistic functional form for population is \( g(n) = 0.3n \left(1 - \frac{n}{1000}\right) \), therefore the growth of the radius is represented by \( g(r) = 0.3\pi r^2 \left(1 - \frac{\pi r^2}{1000}\right) \).
5.2 Economic variables

$c(r_t)$ Unit cost function, assumed to be a decreasing function of the radius, $c'(r_t) < 0$. The initial invasion is an area that is very expensive to contain. We assume our initial $r_0$ is such that eradication of the entire radius is not economically feasible (this is the case on Maui and Hawaii). However, as the invasion expands, it becomes easier to contain it, as the invader runs into unfavorable habitat, rainfall differences, natural barriers, other invasive species, etc.

$D(r_t)$ Damage function, everywhere $D'(r_t) > 0$, and $D''(r_t) < 0$. Because we assume constant density, more invaded space results in more damage.
Note the second derivative – the increase in damage decreases as the radius grows. These specifications of the damage function are possible with the containment model, and are less realistic if we were considering suppression rather than containment.

6. The Model

Dynamic optimal control is used to minimize the sum of control costs and damages at each point in time through containment.

The social planner wishes to minimize the sum of containment costs plus damages over time:

\[
\begin{align*}
\text{Minimize } W &= \int_0^\infty e^{-\eta t} [c(r_t)x_t + D(r_t)] \, dt \\
\text{Or, } \quad \text{Max}\{ -W \} \\
\text{subject to } \dot{r} &= g(r_t) - x_t, \quad x \geq 0, \quad r_0 \text{ given}
\end{align*}
\]

Current value Hamiltonian:

\[
H = -c(r_t)x_t - D(r_t) + \lambda_t [g(r_t) - x_t]
\]

Necessary conditions:

\[
\begin{align*}
\dot{r}_t &= \frac{\partial H}{\partial \lambda_t} = g(r_t) - x_t, \quad (1) \\
\dot{\lambda}_t &= r_\lambda_t - \frac{\partial H}{\partial r_t} = r_\lambda_t - [-c'(r_t)x_t - D'(r_t) + \lambda_t g'(r_t)] \\
&= r_\lambda_t + c'(r_t)x_t + D'(r_t) - \lambda_t g'(r_t) \quad (2) \\
\frac{\partial H}{\partial x_t} &= -c(r_t) - \lambda_t \leq 0 \quad (3)
\end{align*}
\]

From this problem, we can derive an optimal containment policy path for the case where it is uneconomic to eradicate the initial population.

Rearranging (2),

\[
\dot{\lambda}_t - D'(r_t) - c'(r_t)x_t = \lambda_t [r - g'(r_t)] \quad (4)
\]

The \textit{marginal net benefits} of containment are composed of the reduction in damages from the containment, adjusted by the marginal discounted shadow price of resource

\[
\text{The } \textit{marginal cost} \text{ component includes the discounted shadow price of resource containment, adjusted by the marginal}
\]
a more contained population, growth in radius
less the marginal cost of that containment

From (3),

\[ \dot{t} = -c(r_t) \]
\[ \dot{t} = -c'(r_t) \dot{r} \]  \hspace{1cm} (5)

Equation 5 describes the shadow price of invasive species containment. The shadow price should be equal to the (negative) cost of containment at time \( t \). This is the cost savings from containment. Furthermore, the change in the shadow price should be equal to the (negative) marginal cost times the change in radius over time.

Plugging these conditions into (4),

\[ -c'(r_t)(\dot{r}_t) - D'(r_t) - c'(r_t)x_t = -c(r_t)[r - g'(r_t)] \]

Rearranging,

\[ -c'(r_t)(\dot{r}_t + x_t) - D'(r_t) = -c(r_t)[r - g'(r_t)] \]  \hspace{1cm} (6)

From our equation of motion \( \dot{r} = g(r_t) - x_t \),

\[ g(r_t) = \dot{r} + x_t \]  \hspace{1cm} (7)

Plugging (7) into (6) and rearranging,

\[ c(r_t) = \frac{c'(r_t)g(r_t) + D'(r_t)}{[r - g'(r_t)]} \]  \hspace{1cm} (8)

The left hand side of Equation (8) describes the present value cost of containment. This should be equal to the marginal benefits of containing a unit of invasives from spreading. This marginal benefit component consists of future damages avoided by containing the marginal unit and the saved marginal cost of additional growth of this unit. Because we are describing the present value cost of containing the marginal unit perpetually, the term on the right hand side should be divided by the own rate of interest, which is the interest rate minus the marginal growth.

Equation (8) describes the interior steady state of the system. To see this, observe that when \( \dot{r} = 0 \), \( g(r_t) = x_t \). Replacing these conditions in Equation (6) will result in Equation (8). An interesting characteristic about Equation (8), which describes the optimal path in any given period, is that it does not contain \( \dot{r} \). Therefore, the radius will not be changing.
along the optimal path. The implication is that the steady state should be reached immediately. This will be the case when \( x > 0 \).

Therefore, the position of \( r_0 \) in relation to \( r_{ss} \) will dictate the optimal path of \( x \). If \( r_0 = r_{ss} \), then it is optimal to do nothing and remain at that level of containment, harvesting only the growth associated with \( r_{ss} \) every period, or \( x_{ss} \). If \( r_0 > r_{ss} \), the optimal program requires the bang-bang solution of immediate harvest \( x \) to reach \( r_{ss} \). If \( r_0 < r_{ss} \), the optimal program is also bang-bang and entails first doing zero harvesting, \( x = 0 \), letting the circle grow at its natural growth rate until \( r_0 = r_{ss} \), at which point harvest will equal the steady state level, \( x_{ss} \). This is the most direct approach to the steady state.\(^7\)

**Figure 4.** Bang-bang containment

Figure 4 illustrates optimal harvesting policy given the size of the initial radius. If the invasion happens to begin characterized by the steady state radius, harvest will be constant over time, equal to the growth of the radius at steady state, \( x_{ss} \). If the invasion is larger than optimal, the entire difference between the current size circle and the optimal circle will be immediately harvested, followed by \( x_{ss} \) forever after. If the invasion is smaller than optimal, no harvesting should occur until the steady state is reached, after which \( x_{ss} \) will be harvested each period.

The intuition driving the result to reach steady state immediately is that there is no benefit from delaying arrival. In ordinary renewable resource problems, the species to be harvested generally has economic value. For example, in fishery problems the stock is not depleted to the steady state immediately, as this would result in movement down the demand curve and a smaller marginal benefit. Because in this case there is no benefit derived from harvesting the trees, the optimal program requires immediate arrival to the steady state.

\(^7\) See Hanley et al. (1997), pp. 288-289 for a similar result. In their case, the fishery’s objective function is to maximize discounted profits given a fixed price. Because there is no benefit of increasing price from harvesting the stock more slowly, they also find the result that the steady state should be reached as rapidly as possible. If the initial stock is below equilibrium the producer harvests nothing until the equilibrium is reached, as this is the most rapid approach to the steady state.
7. Results

7.1 Three Potential Locations of the Steady State

Above we show that the steady state should be reached by the fastest possible route. Below we show that this steady state can be located at several places along the growth function, depending on the magnitudes of various parameters in the model.

We rearrange Equation 8 again to obtain:

\[ g'(r_t) = r - \frac{c'(r_t)g(r_t) + D'(r_t)}{c(r_t)} \]  \hspace{1cm} (9)

Equation (9) illuminates the possible locations of the steady state. This will depend on the magnitude of the second term on the right hand side. All components are unambiguously positive except the marginal cost, which is negative. To determine the possible positions of the steady state, we compare the magnitude of the second term on the right hand side to the discount rate.

There are a total of 3 possibilities for the location of the steady state, depending on the magnitude of \( \frac{c'(r_t)g(r_t) + D'(r_t)}{c(r_t)} \) in relation to \( r \). Figure 5 depicts the position of these three possibilities, \( r_{s1} \), \( r_{s2} \), and \( r_{s3} \).

**Figure 5.** Potential location of the steady state

The first case to consider is when \( \frac{c'(r_t)g(r_t) + D'(r_t)}{c(r_t)} = r \). Then, \( g'(r_t) = 0 \) and the steady state will occur at \( r_{s1} \), or at the maximum growth rate of the species. In this case, decreasing the total area will be more costly than the damages avoided, and increasing the area would lead to higher damages than cost savings.
The second case is when \( \frac{c'(r_i)g(r_i) + D'(r_i)}{c(r_i)} < r \). Then, \( 0 < g'(r_i) < r \) and the steady state will occur to the left of \( r_{as1} \). Because \( g'(r_i) \) is above zero and positive, the optimal radius size will be somewhere near \( r_{as2} \), in the range of increasing growth. Under this scenario, damages are high relative to containment costs.

Finally there is the case where \( \frac{c'(r_i)g(r_i) + D'(r_i)}{c(r_i)} > r \). Then, \( g'(r_i) < 0 \) and the steady state occurs beyond the maximum growth rate, in the neighborhood of \( r_{as3} \). This case requires the largest invaded area under the optimal program. Damages are low relative to containment costs.

### 7.2 Comparative Statics

Because Equation (9) is a closed form solution, we can directly discuss comparative statics using this expression.

If the cost of containment \( c(r_i) \) increases, then the invasion should be further contained so the total area is smaller. If the cost of containment decreases, the invasion area should be allowed to grow larger.

Because damages are strictly concave, an increase in marginal damage means there are less total damages. Therefore, an increase in marginal damage requires an increase in the contained area. Likewise, a decrease in marginal damage implies higher total damage, and it is optimal to reduce the size of the invasion.

Finally, if the discount rate \( r \) increases, optimal policy will call for a reduction of the invasion size, while a decrease in \( r \) will increase the optimal size of the contained area.

### 8. Implications, Limitations, and Directions

This exercise illustrates important tradeoffs to consider when determining strategies for invasive species containment. The analysis shows that the optimal size of an invasion depends on the cost of containment the damages associated with its size, and the discount rate employed. Below we describe how policy can be improved by consideration of some important economic consequences.

First, it may be beneficial to do no control for some time period while the invasion grows to its optimal size. This is due to the fact that when an invasion is beginning, growth is vigorous. Although damages increase with area contained, a larger area may require less harvesting in the steady state. Current policy generally requires immediate reduction of a new population, disregarding any potential benefit from waiting.

Comparative statics reveal that if costs of containment increase, a smaller area should be contained. If costs go down, a larger area should be contained. Furthermore, if damages
increase, a smaller area should be contained, and if damages decrease, a larger area should be contained. These results are both intuitive and instructive.

The purpose of this model was to take a first look at the economics of containment and to explicitly include space in a model of optimal invasive species control. The simplicity of the model required many assumptions which limit the generality of the model. The assumption of constant density is somewhat heroic but may not make a significant difference in the results. The model assumes that the invasion is limited to one area, neglecting high search costs to find other potential invasions. It also assumes you will know the size of the radius associated with the highest growth rate. Although this can be modeled biologically, whether or not scientists currently have an adequate understanding of this type of population biology is not clear.

The utility of this research would be greatly enhanced by simulating the model, perhaps using biological and economic parameters from the Miconia populations on either Hawaii or Maui. It would also be useful to integrate the economics of satellite populations into this analysis. Especially for the case of Miconia, consideration of the core’s offspring populations will be particularly important in identifying optimal containment programs.

References


Pitafi, B and J Roumasset (2005) The Resource Economics of Invasive Species, manuscript.